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## STRESS STATE OF MULTIPLE TUNNEL LININGS CONSTRUCTED IN URBAN AREAS WITH THE APPLICATION OF GROUTING\*\*\*

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### 1. Introduction

An analytical method for the design of a finite number of multiple circular shallow tunnel linings of different radii including the ones constructed with the application of grouting has been developed at Tula State University. The method allows the linings' stress state and bearing capacity to be evaluated under the actions of the soil own weight and the weight of buildings or structures both existing on the surface before the tunnels driving and erected nearly to the already existing tunnels. The method is based on the same approach as methods for calculations of a single circular [1, 2] and non-circular tunnel [3, 4] that is on consideration of tunnel linings and the surrounding soil mass as elements of a united deformable system and on analytical solutions of the corresponding plane problems of elasticity theory applying a special technique for approximate taking the 3D character of problems into account.

### 2. The design method

The general design scheme of plane problems of elasticity theory for a weighty semi-infinite linearly deformable medium simulating the soil mass weakened by an arbitrary number of arbitrarily located circular openings supported by double-layer rings (the external layers simulate zones of grouted soils, the internal ones simulate tunnel linings) is shown in Figure 1.

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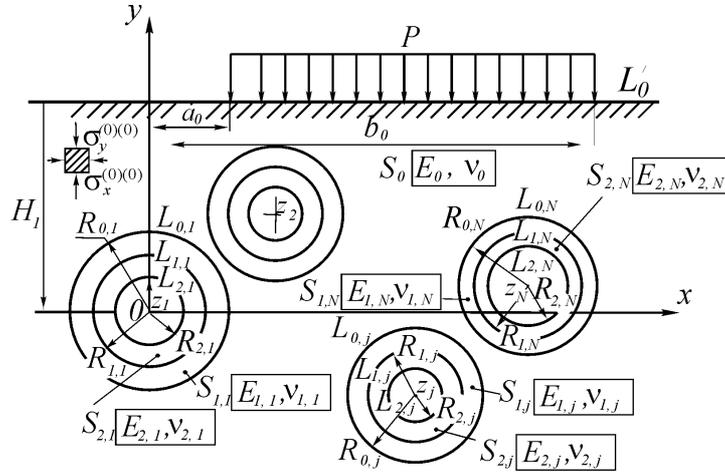


Fig. 1. The design scheme

The medium  $S_0$  and layers of rings  $S_{m,j}$  ( $m = 1, 2; j = 1, \dots, N$ ) undergo deformation together i.e. conditions of continuity of vectors of displacements and complete stresses are satisfied on the contact lines  $L_{m,j}$  ( $m = 0, 1; j = 1, \dots, N$ ). The internal outlines  $L_{2,j}$  ( $j = 1, \dots, N$ ) of the rings are free from loads.

The action of the soil own weight is simulated by a presence of initial stresses in the medium  $S_0$  and layers  $S_{1,j}$  ( $j = 1, \dots, N$ ) determined by formulae

$$\begin{aligned} \sigma_y^{(m)(0)} &= -\gamma(H_1 - y), \\ \sigma_x^{(m)(0)} &= -\lambda\gamma(H_1 - y) \end{aligned} \quad (1)$$

where:

- $\gamma$  — is the soil unit weight,
- $\lambda$  — is the lateral pressure coefficient in an intact soil mass,
- $H_1$  — is the depth of the tunnel the centre of which coincides with the centre of co-ordinates system.

The weight of a building or structure on the surface is simulated by a vertical load of the  $P$  intensity uniformly distributed along an arbitrary part of the semi-plane straight boundary  $L'_0$ .

There are two cases under consideration — when a new building is erected near the already existing tunnels and when the building was erected before the tunnels driving. In the latter case the ground displacements appearing before the tunnels construction are excluded from the corresponding boundary conditions.

The described problems of elasticity theory have been solved with the application of the complex variable analytic functions theory [5], apparatus of the analytical continuation of Kolosov-Muskhelishvili's complex potentials characterizing the lower semi-plane stress-strain state out of the openings restricted by circumstances  $L_{0j}$  ( $j = 1, \dots, N$ ), into the upper semi-plane [6] and the method of solving the problems for an infinite plane weakened by a finite number of circular openings supported by multi-layer rings [7]. Such an approach allows to reduce the solution of the problems considered to an iteration process of a good convergence [1], in every approximation of which the solutions of problems for every double-layer ring supporting a single opening in a whole plane are sequentially applied at the boundary conditions including some additional items reflecting the influence of the rest openings and the straight boundary of the semi-plane. These items are represented in the form of the Laurent series the unknown coefficients of which supposed to be equal to zero in the first approximation, are then specified on each step of iterations.

For approximate taking the influence of distances  $l_j$  ( $j = 1, \dots, N$ ) between the lining being constructed in every tunnel and the tunnel face into account the correcting coefficients are introduced into the results of the linings design under the actions of the soil own weight and the weight of the building existing on the surface before the tunnel driving. These coefficients are determined by empirical formulae obtained on the basis of numerical modeling the 3-D axisymmetric problem using the finite elements method [8]

$$\alpha_j^* = 0.6 \exp(-1.38l_j / R_{0,j}), (j = 1, \dots, N) \quad (2)$$

The influence of time-dependent soil or rock properties is taken into account on the basis of the linear hereditary creep theory applying the method of variable modules according to which deformation characteristics of soil kept in solutions of problems of elasticity theory are represented as time functions [9]. The influence of consequence of tunnels driving may be taken into account in the same way as described in the work [7].

But there is a serious problem connected with taking into account the 3-D character of loads caused by the weight of buildings due to their limited sizes in the direction along the tunnel axes and possible 3-D locations of several buildings with respect to the tunnels. In these cases it is necessary to determine the stress state of every lining in different tunnel cross-section along the axis. With this aim an approximate technique is proposed the validation and application of which for the design of multiple shallow circular tunnel linings constructed with soil grouting in build-up areas are described below.

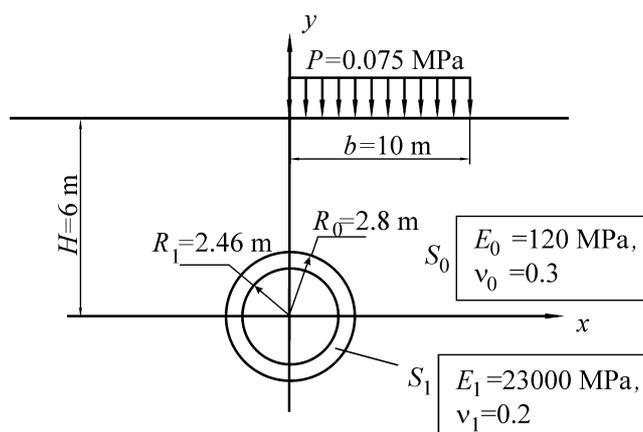
### 3. Validation of the approximate technique

The technique proposed for taking into account the 3-D character of problems concerning the action of the weight of buildings or structures on the surface, is based on the hypothesis that the relation  $k$  between stresses in the ring simulating the tunnel lining,

obtained from the solution of a 3-D problem to the ones given by a 2-D problem is approximately the same as a similar relation between vertical stresses appearing in the point of a solid half space (without tunnels) corresponding to the center of the tunnel cross-section considered caused by the surface load uniformly distributed on the rectangular square  $b \times l$  and on the infinite strip of the width  $b$ . Such a suggestion allows the stresses appearing in any lining cross-section along the axis of each tunnel to be defined introducing some correcting multipliers  $k$  into the results of the 2-D problem solution. The coefficient  $k$  for a certain tunnel cross-section may be determined in way described in the paper [10] using so called “method of corner points” usually applied for determining the stress distribution in the ground under foundations [11].

For validation of this hypothesis the results obtained by the approximate technique have been compared with the data from 3-D numerical modeling [10]. The results of comparison with the data from physical modeling on the equivalent materials carried out by D. Golitsynskiy, Yu. Frolov and V. Kavkazsiy (St.-Petersburg Transport University, Russia) are given below.

The design scheme and input data considered for comparison are shown in Figure 2.

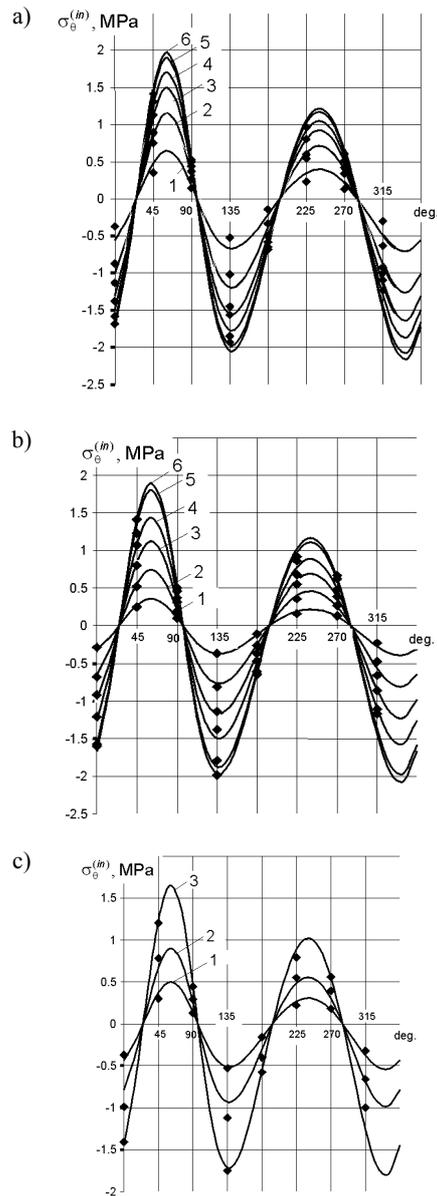


**Fig. 2.** The design scheme and input data considered for comparison

It is assumed that the building is erected nearby the already existing tunnel. The stresses were determined in the lining cross-sections 1, 2 and 3 located correspondingly under the centre of the building and on the distances 3.63 m and 8.51 m from the building centre.

The distributions of circumferential stresses appearing in the cross-section 1 along the lining internal outline obtained by an approximate approach are given in Figure 3 a by solid lines. Curves 1, 2, ..., 6 correspond to the building sizes  $l = 3, 6, 9, 12, 18$  and  $24$  m. The same stresses in the points  $\theta = 0, 45, 90, \dots, 315^\circ$  found from physical modeling are shown by black points. Identical results showing a satisfactory agreement between compared

stresses have been also obtained for the lining cross-sections 2 (Fig. 3 b) and 3 (Fig. 3 c). In the last case curves 1, 2, 3 correspond to the building sizes  $l = 12, 18$  and  $24$  m in the direction along the tunnel axis.



**Fig. 3.** Comparison of stresses in the lining cross-sections 1, 2, 3 obtained by an approximate technique with the ones found from physical modelling

The results of comparison given in Figures 3a, b, c as well as published in the paper [10] show that the proposed approximate technique may be applied for determining the tunnel linings stress state taking the 3-D character of surface loadings into account by introducing some correcting multipliers into the results of solutions of the corresponding 2-D problems determined in the way described above.

The approach proposed allows also a presence or erection of several buildings arbitrarily located with respect to the tunnels to be taken into account for determination of the linings stress state and evaluation of their safety factors in any cross-section along the tunnel axis.

The safety factor of the lining cross-section is determined by formula

$$k_s = \min \left( \frac{R_{bc}}{|\sigma_{\theta}^{(in)(c)}|_{\max}}, \frac{R_{bt}}{\sigma_{\theta\max}^{(in)(t)}} \right) \quad (3)$$

where:

- $R_{bc}, R_{bt}$  — resistances of concrete to the compression and tension,
- $|\sigma_{\theta}^{(in)(c)}|_{\max}, \sigma_{\theta\max}^{(in)(t)}$  — maximal compressive (negative) and tensile (positive) stresses on every lining internal outline caused by the joint action of the soil own weight and weight of buildings existed before the tunnel driving or erected nearby to the already constructed tunnel.

#### 4. Example of the design

Three tunnels constructed under urban area are considered. Sizes and locations of tunnels cross-sections are shown in Figure 4.

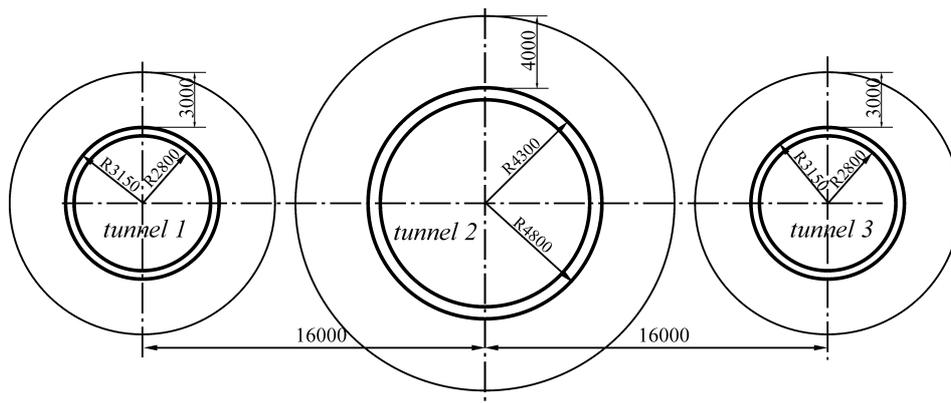
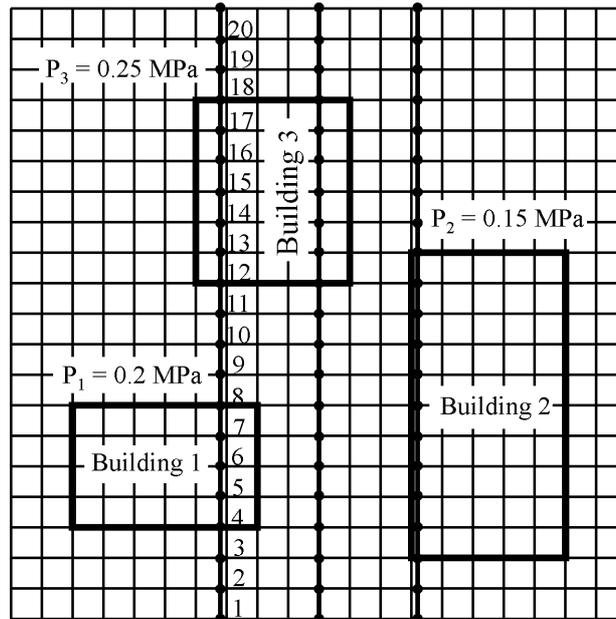


Fig. 4. The tunnel complex cross-section

The lining undergoes the action of the soil own weight and the influence of three buildings existed before the tunnels driving. The plan of building locations respectively to the tunnel axis is shown in Figure 5 (one grid side corresponds to 5 metres).



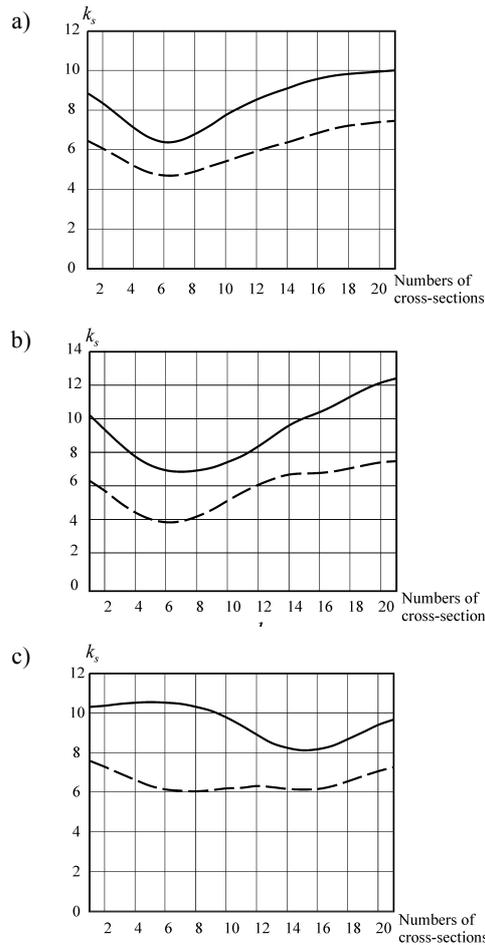
**Fig. 5.** Locations of the buildings

The input data for the design are the following:

- $H = 18$  m,
- $\lambda = 0.54$ ,
- $\gamma = 0.018$  MN/m<sup>3</sup>,
- $E_0 = 1000$  MPa,
- $\nu_0 = 0.35$ ,
- $E_{1,1} = E_{2,1} = E_{3,1} = 3000$  MPa,
- $\nu_{1,1} = \nu_{2,1} = \nu_{3,1} = 0.35$ ,
- $E_{1,2} = E_{2,2} = E_{3,2} = 23,000$  MPa,
- $\nu_{1,2} = \nu_{2,2} = \nu_{3,2} = 0.2$ ,
- $R_{bc} = 8.5$  MPa,
- $R_{bt} = 0.75$  MPa,

- $l_1 = l_3 = 1 \text{ m}$ ,
- $l_2 = 1.5 \text{ m}$ .

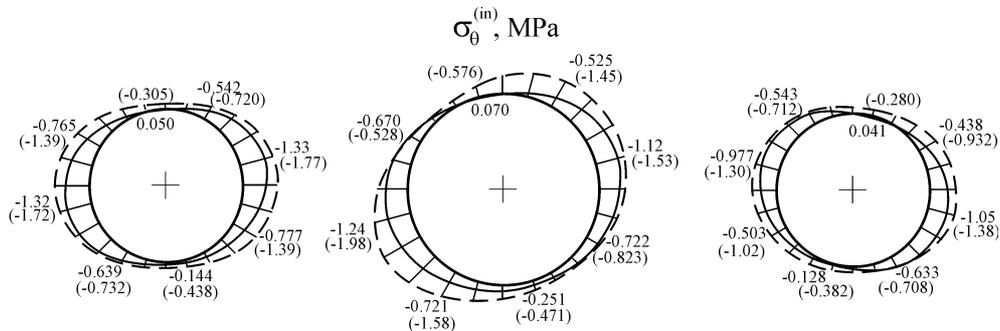
Distribution of safety factor  $k_s$  along the axes of tunnels 1, 2, 3 are shown in Figure 6a, b, c correspondingly. Dash lines correspond to the case when grouting is not used.



**Fig. 6.** Distribution of safety factor  $k_s$  along axes of tunnel 1 (a), tunnel 2 (b), tunnel 3 (c)

One can see from Figure 6, that applying of grouting increases the minimal lining safety factors from 4.70 to 6.45 (tunnel 1), from 3.84 to 6.86 (tunnel 2), from 6.06 to 8.11 (tunnel 3).

Distributions of circumferential stresses (in MPa) appearing in points of the internal lining outlines in the most dangerous cross-sections 6 for tunnel 1, 7 for tunnel 2, 16 for tunnel 3 are presented in Figure 7. Dash lines show stresses corresponding to the case when linings are constructed without grouting.



**Fig. 7.** Distributions of circumferential stresses in most dangerous cross-sections

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