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# AN ELEMENTARY DERIVATION OF KNOTHE'S FORMULA CONCERNING SUBSIDENCE OF A POINT UNDER THE INFLUENCE OF EXPLOITATION

It is common knowledge that underground extraction of minerals causes, among other things, deformations of the surface of an extraction area. Those deformations take the shape of the so-called subsidence trough.

The shape of the subsidence trough at a fixed moment of time $t$ can be described geometrically as a two-dimensional surface immersed in Euclidean space $R^3$ Such a surface is usually given by an implicit function

$$E(x, y, z; t) = 0 \tag{1}$$

where $(x, y, z)$ stands for the position in $R^3$ of any point of the surface (1) at the moment of time $t$ Here $x$ and $y$ stand for horizontal position of a point, while $z$ stands for its vertical position at the moment $t > 0$.

In what follows we restrict our considerations to the so-called planar case, that is we consider the section of the surface (1) with the plane $y = y_0$ = const. Moreover, we consider only the part of the profile of the subsidence trough in the direction of the exploitation. That part of the profile can be described, at the moment time $t > 0$ by

$$F(x, y, z; t) = 0 \tag{2}$$

here $x$ and $z$ denote respectively, position of any point along $x$-axis and its subsidence along $z$-axis at the time $t$.

In our further consideration we interpret the subsidence trough's profile (2) as a rheonomous constraints [1]. Consequently, to get the motion of an observed single point of trough one has to consider its free motion (that is the motion in the absence of the constraints) which is in accordance with the rheonomous constraints.

One can calculate the function $w(t)$ described the law of the subsidence (at the moment $t$) of the observed single point provided that one knows the rheonomous constrainsts, i.e. the function $F(x, z; t)$ in (2), described the shape of trough, and the equation of the free motion of the point under discussion.

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Below we apply the above considerations to some special cases of the rheonomous constraints $F$ and free motion of the observed point. Let the instantaneous trough be described by

$$F(x,z;t) = w_0(1 - \exp - (x^2/t)) - z = 0 \quad (3)$$

for $x > 0$ and $t > 0$, where $w_0 < 0$ is a constant value (denoting maximal subsidence of trough). Thus the part of the trough's shape in the direction of exploitation is described explicitly by

$$z(x,t) = w_0(1 - \exp - (x^2/t)) \quad (4)$$

for $x > 0$ and $t > 0$.

We are now in a position to derive the known Knothe's formula concerning subsidence of a single point of trough.

## 1. Fact

If the motion of the observed single point is subjected to the constraints (4) and its free motion is given by the equation $x(t) = \sqrt{ct}$ $(c > 0)$ then the subsidence of that point is governed by

$$w(t) = w_0(1 - \exp - ct), \text{ for } t \geq 0 \quad (5)$$

Indeed, it suffices to insert into (4) the equation $x(t) = \sqrt{ct}$ $(c > 0)$ then as the result one gets $z(\sqrt{ct}, t) = w(t)$, that is (5).

## 2. Final remarks

1. The obtained formula (5) was at first derived by means of an ordinary differential equation of first order in [2], and [3].

2. It should be noted that all the instantaneous troughs given by (4) attain practically the very same maximal subsidence value $w_0$. This is rather unexpected property of the non-stationary subsidence trough and seems to be of paradoxical nature.

## References


[1] Gutowski R.: *Analytical Mechanics (Part I)*. [in:] Zorski H. (ed.), Foundations of Mechanics, PWN Warszawa, and Elsevier Amsterdam–Oxford–New York–Tokyo 1992

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[3] Knothe S.: *Influence of time on the shapping of subsidence trough*. Archiv. of Mining Sciences, t. 1, z. 1, 1953 [in Polish] [Wpływ czasu na kształtowanie się niecki osiadania]