Piotr Parzych*

Modelling of Urban Estates' Values

1. Introduction

Ground and Building Cadastre is important place for Land Information System data collecting. Prices' and values' register is its integral component. The Surveying and Mapping Law provides rules on Ground and Building Cadastre managing. According to the Law it is run as a computer database. It also is one of few Spatial Information Systems having so high computerization level.

The outlines concerning cadastre, including the prices' and values' register for real estates (RCiWN) are defined by The Order of Ministry of Regional Development and Buildings. The detailed outlines for establishing and managing cadastre are given in the Instruction G-5.

According to the order of ministry of regional development and buildings, county chief is in charge of prices' and values' register for real estates. Real estates' prices derive from authenticated deeds. Real estates' values established by estates' expert come from valuation records, for its extracts are sent to cadastre.

The following data are recorded in the prices' and values' register:

- real estate's price and value,
- real estate's address,
- numbers of parcel being estate's components,
- estate's type,
- estate's area,
- the date of authenticated deed signing or the date of real estate valuation,
- other data concerning real estates.

Urban real estates are the particular components of ground property that means building elements. They are characterized by the ground components great complication. The description of urban real estates should include the following

* AGH University of Science and Technology, Faculty of Mining Surveying and Environmental Engineering, Krakow, Poland

geometrical parameters: urban estates area, buildings of all types' usable areas, rooms of industrial use and warehouses areas and capacities, umbrella roof's usable areas and other data.

Apart from geometrical description, some characteristic parameters of urban estates that may have significant influence on their market value should be taken into consideration. They are localization, building vicinity, building technical state and urban estate way of exploitation. The most important attribute for sold urban estate is its transaction price. This price includes prices of all urban estate's components.

There is no possibility to define fragmentary prices for urban estates components during the transaction process and preparation of authenticated deed. It requires the prior defining of proportion coefficients between their market values. These coefficients may be calculated with advanced statistical analysis for urban estates' market.

The urban estate's value estimated together with mortgage loan decision and possibility of its changes during mortgage paying off time is vital for mortgage loan securing. The changes of estate's market value during mortgage loan paying off time may be caused by market factors or estates components technical and functional use. The market factors analysis and estate's use analysis must be conducted separately for land and each building. To enable such analyses we should estimate unit prices for ground and other estates particular components (urban elements).

This solution concerns polish real estates' market in particular, for appropriate legal regulations concerning cadastral value are being formulated there. The urbanized estates' valuation methods described in [15] and [17] always refer to the whole urbanized estate, so algorithms presented in this work are unique. The methods of market analysis and estates' valuation may be used not only in Poland, but abroad as well.

If the number of transactions (transaction prices) is higher than the unit prices number for each building and other urban estates installations and their attributes accepted for market analysis and estates valuation, than unit prices and weight coefficients estimation will be based on Gauss–Markov parametric model.

Define the following symbols as:

C_{Tj} – transaction price for whole j-estate,

S_i – the area of every i-element (parcel, parcel parts having defined soil classes, flat or building usable areas or whole building),

c_i – i-element unit price,

$a_1, a_2, ..., a_j$ – attributes accepted for urban estates estimation values,

$k_1, k_2, ..., k_j$ – the weight coefficients of attributes accepted for urban estates valuation.

Then the parametric model for unit prices and attributes will get the following formula:

$$C_{Tj} = S_1 \times c_1 + S_2 \times c_2 + \ldots + S_i \times c_i + a_1 \times k_1 + \ldots + a_s \times k_s \tag{1}$$

The estimated parameters in this formula are

- the unit prices coefficients (c_i) of estate's element,
- the weight coefficients (k_s) of elaborated attributes.

2. Valuation Model Parameters' Estimation

The set of equations (1) written as the matrix, for similar estates transaction prices takes the following form:

$$\begin{bmatrix} C_{T1} \\ C_{T2} \\ \vdots \\ C_{TJ} \end{bmatrix} = \left\{ \begin{bmatrix} S_{11} & S_{12} & \cdots & S_{1i} \\ S_{21} & S_{22} & \cdots & S_{2i} \\ \vdots & \vdots & \ddots & \vdots \\ S_{j1} & S_{j2} & \cdots & S_{ji} \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1i} \\ a_{21} & a_{22} & \cdots & a_{2i} \\ \vdots & \vdots & \ddots & \vdots \\ a_{j1} & S_{j2} & \cdots & a_{ji} \end{bmatrix} \right\} \times \left\{ \begin{bmatrix} c_1 \\ c_2 \\ \cdots \\ c_i \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ \cdots \\ k_s \end{bmatrix} \right\} + \begin{bmatrix} \delta_{T1} \\ \delta_{T2} \\ \cdots \\ \delta_{Tj1} \end{bmatrix} \tag{2}$$

where:

- indices 1–i point out elements of similar estates' elaborated group,
- indices 1–s point out similar estates' elaborated attribute,
- indices 1–j point out the following number of similar estate's sell transaction number.

The set of equation above has solution in real numbers if the transaction number (j) is higher than number ($i + s$), that describes number of estate's elaborated elements and attributes.

After using block matrices notation, equation set (2) has the following form:

$$\{C_T\} = \{[S] \quad [a]\} \times \begin{Bmatrix} [c] \\ [k] \end{Bmatrix} + \{\delta_T\} \tag{3}$$

where:

$\{C_T\}$ – transaction prices vector, for whole estates,
$[S]$ – rectangular vertical matrix, containing areas of elements for elaborated estates,
$[a]$ – rectangular vertical matrix, containing attributes of elaborated urban estates,
$\{c\}$ – unit prices vector for elaborated estates' corresponding elements,
$[k]$ – weight coefficients vector for elaborated attributes,
$\{\delta_T\}$ – vector of random remainder (differences between transaction prices and model values).

The estimators of vector $[\hat{c}]$ and vector $[\hat{k}]$ may be written as following block matrices:

$$\begin{Bmatrix} [\hat{c}] \\ [\hat{k}] \end{Bmatrix} = \begin{Bmatrix} [S^T \cdot S] & [S^T \cdot a] \\ [a^T \cdot S] & [a^T \cdot a] \end{Bmatrix}^{-1} \times \begin{Bmatrix} [S]^T \\ [a]^T \end{Bmatrix} \times \{C_T\} \tag{4}$$

After including the estimators of vector $[\hat{c}]$ and vector $[\hat{k}]$, the vector of random remainder deviation $\{\hat{\delta}_T\}$ we can get from the following formula:

$$\{\hat{\delta}_T\} = \{C_T\} - \{[S] \quad [a]\} \times \begin{Bmatrix} [\hat{c}] \\ [\hat{k}] \end{Bmatrix} \tag{5}$$

The remainder variance $(\hat{\sigma}_0^2)$ derived from parametric with taking weight coefficients into account will have the formula of:

$$\hat{\sigma}_0^2 = \frac{\{\hat{\delta}\}^T \{\hat{\delta}\}}{j - R\{[S] \quad [a]\}} \tag{6}$$

The covariance matrix of estimated parameters may be written in formula (7):

$$Cov\begin{Bmatrix} [\hat{c}] \\ [\hat{k}] \end{Bmatrix} = \hat{\sigma}_0^2 \begin{Bmatrix} [S^T \cdot S] & [S^T \cdot a] \\ [a^T \cdot S] & [a^T \cdot a] \end{Bmatrix}^{-1} \tag{7}$$

Making the analysis of variance for transaction prices vector for whole estates (3) we may find the covariance matrix of $\{\hat{\delta}\}$ vector. It has the following form:

$$Cov\{\hat{\delta}\} = \hat{\sigma}_0^2 \left(\{I\} - \{[S] \quad [a]\} \times \begin{Bmatrix} [S^T \cdot S] & [S^T \cdot a] \\ [a^T \cdot S] & [a^T \cdot a] \end{Bmatrix}^{-1} \times \{[S] \quad [a]\}^T \right) \tag{8}$$

3. The Valuation of Market Value for Urban Estates

We can make the assessment of any estate's market value from analyzed market using parameters estimated with formula (4).

If we valuate the urbanized estate, which include parcel and buildings of known areas, these quantities may be written as the one row parameters' matrix:

$$[\overline{S}] = [\overline{S}_1 \quad \overline{S}_2 \quad \ldots \quad \overline{S}_i] \tag{9}$$

The market attributes of valuated estate may be set together as the one row matrix, too:

$$[\overline{a}] = [\overline{a}_1 \quad \overline{a}_2 \quad \ldots \quad \overline{a}_i] \tag{10}$$

After taking into account estimated unit prices (c_i) and weight coefficients (k_j), the estimated market value of elaborated urban estate can be expressed by the formula below:

$$WR = \{[\overline{S}][\overline{a}]\} \times \begin{Bmatrix} [\hat{c}] \\ [\hat{k}] \end{Bmatrix} \tag{11}$$

The variance of estimated (using formula (11)) market value for urbanized estate can be defined including covariance matrix (7) as follows:

$$\sigma^2(WR) = \{[\overline{S}][\overline{a}]\} \times Cov\begin{Bmatrix} [\hat{c}] \\ [\hat{k}] \end{Bmatrix} \times \begin{Bmatrix} [\overline{S}]^T \\ [\overline{a}]^T \end{Bmatrix} \times \{[\overline{S}][\overline{a}]\}^T \tag{12}$$

The symmetric confidence interval for urban estate's estimated market value will be calculated from its standard deviation ($\sigma(WR)$) and the quantile of Student's distribution ($t(1-\alpha/2);u)$), for confidence level α and u degrees of freedom. It is presented in formula:

$$\vec{WR} = WR \pm \sigma(WR) \times t(1-\alpha/2;u) \tag{13}$$

4. The Examples of Model Estimation and Urban Estates' Valuation

4.1. Market Information

The application of valuation parametric model will be presented with the example of urban estates. Information from seven authenticated deeds obtained from public notaries was used here.

The following attributes has been taken into account during calculations:

- part of the city – suburbs,
- destination in land use city plan – the land of urbanization low intensification,
- localization – very good (2), good (1),
- access to parcel – good,
- vicinity – very good,
- public utilities – water, electricity, gas, sewage, road,
- standard of components used for building and decorating (building standard) – very high (2), high (1),
- usable area of urbanized buildings from 200 m^2 to 340 m^2,
- area of parcel from 810 m^2 to 1050 m^2.

There are seven urban real estates that were chosen for consideration. They description and attributes are listed below.

1. Built up estate with commercial building. Building usable area is 260 m^2. Parcel area is 850 m^2. Attributes: localization – very good (2), building standard – high (1), price – 1,570,000 zł (402,000 EUR – 1 EUR = 3.9 zł).
2. Built up estate with commercial building. Building usable area is 300 m^2. Parcel area is 970 m^2. Attributes: localization – good (1), building standard – high (1), price – 1,600,000 zł (410,000 EUR).
3. Built up estate with commercial building. Building usable area is 220 m^2. Parcel area is 760 m^2. Attributes: localization – very good (2), building standard – very high (2), price – 1,450,000 zł (372,000 EUR).
4. Built up estate with commercial building. Building usable area is 320 m^2. Parcel area is 910 m^2. Attributes: localization – very good (2), building standard – high (1), price – 1,800,000 zł (462,000 EUR).
5. Built up estate with commercial building. Building usable area is 200 m^2. Parcel area is 810 m^2. Attributes: localization – good (1), building standard – high (1), price – 1,200,000 zł (308,000 EUR).
6. Built up estate with commercial building. Building usable area is 340 m^2. Parcel area is 1050 m^2. Attributes: localization – very good (2), building standard – high (1), price – 1,900,000 zł (487,000 EUR).
7. Built up estate with commercial building. Building usable area is 290 m^2. Parcel area is 880 m^2. Attributes: localization – good (1), building standard – high (1), price – 1,570,000 zł (397,000 EUR).

There are two factors influencing estates' prices in the examples given above. They are localization and building standard.

4.2. The Estimation of Valuation Model's Parameters

The information obtained from deeds is basis for 7 equations written according to formula (1). Every equation will have two unit prices and two weight coefficients. The matrix [*S*] of parcel areas and building usable areas and the matrix [*a*] of market attributes and the matrix $\{C_T\}$ of transaction prices have the following form:

$$[S] = \begin{bmatrix} 850 & 260 \\ 970 & 300 \\ 760 & 220 \\ 910 & 320 \\ 810 & 200 \\ 1050 & 340 \\ 880 & 290 \end{bmatrix} \quad [a] = \begin{bmatrix} 2 & 1 \\ 1 & 1 \\ 2 & 2 \\ 2 & 1 \\ 1 & 1 \\ 2 & 1 \\ 1 & 1 \end{bmatrix} \quad \{C_T\} = \begin{Bmatrix} 1570000 \\ 1600000 \\ 1450000 \\ 1800000 \\ 1200000 \\ 1900000 \\ 1550000 \end{Bmatrix}$$

From the estimation of this model's parameters according to formulas (4)–(8) unit prices, attributes weight coefficients and random remainders, calculated for every transaction were obtained. The unit parcel and urban buildings' prices and weight coefficients are as follows:

$$\hat{c}_G = 349{,}43 \text{ zł/m}^2, \quad \hat{c}_B = 3547{,}27 \text{ zł/m}^2, \quad k_1 = 137789, \quad k_2 = 65959.$$

The random remainders (differences between transaction prices and model values) are presented in the table 1.

Table 1. The random remainders

Transaction number	Random remainder	Calculations
1	9157	$\sum_{i=1}^{7} \sigma_i^2 = 4973950134$
2	–6817	
3	–3463	
4	5355	
5	3759	
6	–14511	
7	10044	

The remainder variance ($\hat{\sigma}_0^2$) obtained from Gauss–Markov model estimation and the covariance matrix for unit prices and weight coefficients have values as follows:

$$\sigma_0^2 = 1657983378,$$
$$\sigma_0 = 12876.$$

The covariance matrix obtained for all estimated parameters have the following values:

$$Cov\begin{Bmatrix}[\hat{c}]\\ [\hat{k}]\end{Bmatrix} = 1657{,}983\begin{bmatrix} 31.02 & -92.34 & 1561.85 & -3856.94 \\ /-/ & 295.00 & -7277.59 & 10854.97 \\ /-/ & /-/ & 898922.81 & -69565117 \\ /-/ & /-/ & /-/ & 1424580.04 \end{bmatrix}.$$

The model's estimated parameters presented above are basis for the market value of similar estates estimation.

4.3. The Valuation of Similar Estates

As a subject of valuation we choose estate similar to estates elaborated in market analysis presented above. This is the commercial estate, which has usable area of 260 m^2. The parcel it is situated at has area of 980 m^2. The localization is good (1) and building standard high (1). Applying formula (11) we obtained estates market value as follows:

$$WR = \{[\bar{S}][\bar{a}]\} \times \begin{Bmatrix}[\hat{c}]\\ [\hat{k}]\end{Bmatrix} = [980 \quad 260 \quad 1 \quad 1] \times \begin{bmatrix} 349.43 \\ 3547.27 \\ 137789 \\ 65959 \end{bmatrix} = 1468480 \text{ zł}.$$

After performing operation on the proper matrices, the standard deviation of estimated market value was calculated below:

$$\sigma^2(WR) = 1657.983 \times [980 \quad 260 \quad 1 \quad 1] \times \begin{bmatrix} 31.02 & -92.34 & 1561.85 & -3856.94 \\ /-/ & 295.00 & -7277.59 & 10854.97 \\ /-/ & /-/ & 898922.81 & -69565117 \\ /-/ & /-/ & /-/ & 1424580.04 \end{bmatrix} \times \begin{bmatrix} 980 \\ 260 \\ 1 \\ 1 \end{bmatrix},$$

$$\sigma^2(WR) = 1377460000 \Leftrightarrow \sigma(WR) = 37114 \text{ zł}.$$

Applying Student distribution quantile for 3 degrees of freedom and confidence level $(1-\alpha) = 0.95$, that means $t(0.95;3) = 3.2$, the symmetric confidence intervals for estimated estate's market value equals

$$\vec{WR} = 1\,468\,480 \pm 118\,765 \text{ zł}.$$

The variance analysis, which was made in the example presented above shows that estimated estate's market value has confidence interval ranging about 16% of its value.

5. Conclusions

The presented algorithm of urban estates' market value modelling can enable prices' and values' register data enriching. This could help in municipal estates' management and should result in increasing commune income. This is particularly important when cadastral value determination for all estate's components is necessary. It has also great significance during changes in legal regulations concerning land given in perpetual use value alterations. Determination of such value is necessary for calculating corresponding charges.

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