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## Quality Control of Geodetic Networks at Leica Geo Office\*\*

### 1. Introduction

Determination of the results quality, mainly influenced by gross and systematic errors, remains a principal issue in geodetic measurements. There are several methods to detect gross errors [5]. One of these methods used by W. Baarda and developed by P. Teunissen has been known under the name of Delft Method since it originated from the university where it was created and developed. This very method was implemented within Leica Geo Office (LGO) application. Theoretical foundation of this method as well as tools used in its application may be found in reference literature pertaining to the method alone [1, 2] and describing (among the other issues) problems around testing of statistical hypotheses, being its integral part [4, 5]. This paper discusses briefly the selected aspects of theories connected to the presented method and helpful in reading of the following part of the dissertation. A practical presentation of minimal detectable bias (MDB) and bias to noise ratio (BNR) terms used in the Delft method is shown here, using a sample of networks measured with GPS receivers. Mutual relations between described terms and interpretation of results were also included.

### 2. Definitions of Precision, Accuracy and Reliability

Quality of the geodetic network may be defined by expressions of precision, accuracy and reliability. Precision pertains to closeness of repeated observations around the average taken from sample, while accuracy relates to proximity of observations around true value. The true value is not usually known, therefore we

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\*\* Paper developed as part of statutory research, contract No. 11.11.150.312

use the most probable value. A standard deviation which corresponds to stochastic variability of observation is introduced by applying so called the least squares method. Therefore a precision of network may be defined as effect of stochastic variability of network observations on the coordinates of its points. Absolute and relative standard error ellipses, also called confidence ellipses, are applied to demonstrate the precision.

Reliability may be defined as the ability to detect observations characterized by gross or systematic error as well as an estimate of the effect of not detected gross and systematic errors on the estimated values.

The mentioned definition of reliability combines two aspects of this term, pertaining to internal reliability and external reliability respectively.

The internal reliability is determined by the minimum detectable bias (MDB). In another words, MDB represents a value of the least observation error possible to be detected using statistical test with probability equal to test power  $\beta$ . High MDB indicates a small possibility of observation or coordinate control; hence the higher MDB, the lower is the network reliability. The value of MDB will not be calculated in case observations cannot be controlled.

External reliability, applied to determine the effect of undetected gross and/or systematic error of observation on adjusted coordinates, is expressed by the parameter of BNR (Bias to Noise Ratio). BNR is a dimensionless parameter, binding the effect of a single observation on all coordinates. Practical interpretation may be presented if we consider BNR to be an upperbound of a ratio of observation MDB effect on coordinate  $X$  to a posteriori standard deviation of this coordinate [4]; thus

$$\left| \frac{\nabla_X}{\sigma_X} \right| \leq \text{BNR} \quad (1)$$

BNR parameter may be interpreted as a relation between reliability and precision. Mutual relations between described terms are illustrated on figure 1 [8].

Figure 1 shows that high accuracy is related to high reliability of measurement results since both terms pertain to proximity to the true value. There is, however, an essential difference between them, since in the "Delft Method" the reliability is connected to statistical testing procedures (contrary to heuristic methods), thus enabling to determine the effect of gross and systematic errors on adjusted results and their detection. The adjusted result of the geodetic network observation should be precise and reliable, but the precision and reliability do not have to walk in pairs (Fig. 1) due to systematic errors; therefore the increase in number of observations will not improve the quality of calculation results. Thus less precise measurement may be more reliable due to concentration of results around the true value.

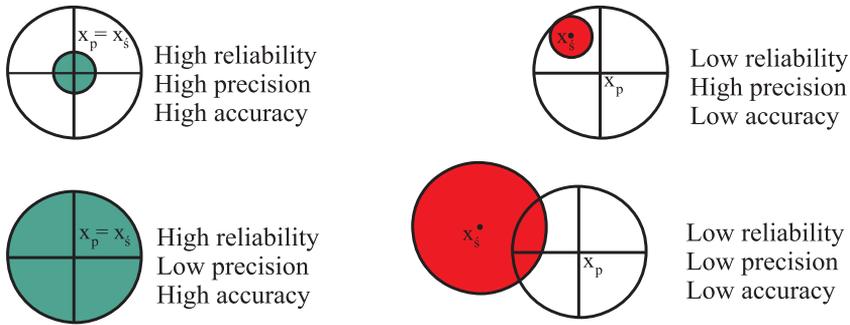


Fig. 1. Relations between accuracy, precision and reliability

### 3. Statistical Tests

Statistical tests are an essential foundation in the process of geodetic network quality control. They are performed basing on corrections calculated with use of the least squares method. Application of statistical tests for determination of observation result quality is used to detect the observations that are odd due to gross and/or systematic errors. Test effectiveness will then depend on the geodetic network reliability. The more reliable network, the higher probability of detection of those observations that appear odd after application of statistical tests and related procedures. Applicable statistical procedures fundamentally include testing of statistical hypotheses, widely described in references, e.g. in [3, 6]. In case of the Delft method there is an assumption indicating that the observation corrections are random variables with normal distribution, and a value of standard deviation is known.

Three mentioned below statistical tests are used for the statistical testing procedure:

- overall  $F$ -test – related to the ratio of *a posteriori* and *a priori* variance coefficients;
- $W$ -test – a single dimension local model test, applied individually to an observation;
- $T$ -test – a multi-dimensional  $W$ -test (2D or 3D, applied depending on the dimension of the tested value).

Zero hypothesis of  $F$ -test assumes there are no gross errors in the observations. Should this hypothesis be rejected it is necessary to find the reason for such rejection. Conventional alternative hypothesis assumes presence of one gross error in the observations. Therefore the observations are individually tested by using  $W$ -test. Both tests are carried out with the same test power  $\beta$  (i.e. using the method B). In case of satellite measurements the observations are formed as spatial vectors. For

that reason it appears necessary to use multidimensional  $T$ -test since individual testing of vector components with  $W$ -test is not sufficient. The  $T$ -test may also be suitable for reference station coordinate control. The  $W$ -test is more effective in detection of any reference station coordinate entered incorrectly, while the  $T$ -test would appear to be more effective in case of small reference station coordinate deformations. However, it cannot be applied to find a direction of the reference station movement. The  $T$ -test is bound with  $F$ -test by the same test power  $\beta$ , but it has its own significance level  $\alpha$  and its own critical values.

LGO also applies  $W$ -test to detect errors resulting from the height of GPS antenna. The calculations are performed separately for all of particular components  $X$ ,  $Y$  and  $H$ .

#### 4. Interpretation of Internal and External Reliability

A simple, arbitrarily assumed, network pattern in form of a square was analyzed (see Fig. 2) and the measurements were carried out with 4 GPS receivers. Components of 6 vectors, represented by arrows in figure 2, were calculated based on the network coordinate points. These calculations were performed in several variants.

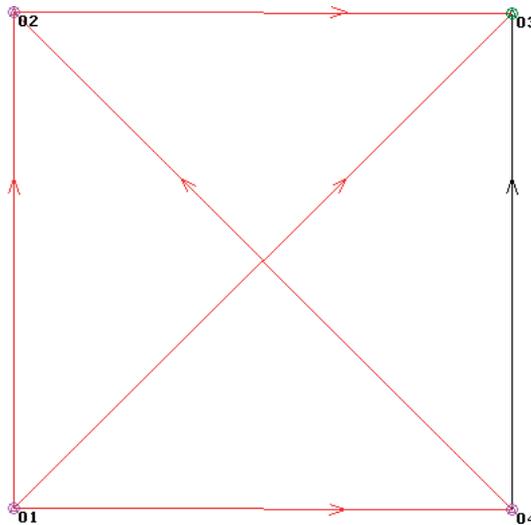


Fig. 2. Sketch of GPS test geodetic network

Variant A calculation result, obtained without interference into observations, rendered a value of  $MDB = 0.0000$  m, due to the method used for defining of the vector components as well as assumed zero values for size of a stochastic model.

Therefore one of observation points in variant B was interfered with by introduction of the minimum value ( $DX_{01-02} = 0.0001$  m), since the value  $MDB$  from variant A. indicates that in practice each outlier observation would be detected. A level of significance  $\alpha_0 = 0.05$  was taken for the purpose of testing, therefore statistical tests were given critical values of  $W = 1.96$  and  $T = 1.89$ , so that the outlier observation was therefore detected (see Tab. 1) and a correct value of estimated gross error was obtained. The value of  $BRN = 3.2$  means that the effect of  $MDB$  value equal to  $0.0001$  m on any of the network coordinates is lower than  $3.2\sigma_{a\text{ posteriori}}$  of a given coordinate, thus rendering a value of  $3.2$  mm in our example. It is important that  $BRN$  remains homogenous for the whole network. Otherwise, it may happen that the reliability of network will depend basically on a single observation. The important feature of both  $MDB$  and  $BRN$  is that they are not depending on a selection of base station.

**Table 1.** Result of statistical tests upon imposing a single outlier observation point

Station 01	Target 02	$MDB$	$BRN$	$W$ -test	$T$ -test
	DX	0.0001 m	3.2	-3.00	3.00
	DY	0.0001 m	3.2	0.00	
	DZ	0.0001 m	3.2	0.00	

A gross error equal to  $0.050$  m ( $DX_{04-02} = 0.0500$  m) was introduced in variant C. The obtained value of  $MDB$  equaled then  $0.0433$  m and the estimated gross error was calculated correctly. A subsequent gross error having a value  $<0.0433$  m ( $DX_{01-02} = -0.0420$  m) was introduced in variant D. The value of  $MDB$  was increased to the level of  $0.07$  m with correct detection of the outlying observation. Instead of introducing just one gross error (variant D), two gross errors were imposed in variant E (in addition  $DY_{03-04} = -0.0420$  m). The  $MDB$  value increased to the level of  $0.07$  m, and no gross error was detected by using statistical tests in vector 03-04 with rejected result of the  $F$ -test.

In practice, the analysis of final product differences or equalized coordinates is significant here in the event of tampered with observation system points (Tab. 2).

**Table 2.** Differences in equalized coordinates from various variants of tampering with the network observation points

No.	variant C			variant D			variant E		
	$dx$	$dy$	$dz$	$dx$	$dy$	$dz$	$dx$	$dy$	$dz$
01	0.0000	0.0000	0.0000	0.0118	0.0000	0.0000	-0.012	0.001	0.000
02	-0.0095	0.0000	0.0000	-0.0022	0.0000	0.0000	0.002	-0.001	0.000
03	0.0000	0.0000	0.0000	-0.0012	0.0000	0.0000	0.001	-0.012	0.000
04	0.0095	0.0000	0.0000	-0.0083	0.0000	0.0000	0.008	0.012	0.000

The gross errors introduced in these observations, one of 5.0 cm and two of 4.2 cm, caused a deformation of the equalized coordinate points at the level of one centimeter. Standard deviations for all coordinate points were maintained at the level of 5 mm, and for all observations at the level of 8 mm. Interpretation of *BNR* indicates that undetected gross error of 4.34 cm will influence the coordinates by less than 1.54 cm.

## 5. Interpretation of Statistical Test Results

It should be remembered that statistical tests render the results which are probable at the determined level of significance with a defined test power, but such results are not absolutely certain. Taking into consideration or rejecting the hypotheses it is possible to make errors of the first and second kind.

An error of the first kind is made when the zero hypothesis is rejected, while it is true, and probability of making this error may be described as a level of significance  $\alpha$ . An error of the second kind is made when the zero hypothesis is assumed, while it is false, and probability of making this error may be described as a test power  $\beta$ .

Statistical tests applied by the Delft method are related with each other by the same test power. Results of those tests should be interpreted together, since rejection of the results by a particular test may be caused by various factors, such as gross errors, and incorrect mathematical and/or stochastic model.

Standard levels of significance of 0.05, 0.01, 0.001 were assumed in analysis of the network shaped as a square, while the test power, considered standard for this method, was selected to be 0.80 and 0.90. The table 3 shows results pertaining to variant C with one gross error; while the last row represents data for variant D with three gross errors. The letters attached to numerical values denote: A – acceptance, R – rejection.

Zero hypothesis within overall *F*-test in variant D was rejected, thus gross errors were detected in those observations. If *F*-test rejects zero hypothesis, then specific types of observations (such as distances, directions, zenith angles, altitude variations) are also rejected and that denotes an error of the mathematical model. Due to the changes of test critical values, in our tested example the change of test power is not that significant for calculation results as is variation of the level of significance. For this reason the statistical tests did not detect any gross error in observation DX02-04. Establishing a lower level of significance results in more difficult rejection of zero hypothesis and increases credibility of an alternative hypothesis.

**Table 3.** Comparison of statistical test values as well as *MDB* and *BNR* values

$\alpha_0$	0.05	0.05	0.01	0.01	0.001	0.001	0.05
$\beta$	0.80	0.90	0.80	0.90	0.80	0.90	0.80
$W_{crit}$	1.96	1.96	2.58	2.58	3.29	3.29	1.96
$T_{crit}$	1.89	1.91	2.83	2.83	4.24	4.24	1.89
$F_{crit}$	1.20	1.16	1.53	1.53	2.01	1.99	1.20
$F$ -test	0.47 A	1.25 R					
$MDB_{02-04}$ [cm]	4.66	5.02	5.29	5.29	6.89	7.08	7.07
$BNR$	3.2	3.7	2.7	2.7	3.2	3.6	3.2
$W$ -test	-3.00 R	-3.00 R	-3.00 R	-3.00 R	-3.00 A	-3.00 A	-2.61 R
$T$ -test	3.00 R	3.00 R	3.00 R	3.00 R	3.00 A	3.00 A	2.28 R

In case a gross error is being indicated by the results of statistical tests at any observation point, it may be removed from calculations and then analyzed for the effect of such action in the final calculation of results.

## 6. Conclusions

Algorithms used to design and develop geodetic measurements should enable the following:

- a control of propagation of random errors present in observations onto coordinates, while meaningfully explaining the precision;
- identification of gross and systematic errors in observations;
- determination of data sensitivity to those errors, meaningfully explaining the reliability.

Both mathematical and stochastic models could be distinguished but not investigated separately within the least squares (LS) method. While the mathematical LS model renders the most probable results based on available data, the statistical tests of stochastic model could check the results for presence of gross errors, and the parameters describing both precision and reliability determine quantitatively the quality of results.

LS attempts to spread the effect of gross errors. Since the variance – covariance matrix is not diagonal, a single gross error may affect all the corrections. In case several gross errors are in existence, their influence may be coinciding, therefore one gross error hides the other. Such error may be a reason to reject a good observation point. Increase in number of observations may secure lower absorption of gross errors. Absence of redundant observations will introduce gross errors in their full dimension. Moreover, the presence of redundant observations is a deciding

factor for possibility of carrying statistical tests, thus determining reliability of measurement results.

Test effectiveness depends then on the network reliability. The more reliable is the network, the higher probability exists to detect gross errors with the use of statistical tests. A control of geodetic network quality could be carried out in the very design phase and that allows to select the appropriate optimum project design related to requirements of customer as well as to measurement costs.

The Delft method used by LGO is a good tool for calculations and analysis of observations. It is one of the many methods applicable in order to detect gross errors and outlier observation points. However, none of those methods can give absolute certainty with reference to correctness of the obtained results.

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