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## FORMAL ANALYSIS OF USE CASE DIAGRAMS

*Use case diagrams play an important role in modeling with UML. Careful modeling is crucial in obtaining a correct and efficient system architecture. The paper refers to the formal analysis of the use case diagrams. A formal model of use cases is proposed and its construction for typical relationships between use cases is described. Two methods of formal analysis and verification are presented. The first one based on a states' exploration represents a model checking approach. The second one refers to the symbolic reasoning using formal methods of temporal logic. Simple but representative example of the use case scenario verification is discussed.*

**Keywords:** UML, use case, formal model, verification, model checking, temporal logic, semantic tableau

## FORMALNA ANALIZA DIAGRAMÓW PRZYPADKÓW UŻYCIA

*Diagramy przypadków użycia odgrywają znaczącą rolę w modelowaniu systemów z wykorzystaniem UML. Staranne i dokładne modelowanie ma zasadnicze znaczenie w postępowaniu umożliwiającym uzyskanie poprawnej i efektywnej architektury systemu. Artykuł odnosi się do formalnej analizy diagramów przypadków użycia. Został zaproponowany model formalny przypadku użycia, a także opisano odpowiednie konstrukcje dla relacji występujących pomiędzy przypadkami użycia. Zostały przedstawione dwie formalne metody ich analizy i weryfikacji. Pierwsza oparta jest na eksploracji stanów i reprezentuje podejście nazwane weryfikacją modelową. Druga odwołuje się do wnioskowania symbolicznego z wykorzystaniem logiki temporalnej. Został pokazany prosty i reprezentatywny przykład weryfikacji pewnego scenariusza przypadku użycia.*

**Słowa kluczowe:** UML, przypadek użycia, model formalny, weryfikacja, weryfikacja modelowa, logika temporalna, metoda tablic semantycznych

### 1. Introduction

Use cases are an important part of UML being a coherent story about system's behavior. They are used for documenting system requirements. They may also be used for communication both between various participants in a software project, i.e. system developers, its future users and owners. Test cases derived from use cases are

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usually the criteria of a system acceptance. Use cases appear to be relatively easy-to-perception, even for people not familiar with the information technology. Use cases enable understanding the system though they do not fall too much in the implementation details. On the other hand, errors committed in the course of the preparation of the use cases' documentation may have far-reaching consequences, c.f. [12]. This may be manifested not only in the design phase of a well-balanced architecture of the system, but even in the implementation phase. Thus, the possibility of formal analysis of the diagrams can be an effective counterweight to the presented above threats. In addition, inference based on a formal approach allows the verification of the desired property of the modeled system. As the primary artifact of requirements documentation use cases appear in two complementary forms:

1. as graphical diagrams enumerating use cases and actors (usually users and external systems) and showing relations between them;
2. as sets of scenarios describing from the actor's perspective flows of events in the system.

Whereas the use case diagrams has strictly formalized syntax, several techniques could be used to document scenarios describing the system behavior. They include, for example, informal natural language, restricted natural language, Activity Diagrams or Message Sequence Charts.

The purpose of this paper is to present the possibility of formal analysis and verification of use cases of UML. This will be implemented in such a way that first a formal model of a use case is proposed. It includes a number of important elements and it's creation is necessary because it can help to find a reliable basis for further discussion and formal verification. Construction methods for typical relationships between use cases are presented, too. It seems obvious that the finite state machines is suitable as a semantic model for use cases, but there is a gap between natural language and finite state machines and the intention of this work is to provide a step in this direction.

The second purpose of this paper is to show the possibility of formal analysis of use cases. Use cases can also describe the system requirements and, therefore, formal verification of these requirements at the initial phase of the system modeling reduces production costs throughout the whole software development cycle. There are two basic methods that can be used for a formal use cases' verification. The first one is based on a states' exploration, i.e. the set of reachable states is determined and analyzed. This method represents a model checking approach. The second one refers to the symbolic reasoning using formal logic and the expressive language of temporal logic and the method of semantic tableau is used to prove system's properties. It has been shown a common example to illustrate these two approaches. Model checking was discussed very thoroughly in this paper. Logical approach has been outlined here and in more detail shown in the paper [13].

The use case diagrams have already been under consideration in the context of their formal analysis and verification. In work [1], an approach in which the emphasis

on description of behavior between actors and use cases through the so-called contracts is presented. In work [19] a kind of a review technique for the use case diagrams is proposed. Work [16] contains the proposed division of scenarios, but it has no formal model and consequently there is a lack of reliable mechanism for formal reasoning about the modeled system. In work [4] consistency between use case scenarios and sequence diagrams is checked. Work [11] provides a very thorough and detailed survey of selected issues concerning use cases. The dualism of representations and informal character of scenarios documentation implies several difficulties in reasoning about the system behavior and validating the consistency between diagrams and scenarios description. Several research works in this field attempted to translate scenarios or formulate them in more formal language. Work [3] includes the transition of use cases to finite state machines, however the formal model presented in our work is more detailed and also shows the specific context of the formal verification. Work [20] shows the formal analysis of use cases, however, it has been used the Petri Nets formalism. The details of the models differed depending on the assumed purpose. They include relations with external objects from the class hierarchy [14], automatic generation of test cases [2], validation use cases against the regulatory constrains [18], tracking changes in use case based on FSM equivalence [3]. It should be mentioned, that in use case modeling methods has no systematic approach that would fully cover relations between use cases and reflect their presence in the models.

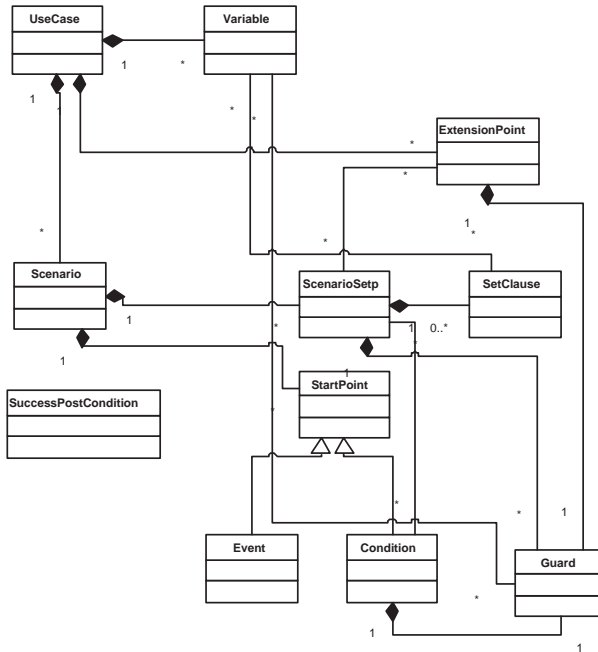
The rest of the paper has been organized as follows. First, the metamodel of the use case is be presented. The next step is to propose a formal model. Another point of work focuses on relationships occurring on the use case diagrams. The analyzed example is contained in a separate section. It includes a detailed and formal analysis using two different approaches mentioned above.

## 2. Metamodel

The use case diagram ([10, 7]) consists of actors and use cases. Actors are environment of the modeled system and use cases form this system. *Actors* are objects which interacts with the target system. *Use cases* are services and functionalities which are used by actors. Use cases should meet actors' requirements. The use case diagram both identify external objects' expectations for the system and specific features of the system. However, the diagram does not refer to the details of the system implementation.

Use cases can be linked by relationships. There are three basic relations between use cases: generalization and two standard stereotypes, i.e. «include» and «extend». Relations between uses cases are an important tool for managing complexity of scenarios. If several consecutive steps are repeated in a number of use cases or they can be treated as logically closed functionality (as logging to a system) they may be extracted to form a stand alone use case that may be called from a basic use case like a function. This situation corresponds to include relation. A use case linked with extend relation represents variant or exceptional behavior to the scenario of a basic

use case. Its occurrence depends on the satisfaction of certain conditions (extension points). Extending use cases can be treated as a set of alternative scenarios that was important enough to be transformed into stand alone use cases visible on the diagram and separately documented. The included use case must augment the execution of basic use case while the extended use case might be used to augment the execution.



**Fig. 1.** An object oriented metamodel of the use case model

The use case diagram is a very descriptive technique to compare with the other UML diagrams and it needs more details to describe its behavior. To achieve this it is necessary to create a narrative for each case of use case. There is no standard for content of use case and natural language for this kind of description is accepted. However, in order to achieve adequate accuracy and precise description it is assumed that documentation should contains some important elements. Besides the obvious element such as the name of a use case, its scenario should contains some assumption and (pre-)condition for a beginning, precise dialog expressed in some steps for use case body, and (post-)conditions for its termination. Both assumption and precondition describe the desired state of the system which must be satisfied before the use case is executed. The main difference is that assumption is not tested and it is assumed that it is true while the precondition is tested and the execution of use case depends on the result of this test, i.e. use case may be never executed. Use case scenario can also define the internal variables. These internal variables can then be the attributes of the respective classes implementing the system. Steps can contain assign instruc-

tions which assigning new values to these variables. The use case scenario can contain initiation triggers. They define situations when the use case must start, i.e. they identify situations when external events cause an use case invocation. Sometimes, there are many ways to end use case and the termination and postcondition describe these situations. Termination is a list of success/failure and final options. Postcondition is a kind of formal commitment which is satisfied when use case ends. Figure 1 shows a sample object-oriented metamodel of the use case model.

### 3. Formal model

To perform formal validation of use cases we transform its specification to a transition system. Each step in the use case scenario is modeled as a transition between states. Transitions can be labeled by guards and assignments to variables, whereas states can be labeled with Boolean functions representing conditions: extension points, pre- and postconditions.

To avoid defining syntax of expressions appearing in a use case specification, we convert them to a set of Boolean variables and assignments of values to variables. Each Boolean variable corresponds to a test expression appearing in guards or extension point conditions of use case specification, e.g.:

$$\begin{aligned}
 v_1 &\equiv \textit{payment} = \textit{cash} \\
 v_2 &\equiv \textit{payment} = \textit{creditcard} \\
 v_3 &\equiv \neg \textit{queue.empty}() \\
 v_4 &\equiv \textit{sessionexpired} \\
 v_5 &\equiv \textit{debit} > 0
 \end{aligned}$$

Let  $V = \{v_1, v_2, \dots, v_n\}$ , where  $n > 0$ , be an enumerable set of Boolean variables. Function  $Val: V \rightarrow \{\textit{true}, \textit{false}\}$  is a valuation function. We will denote by  $Val(i)$  (or simply  $v_i$  where this does not imply ambiguities) a value of  $i$ -th variable.  $Pred(V)$  is a set of predicates defined over a set of variables  $V$ .

Let  $C$  be a set of logical constraints describing dependencies between variables. Any valid valuation function must satisfy constraints  $Val \models C$ . Constraints should allow undefined values.

Let us consider the following example. For variables  $v_1$  and  $v_2$  corresponding to test expressions  $\textit{payment} = \textit{cash}$  and  $\textit{payment} = \textit{creditcard}$  constraints should be defined as:

$$C = (\neg v_1 \wedge \neg v_2) \vee (v_1 \wedge \neg v_2) \vee (\neg v_1 \wedge v_2)$$

The above formula specifies that  $\textit{payment}$  should be undefined (neither by cash, nor by card) or cash only or card only.

A modification (assignment) function  $f: Val \rightarrow Val$  changes the valuation of logical variables. It is assumed, that for any  $Val_2 = f(Val_1)$  if  $Val_1 \models C$  then  $Val_2 \models C$ . A set of modification functions is denoted by  $F$ .

**Definition 1.** Use case model UCM is a tuple  $UCM = \langle S, T, V, Assign, G, s_0, Cond, Val_0 \rangle$ , where

$S$  – is a set of states,

$T$  – where  $T \subseteq S \times S$  is a transition function,

$V$  – is a set of Boolean and atomic variables,

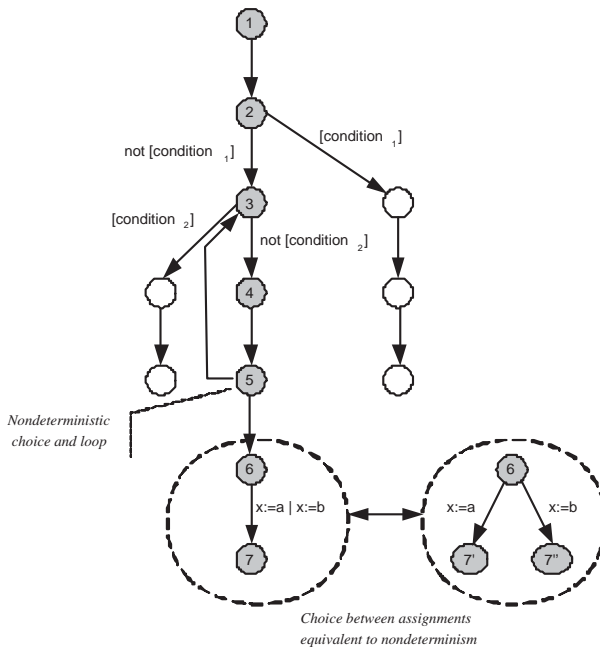
$Assign$  – is a partial function:  $T \rightarrow 2^F$ , i.e. any transition can be linked with a set of assignments,

$G$  – is a partial function  $T \rightarrow Pred(V)$ ,

$s_0$  – is the initial state,

$Cond$  – is a partial function  $S \rightarrow Pred(V)$ , i.e. this function represents post- and preconditions that can be assigned to states,

$Val_0$  – is the initial valuation of variables, satisfying  $cardPred(V) = Cond(s_0)$  and  $Pred(Val_0) = true$ .



**Fig. 2.** Use case model

Figure 2 shows typical patterns that can appear in UCM:

- deterministic choice guarded by expressions at states 2 and 3,
- loops (states 3-4-5) with accompanying nondeterministic choice – an actor performs a step 5-6 of main scenario or optionally a step 5-3,

- nondeterministic assignment – transition 6-7 is labeled with two assignments  $x := a$  and  $x := b$  what corresponds to two transitions 6-7' labeled with  $x := a$  and 6-7'' labeled with  $x := b$ .

## 4. Modeling relations in use case specifications

Use case specification usually takes the form of a scenario expressed in natural language. In the presence of includes, extends or inheritance relations the scenario should be consistent with constraints imposed by those relations. The include relation implies calling the included use case in a step of the basic use case. The extends relation corresponds to linking scenarios of extending use case at a certain extension point. We treat the inheritance as an obligation that a specialized version of the some general use case preserves its set of pre- and postconditions.

### 4.1. Include

If use case A includes use case B it implies an obligation. In a step of main or alternative scenario of A the use case B should called. This corresponds to replacing transition in a model of A by the model of B (Figure 3) and linking initial and final states. If the included use case defines variables, they are inserted to the set of variables of the model of main use case. This allows checking failures in included use case after calling it by appropriate guards in the scenarios of main use case.

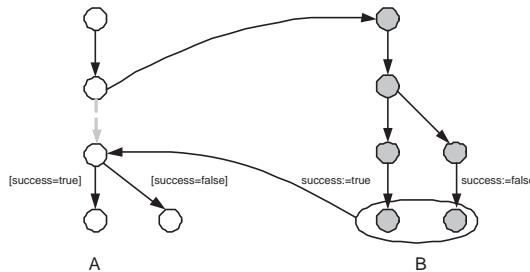


Fig. 3. Modeling include relation

### 4.2. Extends

Extends relation between use case A and B implies, that at a certain extension point the main scenario of A forks and possibly joins at a state indicated in B specification. As the result models of both use cases are merged (states, transitions and variables). Extending use cases are treated similarly to alternative scenarios (Figure 4). They specify alternative branches in the model.

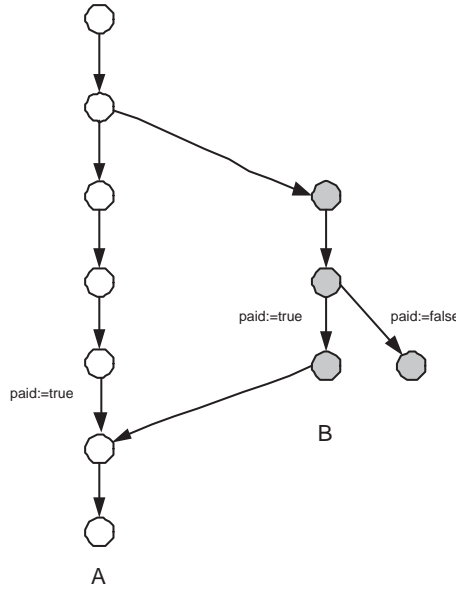


Fig. 4. Modeling extends relation

### 4.3. Inheritance

The inheritance relation between two use cases: the base and the derived implies that the derived use case delivers the same results as the base use case while manifesting different behavior. The typical example is shown in Figure 5. An abstract use case Identify User has two specializations: Identify User By Retina Scan and Identify User By Fingerprint. Those methods of identifications base on different principles and have different steps, however both result the same result: the status of user identification.

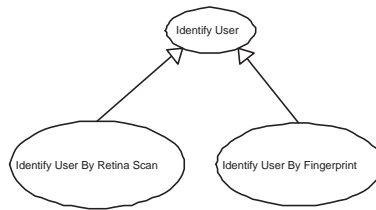


Fig. 5. Inheritance between use cases

Figure 6 shows a model of the use case Identify User. For a given precondition (true) it leads to two states annotated with different postconditions *identified = true* or *identified = false*. If an external use case includes it, both models are merged by appropriately linking initial and final states and merging sets of variables (see



Figure 3). Calling use case may then test postconditions referring to variables defined in scope of Identify User.

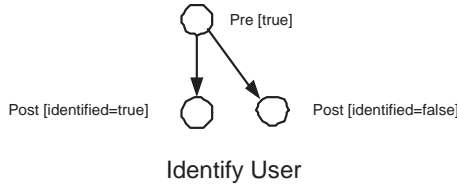


Fig. 6. Model of generalized use case

It is assumed that specialized use cases can be used in any context, where the base use case can be called (included). In Figure 7 a model of specialized use case Identify User by Retina Scan is presented. Although more expanded, it preserves the pre and postconditions. In fact, we may expect that the derived use case has identical preconditions and its postconditions form a subset of postconditions in base use case (derived use case may narrow but not widen the set of postconditions). Moreover, for any trace in the model UCM of derived use case transforming a precondition into a postcondition, analogous trace should exist in the model of base use case.

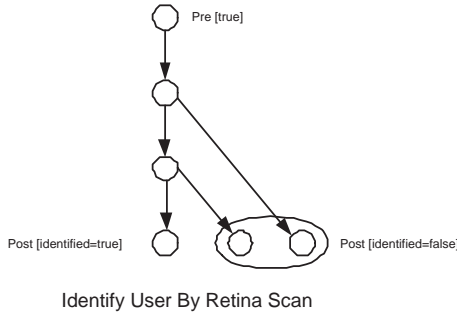


Fig. 7. Model of specialized use case

## 5. Verification of UCM

### 5.1. Correctness of use case model

Let us consider a finite trace representing behavior of UCM. The trace  $s_i$  a sequence of states and valuation of variables starting with the initial values  $s_0$  and  $Val_0$ .  $\sigma = (s_0, Val_0), (s_1, Val_1), \dots, (s_n, Val_n)$ .

For each subsequent pairs  $(s_{i-1}, Val_{i-1})$  and  $(s_i, Val_i)$  the following conditions holds:

1.  $t_i = (s_{i-1}, s_i) \in T$

2.  $G(t_i)(Val_{i-1}) = true$
3. if  $Val_{i-1} \neq Val_i$  then  $f \in Assign(t): f(Val_{i-1}) = Val_i$
4.  $\forall s \in S \setminus \{s_n\} : (s_n, s) \notin T$  or  $\exists s \in S \setminus \{s_n\} : (s_n, s) \in T \wedge G(s_n, s)(Val_n) = false$

**Definition 2.** We define a set of final states of UCM as  $FS = \{s \in S : \neg \exists s_{next}.(s, s_{next}) \in T\}$ . The trace  $\sigma$  is correct iff:

1.  $s_n \in FS$
2.  $Cond(Val_n) = true$

**Definition 3.** Let  $L(UCM)$  be a set of all traces of UCM satisfying conditions (1-4). The UCM model is correct iff:

1.  $\forall \sigma \in L(UCM)$   $\sigma$  is correct
2.  $\forall t = (s_1, s_2) \in T \exists \sigma \in L(UCM),$   
 $\sigma = (s_0, Val_0), \dots, (s_{i-1}, Val_{i-1}), (s_i, Val_i), \dots, (s_n, Val_n) : s_1 = s_{i-1}, s_2 = s_i$

## 5.2. Model verification

Verification of UCM model consists in systematic exploration of its state space and constructing an execution graph  $\Gamma = (V, E)$  modeling its behavior. Each vertex  $v_i \in V$  is labeled with the pair  $(s_i, Val_i)$ . An edge  $e \in E$  in the graph is labeled by a transition  $t \in T$  in UCM and an assignment function  $f \in Assign(t)$ .

Two vertices  $v_1 = (s_1, Val_1)$  and  $v_2 = (s_2, Val_2)$  are linked by an edge  $e_{12} = (t_{12}, f_{12})$  if the following conditions are satisfied:

1.  $t_{12} = (s_1, s_2) \in T$
2.  $G(t_{12})(Val_1) = true$
3.  $f_{12} \in Assign(t_{12})$
4.  $Val_2 = f_{12}(Val_1)$

As it can be observed, the graph  $G$  represents in fact a Kripke structure [6] or it can be easily transformed to it by removing labeling of edges and adding extra self-loops at vertices having no successors. The process of building graph  $G$  follows the method of building occurrence graph for Petri nets described in [17]. It starts with an initial graph  $\Gamma_0 = (V_0, E_0)$ , where  $V_0 = (s_0, Val_0)$ ,  $E_0 = \emptyset$ . At each step a new graph  $\Gamma_i$  is constructed from a graph  $\Gamma_{i-1}$  by adding an edge  $e_i$  and a vertex  $v_i$  to the graph  $\Gamma_{i-1}$ . This is done by selecting a starting vertex  $v_s = (s_s, Val_s)$ , a candidate transition  $t_i \in \{(s_1, s_2) \in T : s_1 = s_s\}$ , checking its guard and selecting an assignment function  $f_i \in Assign(t_i)$ . If the edge  $e_i = (t_i, f_i)$  is not present in  $E_{i-1}$ , graph  $\Gamma_i$  is constructed by calculating  $E_i = E_{i-1} \cup \{e_i\}$  and  $V_i = V_{i-1} \cup \{(s_i, Val_i)\}$ , where  $t_i = (s_s, s_i)$  and  $Val_i = f_i(Val_s)$ . The process terminates if all edges labeled with pairs  $(t_i, f_i)$  are present in the graph  $\Gamma_{i-1}$ .

It can be observed that the graph  $\Gamma$  has following properties:

- its finite, as sets  $S$  and the set valuations of Boolean variables are finite,
- any trace of UCM execution represents a path in the graph.

Correctness of UCM can be checked by checking two properties:

- for each vertex  $v = (s, Val)$  with no outgoing edge  $v$  should satisfy:  $s \in FS$  and  $Cond(Val) = true$ .
- for any transition  $t \in T$  the set of edges  $E$  should contain an edge labeled with  $t$ .

### 5.3. Logic verification

Another way of system's verification is an analysis of correctness using the methods of formal logic. Such analysis enables checking whether our inference procedures preserve truth, which is of fundamental importance in validation of modeled systems. It can be observed increasing of importance of non-classical logics as they are close to the natural inference mechanisms. Particularly noteworthy is temporal logic which originates from modal logic. It allows to reason about the system behavior taking into account its relations with time. However, temporal logic is not focused on time measurement and more important are temporal relationships among events such as precedence and sequence of events.

**Definition 4.** Formal logic is a structure  $\mathcal{L} = \langle \mathcal{F}, \mathcal{I}, \models \rangle$ , where  $\mathcal{F}$  is a set of syntactically correct formulae,  $\mathcal{I}$  is a set of legal interpretations, and  $\models \subseteq \mathcal{I} \times \mathcal{F}$  is a relationship that combines syntax and semantics of a logic.

The formal definition of the syntax and semantics of temporal logic can be found in many works, e.g. [9]. Thus, treating these concepts as defined, let us define further ones.

**Definition 5.** Temporal logic formula  $P$  is satisfied, if there is any interpretation  $I$  such that  $I \models P$ , i.e.  $I$  is a model for formula  $P$ . Formula  $P$  is called valid, or simply tautology, if for every interpretation  $I$  there is  $I \models P$ .

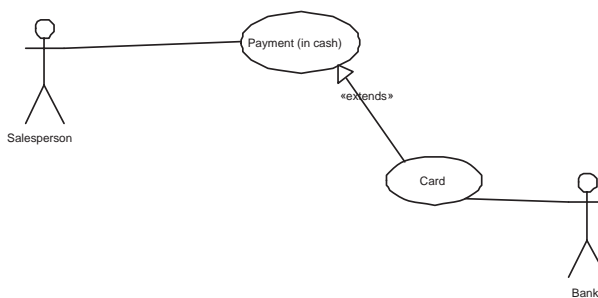
There are many ways of logical inference, but here attention will be paid in two key methods of temporal logic inference about modeled system. The first one is called the *deductive method*. It is based on a logical system which is build in such a way that there are some rules of inference which contain assumptions that lead to the conclusions. This makes possible to introduce a formal scheme which allows to obtain new sentences in a formal way. These new sentences are theorems in a logical system. Therefore, the construction of a system of logical inference requires to propose sentences recognized by the system as its assumptions, i.e. sentences which do not require proofs, and rules of inference. These sentences are schemes which allow transformation of some valid sentences in the other valid sentences. However, this is done within the formal language. On the other hand, the inference rules relate to some metalanguage and they are reliable inference patterns. Construction of a deductive system must take into account a number of its properties, including the structure of time domain, which in general is a fairly complex process. However, axiomatic and deductive system for the *smallest temporal logic* is defined ([5]) as an extension of the classical propositional calculus about axiom  $\Box(P \Rightarrow Q) \Rightarrow (\Box P \Rightarrow \Box Q)$  and inference rule

rule  $\frac{\vdash P}{\vdash \Box P}$  This rule states how to transform a theorem to another one.

An alternative way of inference is the *method of semantic tableau* (e.g. [8]). In this method, an attempt to prove a formula by examining the complementary formula, i.e. its negation, and then lead to a contradiction. The procedure starts by a placing the complementary formula in the root of a tree, and then other subtrees are generated. Building the tree is achieved through an application of some defined rules. Some of the generating rules come from classical logic, but others are specific to temporal logic. The purpose of a such procedure is to find contradictions in various branches of the tree. This, in turn, means that there is an absence of a contradiction for the initial formula. Thus, the tree is called *closed* if in each branch was found a contradiction. The method of semantic tableau can be automated. What's more, the method has an advantage over the deductive method. It follows from the fact that it is easier to find the place of potential error which is simply the appropriate branch of the tree. In addition, the method is characterized by a lower complexity, which arises from the fact that the generated tree has less and less complex formulae at each stage. Proving the desired property of the modeled system consists in the logical implications  $P \Rightarrow Q$ , where  $P$  is a conjunction  $P = p_1 \wedge p_2 \wedge \dots \wedge p_n$  of all formulas which constitute the system specification, and  $Q$  is a formula expressing the desired property. It is interpreted that the examined property is a logical consequence of the system specification. Of course, the key issue is the careful identification of formulas which make up the modeled system.

## 6. Example

Let us consider an example. It deals with the classical problem of the choice of payment method for goods. A common solution to this problem is presented in Figure 8. and the scenario for this problem is presented in Table 1.



**Fig. 8.** Use case implementation using one extended use case

The baseline scenario assumes the implementation of cash payment. However, as an optional extension might be the implementation of payment by credit card. This is an extension of the base case, which occurs as a result of switching and transition control to the extending use case. This solution seems to be a model pattern.

**Table 1**  
Scenario for use case Payment (in cash)

Precondition	
<i>Debit</i>	undefined value
<i>PaymentMethod</i>	undefined value
Postcondition for success	
<i>Debit</i>	$Debit = 0$
<i>PaymentMethod</i>	$PaymentMethod = cash \vee$ $PaymentMethod = card$
Postcondition for failure	
<i>Debit</i>	$Debit > 0$
<i>PaymentMethod</i>	$PaymentMethod = cash \vee$ $PaymentMethod = card$
Main scenario	
1. System displays the customer's account	<b>Debit:= anynonnegativevalue</b>
2. System introduces a standard method of payment	<b>PaymentMethod:= cash</b>
3. Salesperson verifies the choice of method of payment	Guard: [ $PaymentMethod = cash$ ]
4. Salesperson accepts payment and confirms it to the system	<b>Debit:= 0</b>
Alternative	
2.a.	Guard: [ $Debit = 0$ ]
2.a.1. System displays information that no payment is needed and use case terminates	
4.a. Payment cannot be realized	
4.a.1. Use case terminates	
Extensions	
3.a. Payment in card	<b>PaymentMethod:= card</b>

### Model verification

Let us consider the formal analysis of the last solution. Figure 9 shows an example of UCM for the use case Payment (in cash). The conditions in use case specifications:  $debit = 0$ ,  $debit > 0$ ,  $PaymentMethod = cash$  and  $PaymentMethod = card$  are represented by Boolean variables:  $debit\_eq\_0$ ,  $debit\_gt\_0$ ,  $pm\_eq\_cash$ ,  $pm\_eq\_card$ . The constraints  $C$  for the variables are:

$$((\neg debit\_eq\_0 \wedge \neg debit\_gt\_0) \vee (debit\_eq\_0 \wedge \neg debit\_gt\_0)) \vee$$

$$(\neg debit\_eq\_0 \wedge debit\_gt\_0) \wedge ((\neg pm\_eq\_cash \wedge \neg pm\_eq\_card) \vee (\neg pm\_eq\_cash \wedge pm\_eq\_card) \vee (pm\_eq\_cash \wedge \neg pm\_eq\_card))$$

Valuations of Boolean variables ( $debit\_eq\_0$ ,  $debit\_gt\_0$ ,  $pm\_eq\_cash$ ,  $pm\_eq\_card$ ) we will represent by strings containing  $T$  and  $F$  letters. The initial valuation assigns false to all variables, thus corresponding string is  $FFFF$ .

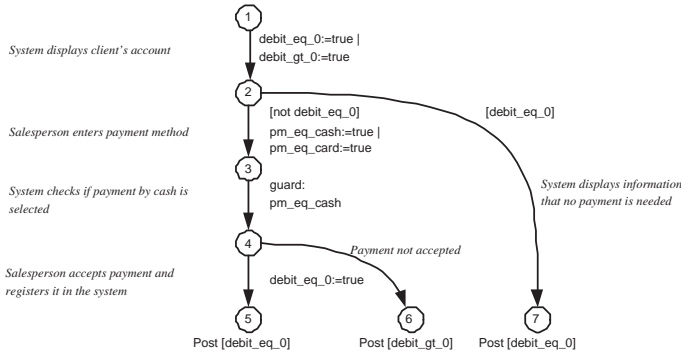


Fig. 9. Use case model example

In the Figure 10  $\Gamma$  graph for the UCM from Figure 9 is presented. Each vertex is annotated with the state number and the string representing valuation of Boolean variables. Edges of the graph are annotated with assignments to the variables.

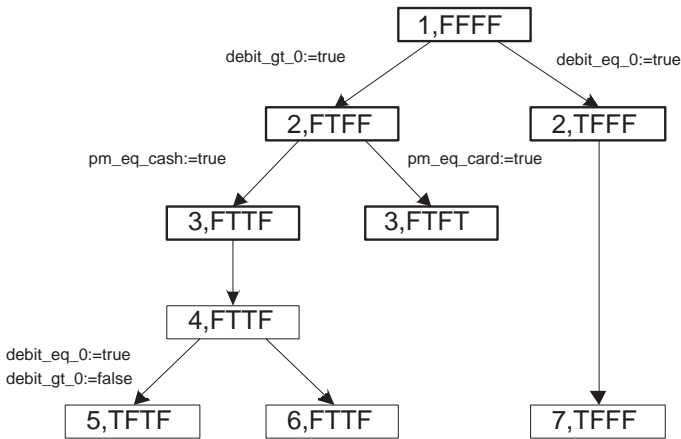


Fig. 10.  $\Gamma$  graph

It can be observed that for the leaf vertices  $[5, TFTF]$ ,  $[6, FTTF]$  and  $[7, TFFF]$  postconditions assigned to states evaluate to true, thus the traces leading to them are correct. The vertex labeled with  $[3, FTFT]$  corresponding to selecting payment by card

has no successor and does not belong to the set of final states  $FS$ . The trace  $[1, FFFF]$   $[2, FTFF]$   $[3, FTFT]$  can be classified as incorrect, what reveals incorrectness of the isolated use case **Payment (in cash)**. This however can be fixed if the extending use case **Card** is defined and its UCM model is merged with the model in Figure 9 as described in section 4.2.

### Logic verification

The use case specification should be carefully constructed and divided to the following items: main scenario, alternative (scenario), extensions, etc. However, the model presented here is relatively small and therefore the specification (Table 1) of the (use case) model will be considered a whole. First, let us consider the possibility of setting a standard form of payment:

$$\diamond pm\_eq\_cash \quad (1a)$$

On the other hand, the customer is left to the choice of the other form of payment:

$$\diamond(pm\_eq\_cash \Rightarrow \diamond(pm\_eq\_cash \vee pm\_eq\_card)) \quad (1b)$$

If the debit is not positive then payment (in card) will not be made:

$$\square(debit\_eq\_0 \Rightarrow \neg\diamond pm\_eq\_card) \quad (1c)$$

It is not possible to change the form of payment in card:

$$\square(pm\_eq\_card \Rightarrow \neg\diamond pm\_eq\_cash) \quad (1d)$$

Now, let us consider some expected liveness and safety properties. The basic safety property says that it is not possible to request two forms of payment:

$$\square\neg(pm\_eq\_cash \wedge pm\_eq\_card) \quad (2a)$$

The basic liveness property express the transition from precondition to postconditions both for success and failures:

$$\neg(debit\_eq\_0 \vee debit\_gt\_0) \wedge \neg(pm\_eq\_cash \vee pm\_eq\_card) \Rightarrow \\ \diamond((debit\_eq\_0 \vee debit\_gt\_0) \wedge (pm\_eq\_cash \vee pm\_eq\_card)) \quad (2b)$$

(Note that undefined value may be treated as a specific variable's value.) Formulas from 1a to 1d belong to the logic model of the specification from Table 1. Formulas 2a and 2b express basic and sample properties of the system. Properties can be verified using the method of semantic tableau which was outlining above (section 5.3). The detailed example of formal verification of use case diagrams using temporal logic and semantic tableau method has been presented in work [13].

## 7. Conclusion

In this paper a formal model of use cases, enabling application of formal methods for a system verification is proposed. There are two ways of formal reasoning about the modeled system. The first method refers to states' exploration while the other one refers to the symbolic inference using temporal logic. An example of use case is analyzed taking into account two complementary ways of verification. Careful and precise modeling of the use case diagrams is very important not only for understanding the system itself, but there is also the ability to automatically, or semi-automatically, generate a number of UML diagrams based on the use cases documentation, c.f. [15].

Future work will include defining a formal language of the use case specification that would allow to define both narrative part in natural language and operations on formally defined sets of variables. Presented in section 2 metamodel indirectly defines the scope of such language. It is planned to develop a tool enabling automatic translation of use case specification to the UCM model. It is planned also to investigate the possibility of automatic, or semi-automatic, generation of temporal logic formulas for the modeled system. Alternative to the model checking presented here is the model checking with temporal logic, i.e. when a model is specified as a transition system but properties are expressed as temporal logic formulas.

## References

- [1] Back R.-J., Petre L., Porres Paltor I.: *Analyzing UML Use Case as Contracts*. Proceedings of UML'99, Second International Conference on the Unified Modeling Language. (Lecture Notes in Computer Science, 1723), Springer Verlag 1999, pp. 518–533.
- [2] Barnett M., Grieskamp W., Schulte W., Tillmann N., Veanes M.: *Validating Use-Cases with the AsmL Test Tool*. Proc. of the 3rd International Conference on Quality Software (QSIC'03). IEEE Computer Society 2003.
- [3] Barrett S., Sinnig D., Chalin P., Butler G.: *Merging of Use Case Models: Semantic Foundations*. 3rd International Symposium on Theoretical Aspects of Software Engineering, IEEE Computer Society 2009, pp. 182–189.
- [4] Bartsch K., Robey M., Ivins J., Lam C.P.: *Consistency Checking between Use Case Scenarios and UML Sequence Diagrams*. International Conference on Software Engineering, Innsbruck, 2004. IASTED/ACTA Press 2004.
- [5] van Benthem J.: *Temporal Logic*. [in:] *Handbook of Logic in Artificial Intelligence and Logic Programming*. vol. 4, Clarendon Press 1993–95, pp. 241–350.
- [6] Clarke E. M. Jr., Grumberg O., Peled D. A.: *Model Checking*. MIT Press 1999.
- [7] Cockburn A.: *Writing Effective Use Cases*. Addison-Wesley 2001.
- [8] D'Agostino M., Gabbay D.M., Hähnle R., Posegga J. (eds): *Handbook of Tableau Methods*. Kluwer Academic Publishers 1999.



- [9] Emerson E. A.: Temporal and Modal Logic. *Handbook of Theoretical Computer Science*, vol. B: Formal Models and Semantics, Elsevier, MIT Press 1990, pp. 995–1072.
- [10] Fowler M.: *UML Distilled*. 3rd Edition. Addison-Wesley 2004.
- [11] Hurlbut R.: *A survey of approaches for describing and formalizing use-cases*. Technical Report 9703, Department of Computer Science, Illinois Institute of Technology 1997.
- [12] Jacobson I.: *Object-Oriented Development in an Industrial Environment*. Proc. of OOPSLA'87, special issue of SIGPLAN Notices. vol. 22, 12, 1987, pp. 183–191.
- [13] Klimek R., Skrzyński P., Turek M.: *Automatic verification of the model at the requirements analysis phase* [in Polish]. 12th National Conference of Software Engineering, Poland, Gdańsk, September 27-29, PWNT 2010, pp. 209–216.
- [14] Kösters G., Six H.-W., Winter M.: *Validation and Verification of Use Cases and Class Models*. 7th International Workshop on Requirements Engineering: Foundations for Software Quality (REFSQ'2001, Proc.), 2001.
- [15] Król T.: *The simulation of use cases* [in Polish]. Master thesis (supervisor: R. Klimek). AGH University of Science and Technology 2007.
- [16] Pohl K., Haumer P.: *Modelling Contextual Information about Scenarios*. Proc. of the Third International Workshop on Requirements Engineering: Foundations of Software Quality REFSQ'97, Barcelona, 1997, pp. 187–204.
- [17] Reisig W.: *Petri Nets – An Introduction*. (EATCS Monographs on Theoretical Computer Science, Volume 4). Springer Verlag 1985.
- [18] Saeki M., Kaiya H., Hattori S.: *Applying a Model Checker to Check Regulatory Compliance of Use Case Models*. Proc. of CAiSE Forum 2009.
- [19] Shen W., Liu S.: *Formalization, Testing and Execution of a Use Case Diagram*. ICFEM 2003 International Conference on Formal Engineering Methods, Singapore, 2003. (Lecture Notes in Computer Science, 2885), Springer Verlag 2003, pp. 68–85.
- [20] Zhao J., Duan Z.: *Verification of Use Case with Petri Nets in Requirement Analysis*. Proc. of the International Conference on Computational Science and Its Applications, Springer Verlag 2009, pp. 29–42.