1. INTRODUCTION

In light of the upcoming gas market liberalization, the storage in underground gas storage should be treated as a commercial activity, providing profit to investors. Gas fuel consumed for the operation of underground gas storage facility, mainly by work of compressors injecting gas into storage, will have to be carefully accounted and bought at market prices. In a competitive gas market, technical and economic optimization of the underground gas storage operation takes on particular significance. Liberalized market requires that the underground gas storage operator manages the operation in the most efficient way, affecting the profitability of service storage provision.

Optimization of operation of underground gas storage in salt cavern (CUGS) is an important operational problem in CUGS management. Despite many works on this subject [6, 16, 17, 20], it is not totally resolved. The reduction of gas storage capacity in a bed of rock salt, called convergence is inevitable, but its size also depends on the cavern depth and the operating storage [5, 19]. Operation practice suggests that the first step is to inject a large cavern convergence, then the lower and finally the shallowest cavern lying. In the withdraw process, the situation is reversed. However, the optimization that takes into account only the speed of storage caverns convergence is not ideal. You may find that maintaining a regime of operation that minimizes the effect of reducing the volume of the storage spaces can increase the cost of the gas installation (surface infrastructure). It is because Injection of the gas to the deepest caverns requires more power compressors.

This occurs in CUGS Mogilno where the system can transport up to 400,000 standard cubic meter per hour [SCm/hr] of natural gas which about 2% is used by the compressor stations to inject gas into storage caverns. It is estimated [13] that the global optimization of the
operation can save up to 20% of the fuel consumed by the gas compression stations. Hence, the problem of minimizing the consumption of fuel gas is of great importance.

In this paper, the authors proposed optimization model to minimize fuel gas consumption during the gas injection into storage caverns. The program also prevents overdependence loss volume of underground salt caverns. Multi-stage injection was used to determine the optimal injection strategy.

2. METHODS OF MANAGING A COMPRESSOR

In the operational practice, control of underground gas storage facility is based on a simple principle. According to [3], the first step is to inject gas to the caverns with a high convergence, then a smaller and at the end to fill caverns with the smallest convergence. In the withdraw process, the situation is reversed. The project [17] attempts to minimize loss volume of underground salt caverns caused by convergence. This research program, however, did not include economic (energy) aspects in the optimization strategy of gas injection into the storage spaces.

Earlier works on the development of optimization algorithms for minimizing the fuel gas consumption in a steady state gas network go back to work [21] from 1968, in which the technique of dynamic programming (DP) for a simple network structures (gun-barrel) was used. In more recent times, [6] DP algorithm is presented that supports the gas network topologies with side branches and turns to model decision variables representing the number of compression stages to lead exploitation of each compressor. On the other hand, in paper [1] non-sequential DP algorithm was developed to handle looped network where the mass flow is constant. In the paper [11] an optimization problem network with loops using the method of GRG (generalized reduced gradient) was considered. Since GRG method is based on an analysis of the gradient, it cannot guarantee finding the global optimum, especially in the presence of discrete decision variables.

In [23], Wu et al. presented mathematical model for minimizing the consumption of fuel gas in one unit compressor. Some of the properties studied in this work have been extended to support multiple units compression station. Optimization techniques have also been used to the transient model [9], [11], and network design [10]. Practical application of these models is limited due to the simplifying assumptions.

3. CAVERN UNDERGROUND GAS STORAGE “CUGS MOGILNO”

As part of the first stage of construction of CUGS Mogilno to 2005 ten storage caverns with a total working capacity of 380 million SCm were completed. During second stage it is planned to build another ten storage caverns. In 2012, the eleventh cavern was incorporated into the operation, and three more are currently in the leaching process.

Brief description of CUGS Mogilno is shown in Table 1.
Caverns are sited at different depths, depending on the intervals occurrence of proper salt in salt dome structure. Average caverns are about 250 m high and less than 400,000 Cm geometric cavern volume. The maximum storage pressure varies depending on the depth of the cavern and is between 9.8 MPa and 21.3 MPa. Similarly, the minimum storage pressure is in the range of 3.3 MPa to 6.4 MPa [5].

The complicated geological salt dome (different foundation depth) affects the large diversity of operating parameters of storage caverns. Capacity of the storage caverns will depend not only on their geometric volume but largely on the gas pressure in the chamber.

<table>
<thead>
<tr>
<th>Specification</th>
<th>Unit</th>
<th>Maximum value</th>
<th>Minimum value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Location</td>
<td>Palędzie Dolne, Poland</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Start of construction</td>
<td>1989</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of caverns</td>
<td>11 (20)*</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Purpose of storage</td>
<td>Peak shaving (seasonal)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Type of formation</td>
<td>Dome</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Geological age</td>
<td>Permian</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average temperature gradient</td>
<td>[K/m]</td>
<td>0.04</td>
<td></td>
</tr>
<tr>
<td>Last cemented casing depth</td>
<td>[m]</td>
<td>1368</td>
<td>584</td>
</tr>
<tr>
<td>Cavern roof depth</td>
<td>[m]</td>
<td>1398</td>
<td>610</td>
</tr>
<tr>
<td>Cavern sump depth</td>
<td>[m]</td>
<td>1567.7</td>
<td>749.9</td>
</tr>
<tr>
<td>Cavern height</td>
<td>[m]</td>
<td>263.9</td>
<td>86.4</td>
</tr>
<tr>
<td>Cavern neck height</td>
<td>[m]</td>
<td>76.43</td>
<td>30.2</td>
</tr>
<tr>
<td>Geometric cavern volume</td>
<td>[10^3 Cm]</td>
<td>562</td>
<td>182</td>
</tr>
<tr>
<td>Total gas cavern volume</td>
<td>[10^6 SCm]</td>
<td>113.7</td>
<td>17.42</td>
</tr>
<tr>
<td>Cushion gas cavern volume</td>
<td>[10^6 SCm]</td>
<td>36.1</td>
<td>6.02</td>
</tr>
<tr>
<td>Working gas cavern volume</td>
<td>[10^6 SCm]</td>
<td>77.6</td>
<td>11.4</td>
</tr>
<tr>
<td>Maximum wellhead pressure</td>
<td>[MPa]</td>
<td>21.3</td>
<td>9.8</td>
</tr>
<tr>
<td>Minimum wellhead pressure</td>
<td>[MPa]</td>
<td>6.4</td>
<td>3.3</td>
</tr>
<tr>
<td>Maximum withdrawal rate</td>
<td>[1000 SCm/hr]</td>
<td>120</td>
<td>200</td>
</tr>
</tbody>
</table>

* after expansion in 2020
Therefore, the total volume in the cavern is connected with the maximum pressure of the
gas in the cavern, and the cushion gas capacity of the cavern is connected with the low pres-
sure gas in the cavern [7, 17]. Geological cross-section through the Mogilno II deposit is in
Figure 1.

![Geological cross-section through the Mogilno II deposit](source – OSM)

**Fig. 1.** Geological cross-section through the Mogilno II deposit (source – OSM)

### 4. OPERATING COSTS ANALYSIS

**OF THE CAVERN UNDERGROUND GAS STORAGE**

Summary of the annual operating costs of the CUGS (both fixed and variable) are shown
in Figure 2.
On the basis of the above graph we can see that the main costs of the CUGS Mogilno operation are variable costs (gas injection / withdrawal). They represent nearly 35% of the total costs. On second place there are costs of services (well services and surface infrastructure installation services), which constitute more than 25% of total costs. Similar costs are employee salaries, amounting to 22%. Figures 2 and 3 show the statement variable costs of injection and withdrawal gas from CUGS.

Authors ranked for these variable costs consumed costs:
- fuel gas use sets of turbocompressors, reciprocating compressor 6-SMH, glycol regeneration system and gas heating;
- electricity use sets of turbocompressors, reciprocating compressor 6-SMH, glycol regeneration system and gas heating;
- oil use sets of turbocompressors and reciprocating compressor 6-SMH;
- methanol, glycol and petrygo use for heating and drying gas while gas withdrawal from the storage caverns.

The costs of these media and materials were estimated on the basis of market prices.

In Figure 3 we can see that 84% of the variable cost is the cost of fuel gas used in various processes related to the injection and reception of gas from Mogilno.

Figure 4 shows the structure of the total consumption of fuel gas in Mogilno. Turbocompressors consume 74% of the fuel gas. If we add to that 7% of gas consumption generated by the compressor reciprocating we reach 81% fuel gas consumption by the compressors used primarily for injecting gas to storage caverns. The cost of fuel gas used is a significant operating cost of CUGS Mogilno.
Therefore, the optimization of the minimum consumption of fuel gas is such an important issue.
5. GAS COMPRESSION STATIONS

Gas compression station is used to raise the pressure of the gas stream supplied from the system and sent to storage caverns or withdraw gas from certain caverns. This is required when free flow is not possible due to the low pressure difference between the pipeline and caverns. Using the compressor is normally required at the end of summer mainly for injection mode, in order to obtain the correct pressure in the caverns for the upcoming winter. For withdrawal mode, it will be normal at the end of winter, when the caverns pressure becomes lower. The two main types of compressor units used in today’s natural gas industry are centrifugal and reciprocating compressor units.

At the CUGS Mogilno, there are two types of compressors:
– two sets of centrifugal turbocompressors,
– one set of reciprocating compressor.

5.1. Fuel gas consumption

Theoretical consumption of fuel gas \( z^i \) while working of \( i \)-th compressor is given by the equation [22]:

\[
  z^i(Q,P_s,P_d,T_s) = \alpha \frac{Q \cdot P_n \cdot Z_{av} \cdot T_s}{T_n \cdot Z_n \cdot \eta_t \cdot \frac{\kappa - 1}{\kappa}} \left[ (\frac{P_d}{P_s})^\frac{\kappa-1}{\kappa} - 1 \right]
\]

where:
\( \alpha \) – a positive constant,
\( \eta_t \) – isotropic efficiency,
\( \kappa \) – isentropic exponent,
\( Q \) – volumetric flow rate gas supplied from the network in standard conditions,
\( P_n, T_n, Z_n \) – gas parameters in standard conditions,
\( T_s, P_s \) – gas suction parameters,
\( P_d \) – gas discharge pressure,
\( Z_{av} \) – average gas compressibility factor.

Function \( z(Q, P_s, P_d, T_s) \) depends on the characteristics of the compressing device, and therefore, in practice, approximation methods are used to determine the function. According to [22], most commonly used approximation functions are polynomials variables \( (Q, P_s, P_d) \) of the 1\(^{\text{st}}\) or the 2\(^{\text{nd}}\) degree. Looking for functions in terms of non-polynomial variables \( (Q, P_s, P_d, T_s) \), we noted that it can also be considered as a function of pressure ratio \( \Pi = (P_d/P_s) \).

The authors have tested several different functions approximating gas consumption comparing the calculation results with the real consumption registered for individual compressors installed in CUGS Mogilno. Satisfactory results were obtained for the function approximating the consumption of fuel gas in the form of:

\[
  z^i(Q) = Q^{\gamma^i} \cdot F^i(\Pi, T_s)
\]

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and:

\[ F'(\Pi, T_s, I_i) = (A'_i(\Pi)^3 + B'_i(T_s)^3 + C'_i(\Pi)^2 + D'_i(T_s)^2 + E'_i(\Pi) + F'_i(T_s) + G'_i(\Pi)^2 T_s + H'_i(\Pi) T_s^2 + I'_i \Pi T_s + J'_i) \]  \tag{3}

where \( b' \) and \( A'_i \) – coefficients which depend on the technical characteristics of the \( i \)-th compressor \((i = 1, \ldots, n)\) determined by least square method.

In the Table 2 there are coefficients appearing in the equation (3) for the fuel gas consumption of one of the compressors occurring in the CUGS Mogilno in performance 45,000–105,000 SCm/hr approximating the gas consumption.

**Table 2**

The values of function coefficients

<table>
<thead>
<tr>
<th>Factor in equation (3)</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( b )</td>
<td>0.869</td>
</tr>
<tr>
<td>( A_s )</td>
<td>-0.119</td>
</tr>
<tr>
<td>( B_s )</td>
<td>-0.412</td>
</tr>
<tr>
<td>( C_s )</td>
<td>5.022</td>
</tr>
<tr>
<td>( D_s )</td>
<td>-8.289</td>
</tr>
<tr>
<td>( E_s )</td>
<td>0.172</td>
</tr>
<tr>
<td>( F_s )</td>
<td>0.021</td>
</tr>
<tr>
<td>( G_s )</td>
<td>18.233</td>
</tr>
<tr>
<td>( H_s )</td>
<td>0.552</td>
</tr>
<tr>
<td>( I_s )</td>
<td>-0.13</td>
</tr>
<tr>
<td>( J_s )</td>
<td>0.003</td>
</tr>
</tbody>
</table>

Figure 5 shows a comparison of the real and calculated fuel gas consumption using power-law model (non-linear). Each function to optimize the form of (3) is set in a range of flow capacity \([Q_0, Q_1]\) depending on the technical characteristics of the \( i \)-th compressor. Assuming that at a certain time values \( \Pi \) and \( T_s \) are constant; and expanding (3) in a Taylor series about the point \( Q_0 \) and neglecting the quadratic and higher order terms, linear approximation function of the fuel gas consumption is obtained:

\[ z^{i,j}(Q) = A_i^{i,j}(\Pi^i, T_s) \cdot Q + B_i^{i,j}(\Pi^i, T_s) \]  \tag{4}

Function \( A_i^{i,j}(\Pi^i, T_s) \) depends on the temperature of the gas delivered to the store and the gas pressure delivered to the store, and the conditions in the \( j \)-th cavern by the value of \( \Pi^j \), \( j = 1, \ldots, m \).

Assuming a constant for a given stage suction temperature \( T_s \) and the value of \( \Pi^j \) equation (4) can be written as:

\[ z^{i,j}(y^{i,j}) = A^{i,j} \cdot y^{i,j} + B^{i,j} \]  \tag{5}

where:

\( A^{i,j} = A_i^{i,j}(\Pi^i, T_s) \),\( B^{i,j} = B_i^{i,j}(\Pi^i, T_s) \), \( i = 1, \ldots, n \), \( j = 1, \ldots, m \) and represents the injection rate of the \( i \)-th compressor to \( j \)-th cavern.
Figure 6 shows a comparison of the real and calculated fuel gas consumption using a linearized power-law model (linear).

Figure 5. The graph of the fuel gas consumption – non-linear function

Figure 6. The graph of the fuel gas consumption – linear function
The average relative error of calculation of fuel gas consumption for the linear model in relation to actual consumption amounts to 2.77%. For 40% of the data maximum relative error was 2%. And for 90% of the data maximum relative error does not exceed 5%.

6. OPTIMIZATION OF GAS INJECTION TO STORAGE CAVERNS

The task of dispatcher is to determine in time varying volume of flue gas flow injected by the \( i \)-th compressor to the \( j \)-th cavern, while a number of conditions resulting from technical, geological and good practice limitations must be fulfilled. In this paper, the authors present the optimization model which task is to find injection strategy that provides for minimum fuel gas consumption.

Injection strategies include the following restrictions:

- pressure limits for each compressor,
- operational limit for each compressor,
- operational limit for each storage cavern,
- pressure limits for each storage cavern,
- geomechanical constraints (cavern volume change due to salt creep),
- thermodynamic constraints (heat exchange between the gas in the cavern and the salt formation at the wall of the cavern, friction gas flow in tubing, etc.).

Gas injection cycle is divided into phases in time, where there is continuous gas injection performance by the \( i \)-th compressor to the \( j \)-th cavern that form a gas injection strategy. Due to the fact, that for a given phase appointments in the subsequent stages are unknown, each stage must be optimized separately, based on the currently prevailing conditions. This means that the value of \( \Pi \) and \( T_s \) are constant at a given time step.

In this approach, each step in the strategy of gas injection into storage caverns is a classic “transportation problem”. So-called classic transportation problem is one of the linear decision models which is widely used in the transportation economics [12].

In order to formulate a mathematical model, it was assumed that the considered system is composed of \( m \) storage caverns and \( n \) compressors. The objective function is the sum of the fuel gas consumption during the injection of gas into storage caverns. Using (5), the following is obtained:

\[
Z(y) = \sum_{i=1}^{n} \sum_{j=1}^{m} z^{i,j} (y^{i,j}) = \sum_{i=1}^{n} \sum_{j=1}^{m} A^{i,j} \cdot y^{i,j} + \sum_{i=1}^{n} \sum_{j=1}^{m} B^{i,j}
\]

Since the second factor in the equation is constant, therefore, to determine the minimum fuel gas consumption, it is sufficient to minimize the function:

\[
O(y) = \sum_{i=1}^{n} \sum_{j=1}^{m} A^{i,j} \cdot y^{i,j}
\]

where \( y^{i,j} \) is decision variable that determines how much gas you need to inject by the \( i \)-th compressor to \( j \)-th cavern, so that you obtain the lowest possible cost for the injection of whole gas stream delivered to CUGS.
In the present case, the set of compressors comprises two turbo compressors (TK31 and TK32), one reciprocating compressor and fictional compressor representing gas injection into caverns using the pipeline pressure (without the use of compressors), as shown in Table 3.

Table 3
Set of compressors in CUGS Mogilno

<table>
<thead>
<tr>
<th>Compressors</th>
<th>No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>The first turbo compressor TK31</td>
<td>(i = 1)</td>
</tr>
<tr>
<td>The second turbo compressor TK32</td>
<td>(i = 2)</td>
</tr>
<tr>
<td>The reciprocating compressor 6 SHM</td>
<td>(i = 3)</td>
</tr>
<tr>
<td>The fictional compressor – injection with pipeline pressure</td>
<td>(i = 4)</td>
</tr>
</tbody>
</table>

In case of injection without compression, using only pressure in the pipeline, which is represented by a fictional compressor No. 4, fuel gas consumption is 0.

Convergence of storage caverns is a harmful phenomenon, since it results in reducing the volume of cavities. The problem is particularly important in the salt dome in which the caverns are located at different depths, have different sizes and shapes, and the rock mass around them is not uniform. In this case, convergences of individual chambers are diverse, and global convergence of storage is highly dependent on the operation scenario [2, 4, 14, 15, 18].

In this paper, a weighting factors \(W_G\) have been introduced to the objective function to determine the sequence of filling storage caverns in terms of susceptibility to the convergence phenomenon. These factors account decline in the monetary value of \(j\)-th cavern caused by losses in their geometric volume (Tab. 4).

Therefore, the objective function determines the minimum consumption of fuel gas, while preventing overdependence loss of underground salt caverns volume:

\[
O(y) = \sum_{i=1}^{n} \sum_{j=1}^{m} A_{i,j} \cdot W_G \cdot y^{i,j}
\]  

(8)

Table 4 presents a set of storage caverns grouped according to depth of foundation.

Table 4
Set of storage caverns in CUGS Mogilno

<table>
<thead>
<tr>
<th>Storage caverns</th>
<th>No.</th>
<th>(W_G)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shallow caverns</td>
<td>(j = 1, 2)</td>
<td>1.61–2.87(\times)10^3</td>
</tr>
<tr>
<td>Average deep caverns</td>
<td>(j = 3, 4, 7, 8)</td>
<td>0.63–0.90(\times)10^3</td>
</tr>
<tr>
<td>Deep caverns</td>
<td>(j = 5, 6, 9, 10)</td>
<td>0.29–0.42(\times)10^3</td>
</tr>
</tbody>
</table>

Solving mathematical optimization model for the current time step is to find the matrix representing the rate of gas injected to the \(j\)-th cavern by use of \(i\)-th compressor:

\[
y = y^{i,j}
\]

(9)

to minimize the objective function (7).
The objective function is subject to the following constrains:

– Sum of gas rates of the $i$-th compressor to $j$-th storage cavern must be equal to the amount of the gas delivered to a magazine:

$$
\sum_{i=1}^{n} \sum_{j=1}^{m} y_{i,j} = Q
$$

(10)

– Capacity of the $i$-th compressor to the $j$-th cavern storage cannot be negative:

$$
y_{i,j} \geq 0
$$

(11)

Technical constraints are:

– The maximum gas injection rate for $i$-th compressor (Tab. 5):

$$
\sum_{j=1}^{m} y_{i,j} \leq Q_{i, \text{max}}^i; \quad i = 1, \ldots, n
$$

(12)

Table 5

<table>
<thead>
<tr>
<th>$Q_{\text{max}}^i$</th>
<th>Value [SCm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q_{1, \text{max}}^1$</td>
<td>140,000*</td>
</tr>
<tr>
<td>$Q_{2, \text{max}}^1$</td>
<td>140,000*</td>
</tr>
<tr>
<td>$Q_{3, \text{max}}^3$</td>
<td>35,000*</td>
</tr>
<tr>
<td>$Q_{4, \text{max}}^4$</td>
<td>400,000**</td>
</tr>
</tbody>
</table>

* depend on actual stage of storage,  
** depend on the maximum capacity of the gas inlet filters.

– The minimal acceptable gas injection rate for $i$-th compressor (Tab. 6):

$$
\sum_{j=1}^{m} y_{i,j} \geq Q_{i, \text{min}}^i; \quad i = 1, \ldots, n
$$

(13)

Table 6

<table>
<thead>
<tr>
<th>$Q_{\text{min}}^i$</th>
<th>Value [SCm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q_{1, \text{min}}^1$</td>
<td>70,000*</td>
</tr>
<tr>
<td>$Q_{2, \text{min}}^1$</td>
<td>70,000*</td>
</tr>
<tr>
<td>$Q_{3, \text{min}}^3$</td>
<td>28,000*</td>
</tr>
<tr>
<td>$Q_{4, \text{min}}^4$</td>
<td>30,000**</td>
</tr>
</tbody>
</table>

* depend on actual stage of storage  
** depend on the lower range limit of accounting measurement instruments
– Maximum performance gas injection into the \( j \)-th storage cavern dependent on the actual pressure in the \( j \)-th cavern and on the characteristics of safety valve installed:

\[
\sum_{i=1}^{n} y_{ij} \leq Q_{\text{w,max}}^j; \quad j = 1, \ldots, m
\]  

(14)

where \( Q_{\text{w,max}}^j \), \( j = 1, \ldots, m \) depend on the actual wellhead pressure in the \( j \)-th cavern (Tab. 7).

### Table 7

<table>
<thead>
<tr>
<th>( Q_{\text{w,max}}^j )</th>
<th>Value [SCm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Q_{\text{w,max}}^1 )</td>
<td>120,000*</td>
</tr>
<tr>
<td>( Q_{\text{w,max}}^2 )</td>
<td>105,000*</td>
</tr>
<tr>
<td>( Q_{\text{w,max}}^3 )</td>
<td>130,000*</td>
</tr>
<tr>
<td>( Q_{\text{w,max}}^4 )</td>
<td>200,000*</td>
</tr>
<tr>
<td>( Q_{\text{w,max}}^5 )</td>
<td>200,000*</td>
</tr>
<tr>
<td>( Q_{\text{w,max}}^6 )</td>
<td>200,000*</td>
</tr>
<tr>
<td>( Q_{\text{w,max}}^7 )</td>
<td>135,000*</td>
</tr>
<tr>
<td>( Q_{\text{w,max}}^8 )</td>
<td>100,000*</td>
</tr>
<tr>
<td>( Q_{\text{w,max}}^9 )</td>
<td>140,000*</td>
</tr>
<tr>
<td>( Q_{\text{w,max}}^{10} )</td>
<td>200,000*</td>
</tr>
</tbody>
</table>

* depend on the actual amount of gas in the \( j \)-th cavern and on the characteristics of safety valve installed.

– The compressors can operate if the following conditions are satisfied.

– The minimum discharge gas pressure for \( i \)-th compressor:

\[
P_{d}^i \geq P_{\text{d,min}}^i; \quad i = 1, \ldots, n
\]

(15)

\[
\sum_{j=1}^{m} y_{i-j} = 0
\]

where \( P_{\text{d,min}}^i \) depend on technical characteristics of \( i \)-th compressor (Tab. 8).

### Table 8

<table>
<thead>
<tr>
<th>( P_{\text{d,min}}^i )</th>
<th>Value [MPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_{\text{d,min}}^1 )</td>
<td>10.0*</td>
</tr>
<tr>
<td>( P_{\text{d,min}}^2 )</td>
<td>10.0*</td>
</tr>
<tr>
<td>( P_{\text{d,min}}^3 )</td>
<td>9.6*</td>
</tr>
<tr>
<td>( P_{\text{d,min}}^4 )</td>
<td>3.7**</td>
</tr>
</tbody>
</table>

* depend on technical characteristics of \( i \)-th compressor, ** depend on the minimum pressure in gas network.
– The maximum discharge gas pressure for \( i \)-th compressor:

\[
P_{d}^{i} \leq P_{w \text{max}}^{j} \quad j = 1, \ldots, m
\]

\[
P_{d}^{2} \leq P_{w \text{max}}^{j} \quad j = 1, \ldots, m
\]

\[
P_{d}^{3} \leq P_{w \text{max}}^{j} \quad j = 1, \ldots, m
\]

\[
P_{d}^{4} \leq P_{s \text{max}}
\]

\[
\sum_{j=1}^{m} y^{i,j} = 0, \text{ if the condition (16) is not satisfied}
\]

where \( P_{w \text{max}}^{j}, j = 1, \ldots, m \) depend on the geomechanical conditions in the \( j \)-th cavern (Tab. 9).

<table>
<thead>
<tr>
<th>( P_{w \text{max}}^{j} ) &amp; ( P_{s \text{max}} )</th>
<th>Value [MPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_{w \text{max}}^{1} )</td>
<td>10.5*</td>
</tr>
<tr>
<td>( P_{w \text{max}}^{2} )</td>
<td>13.0*</td>
</tr>
<tr>
<td>( P_{w \text{max}}^{3} )</td>
<td>17.6*</td>
</tr>
<tr>
<td>( P_{w \text{max}}^{4} )</td>
<td>18.0*</td>
</tr>
<tr>
<td>( P_{w \text{max}}^{5} )</td>
<td>21.3*</td>
</tr>
<tr>
<td>( P_{w \text{max}}^{6} )</td>
<td>21.3*</td>
</tr>
<tr>
<td>( P_{w \text{max}}^{7} )</td>
<td>13.3*</td>
</tr>
<tr>
<td>( P_{w \text{max}}^{8} )</td>
<td>9.8*</td>
</tr>
<tr>
<td>( P_{w \text{max}}^{9} )</td>
<td>11.5*</td>
</tr>
<tr>
<td>( P_{w \text{max}}^{10} )</td>
<td>21.0*</td>
</tr>
<tr>
<td>( P_{s \text{max}} )</td>
<td>8.4**</td>
</tr>
</tbody>
</table>

* depend on the geomechanical conditions in the \( j \)-th cavern,

** depend on the maximum pressure in gas network.

Experience has shown that the discharge pressure should be 0.5 MPa higher than the wellhead cavern pressure due to pressure losses along the valves in the inlet of pipe branched.

Mathematical model of optimization presented above has been tested using real data collected from history of the cavern gas storage Mogilno. Numerical computations were performed by using the program OptIn developed by the authors in a commercial computer algebra program Mathcad.
7. EVALUATION OF THE MODEL – COMPARISON OF SIMULATION RESULTS WITH HISTORICAL DATA

7.1. Case 1

The first case illustrates the situation when the flow rate of gas injected to the CUGS Mogilno from gas system was \( Q_1 = 178,000 \) SCm/hr. However the temperature and pressure of gas delivered (measuring gas station) was \( P_1 = 6.2 \) MPa and \( T_1 = 6.5^\circ C \) respectively. Wellhead pressure for individual storage caverns were: \( P_{k1} = \{4.88, 4.89, 12.3, 10.9, 16.3, 15.9, 8.7, 6.1, 6.9, 15.3\} \) MPa. Caverns pressure indicate that this is the initial phase of the injection period.

Matrix \( Y_{1}^{k1} \) shows the control program suggested by OptIn for step \( k_1 \). For comparison matrix \( D_{1}^{k1} \) presents historical decisions made by dispatcher.

\[
Y_{1}^{k1} = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 146 \\
0 & 0 & 0 & 0 & 32 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}, \text{[SCm/hr]},
\]

\[
D_{1}^{k1} = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 146 \\
0 & 0 & 0 & 0 & 32 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}, \text{[SCm/hr]}.
\]

It can be noticed that both the simulator and the dispatcher to inject given gas quantity chose one turbocompressor and reciprocating compressor. Performance of the two strategies are the same. The difference is, however, in choosing the storage caverns for natural gas injection. The dispatcher injected gas with a turbocompressor to a deep cavern with wellhead pressure 15.90 MPa and with a reciprocating compressor to a medium deep cavern with wellhead pressure much lower 8.7 MPa. The simulator however, injected gas both with turbocompressor and reciprocating compressor to deep cavities No. 9 and 6 with wellhead pressures of 6.91–15.90 MPa, respectively. As a result of these seemingly minor differences, the consumption of fuel gas in the strategy used by the simulator was about 21.10% lower than in the injection strategy used by dispatcher. Results are shown in Table 10.

**Table 10**

Fuel gas consumption for gas injection strategy used by the simulator and the dispatcher

<table>
<thead>
<tr>
<th></th>
<th>Simulator strategy</th>
<th>Dispatcher strategy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fuel gas consumption for turbocompressor I [Cm]</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Fuel gas consumption for turbocompressor II [Cm]</td>
<td>1950</td>
<td>2680</td>
</tr>
<tr>
<td>Fuel gas consumption for reciprocating compressor [Cm]</td>
<td>397</td>
<td>295</td>
</tr>
<tr>
<td>Total fuel gas consumption</td>
<td>2347</td>
<td>2975</td>
</tr>
</tbody>
</table>
7.2. Case 2

The second case illustrates the situation when the flow rate of gas injected to the CUGS Mogilno from gas system was \( Q_2 = 343,000 \text{ SCm/hr} \). Whereas the temperature and pressure of gas delivered (measuring gas station) was \( P_1 = 6.4 \text{ MPa} \) and \( T_1 = 3^\circ \text{C} \), respectively. Wellhead pressure for individual storage caverns were: \( P_{2,k} = \{4.88, 4.88, 12.4, 11.0, 17.8, 17.8, 9.4, 6.7, 6.9, 15.6\} \text{ MPa} \). Caverns pressure indicate that it is the beginning of the spring-summer period.

Matrix \( Y_{2,k}^{*} \) shows the control program suggested by OptIn for step \( k_1 \). For comparison matrix \( D_{2,k}^{*} \) presents historical decisions made by dispatcher.

\[
Y_{2,k}^{*} = \begin{bmatrix}
0 & 0 & 0 & 0 & 40 & 0 & 0 & 0 & 0 & 130 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 83 & 55 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 35 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}, \text{[SCm/hr]},
\]

\[
D_{2,k}^{*} = \begin{bmatrix}
0 & 0 & 84 & 79 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 42 & 33 & 0 & 0 & 0 & 71 \\
0 & 0 & 0 & 0 & 0 & 0 & 34 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}, \text{[SCm/hr]}.
\]

It can be noticed that both the simulator and the dispatcher to inject given gas quantity chose two turbocompressors and reciprocating compressor. Compressor capacity for both strategies are similar to each other. First turbocompressor injected a little less gas for the dispatcher’s strategy 163,000 in relation to 170,000 SCm/hr strategy simulator. For the second turbocompressor situation is reversed. The difference is in the choice of the storage cavern for gas filling. The dispatcher injects gas using compressor to six storage caverns, three of them are deep and three of medium depth. The simulator split injected performance on only three storage cavities, all deep, two with high-wellhead pressure 15.57 MPa and 17.84 MPa and one with low 6.91 MPa. As a result of these differences, the fuel gas consumption in the strategy used by the simulator was 2.6% lower than in the injection strategy used by dispatcher. Results are shown in Table 11.

<p>| Table 11 |
| Fuel gas consumption for gas injection strategy used by the simulator and the dispatcher |</p>
<table>
<thead>
<tr>
<th>Simulator strategy</th>
<th>Dispatcher strategy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fuel gas consumption for turbocompressor I [Cm]</td>
<td>2810</td>
</tr>
<tr>
<td>Fuel gas consumption for turbocompressor II [Cm]</td>
<td>2650</td>
</tr>
<tr>
<td>Fuel gas consumption for reciprocating compressor [Cm]</td>
<td>198</td>
</tr>
<tr>
<td>Total fuel gas consumption</td>
<td>5658</td>
</tr>
</tbody>
</table>
Proposed procedures have been tested on two examples taken from actual data. For the first example with a lower rate of the gas delivered to the CUGS, injection strategies that were received were, at first glance similar to the decision of the dispatcher. After analyzing the fuel gas consumption, it can be seen that the decision of the simulator can save about 20% of the fuel gas. In the second example, a much higher rate volume the gas delivered from the gas network, dispatcher and simulation strategies were more divergent than in the first example. Simulator injected the same amount of gas to less storage caverns than dispatcher. Fuel gas consumption was 2.6% lower for the decision of simulator with respect to the historical gas consumption.

8. CONCLUSIONS

1. Analysis of CUGS operating costs identified the cost of fuel gas consumption as a significant factor in operating costs in underground gas storage in salt cavern Mogilno.
2. An algorithm and a computer program OptIn were developed to optimize the use of compressors for filling underground gas storage (UGS) in salt caverns.
3. The proposed procedure has been tested on two examples taken from actual data. After analyzing the fuel gas consumption it was found that by using a simulator up to 20% of the fuel gas can be saved.
4. The program can be used as a tool to assist decisions on the management of operation UGS in salt caverns.
5. OptIn is independent from the staff experience and brings automation to the process of controlling the CUGS operation.
6. The program can be useful for both, the novice dispatcher, as a program for learning as well as for the specialist, for verifying his decision.

9. NOMENCLATURE

9.1. Latin and Greek Letters

\( A \) directional factor objective function [1]
\( B \) free term linear approximation function fuel gas consumption [SCm/hr]
\( P_w \) gas pressure at wellhead \( j \)-th cavern storage [MPa]
\( P_n \) gas pressure in standard conditions [MPa]
\( P_s \) gas pressure supplied from the network (suction pressure) [MPa]
\( P_{d_i} \) gas discharge pressure of \( i \)-th compressor [MPa]
\( Q \) volumetric flow rate gas supplied from the network in standard conditions [SCm/hr]
\( T_n \) gas temperature in standard conditions [K]
\( T_s \) gas temperature supplied from the network (suction temperature) [K]
\( W_G \) geomechanical ratio of the objective function [1]
$y$ decision variable determines how much gas you need to inject the compressor to cavern, so as to obtain the lowest possible cost for the injection of whole gas stream delivered to CUGS [SCm/hr]

$z$ fuel gas consumption [SCm/hr]

$Z_n$ gas compressibility factor in standard conditions [1]

$Z_{av}$ average gas compressibility factor [1]

$\alpha$ positive constant, which is assumed to be equal to 1 [-]

$\eta_{is}$ isotropic efficiency of the compressor [1]

$\kappa$ isentropic exponent [-]

$\Pi$ ratio of the discharge gas pressure to suction gas pressure (compression) [1]

9.2. Abbreviations

Cm cubic meter

CUGS underground gas storage in salt cavern

DP dynamic programming

GRG generalized reduced gradient

OSM operator systemu magazynowania (storage system operator)

SCm standard cubic meter

UGS underground uas utorage

REFERENCES


