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## Methodology of optimisation of local energy infrastructure development

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### 1. Introduction

For many years economists believed that the best tool to optimize allocation of resources is the market itself. In the 20<sup>th</sup> century however, along with progressing devastation of the natural environment, scientists discovered that in some cases the market alone does not lead us to socially optimal solutions. One of the fields where the free market may and should be rectified is the energy market. Among basic reasons of this state is the existence in energy sector of the problem of external costs and natural monopolies. In Poland in accordance with the “Energy law” the bodies responsible for the creation and enforcement of energy policy are: the government, Energy Regulation Office and local (municipal) authorities. One of the basic means of fulfilling this obligation by municipalities are “Plans of supplying local consumers in heat, electricity and gas” [7]. Those plans (to update at least every 3 years) should draw the main axes of local energy systems development in the way which is coherent with socially optimal solutions. Their importance results from the fact that they should influence other social actors i.e. energy companies and energy consumers, which peruse their own interests. The reality is that the majority of municipalities do not prepare such strategic plans [22], and that these plans that exist are too general to influence the market actor’s behavior [28]. One of the basic reasons of this situation is not applying by local authorities models optimizing local energy systems development. To fulfill such task model should have the following features:

- municipality oriented and comprehensible by local administration,
- consider various forms of final energy,

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- consider consumers, both connected and not connected to energy networks (electricity, heat, gas), including for example consumers of heat with individual boilers,
- ignore details which may locally exist but are out of local authorities control (for example technologies and fuels applied in a big condensing power plants, locally existing but owned by large corporations),
- represent in a detailed way local reality (incl. existing infrastructure, variety of local energy resources, etc.),
- concentrate on social (not private) optimum,
- represent typical behaviors of consumers, parameters of technologies, environmental conditions and existing energy networks features.

This paper presents a methodology of construction of a mathematical model which seems to fulfill all these requirements. The model may be a good starting point to create a software tool which can be used by local authorities in practice.

## 2. Optimization models of local energy systems

In the last three decades a big number of energy models appeared. A fraction of them may be to some degree useful for local authorities although most of them have serious limitations and disadvantages which reduce their capacity to create strategic plans of municipal energy infrastructure development. In this chapter some examples of existing models of local energy systems are presented.

One of the most popular energy models which may be used in the local scale (although it's geographical scope is universal - including region and country) is *Perseus* [5, 11, 14, 27]. It is a dynamic, linear, optimization model which selects the best technologies and fuels to meet the given demand for various energy forms. It represents all phases of energy transformation (extraction and transportation of fuels, energy production, energy transmission and distribution). The objective function of the model is the sum of discounted, yearly costs of the following factors: fuel (extraction, transport), technologies (variable and fixed), investments, transmission and distribution. Constraints of the model may be divided into three groups: technical, environmental and socio-political. The model may be particularly useful in optimizing technologies and fuels of power and heating plants thus seems more useful to energy companies than to local authorities - less interested in technologies of big energy producers and less conscious of them.

A well-known tool for support of local energy systems development, with many application is *Homer* [13, 19]. It has been built by the National Energy Laboratory (USA) and commercial companies. The model (recommended by the World Bank) shows the various effects of the application of different combinations

of fuels and technologies. The main assumption of the model is the balance between energy production and energy demand in each of the 8760 hours of the year. Package is free and accessible in the internet ([www.nrel.gov/homer](http://www.nrel.gov/homer)).

The tools which may support municipal authorities in elaborating or updating long-term plans of meeting local energy needs are *CAPLEP* [14] (Laboratory of Energy Models of Polytechnic University of Torino) and *MARTES* (Goeteborg – Sweden). The main limitation of both models is concentrating exclusively on district heating while neglecting other forms of final energy and residents not connected to the heat network).

Another model which may be helpful to local administration is *MODEST* (*Model for Optimization of Dynamic Energy System with Time-dependent* components [11, 12]. *MODEST* is a linear programming model that minimizes the costs of supplying heat and electricity during the analyzed period. It was used to optimize local energy systems development in several municipalities.

Long-run local energy systems designing may also be supported by *MARKAL* or its successor – *TIMES*. These well-known and frequently applied energy models have a universal geographical range (country, region, local) [21]. Both tools (although useful for energy companies) have some disadvantages from the point of view of local authorities and of the creation of municipal strategic energy plans which are: concentration on network infrastructure (neglecting for example heat for individual-dispersed housing) and considering issues which are out of control of local authorities (big power plants, electricity distribution technical problems, etc.).

An interesting tool which may also be used to plan the development of local energy systems was created in 2008 by Ritsumeikan Global Innovation Research Organisation, Ritsumeikan University (Kioto, Japan) and Faculty of Environmental Engineering of Kitayushu University [24, 25, 26]. It has a MILP form (*Mixed Integer Linear Programming*) and is static (one year period).

The objective function is the sum of the following factors:

- fuel costs,
- investments costs,
- fixed exploitation costs,
- variable exploitation costs.

Restrictions of the model are technological and demand side aspects. The tool considers local climate, tariffs for energy (heat and electricity), technological and economic data of technologies and the demand for energy (divided into seasons and hours). The model selects (for the given local energy system) the cheapest set of technologies and the best way of their use. It has been applied to optimize energy supply to Kitakyushu University Campus. The tool can be applied in case of smaller (then municipality) areas.

Beside the optimization models, also multi-criteria analysis are applied to support the planning of local energy systems development. Although it do not show one best solution for the formulated problem, the multi-criteria approach allows us to see many aspects (of both quality and quantity character) of potential decisions. This methodology leads us to a set of Pareto optimal states. An example of the application of this approach in the process of designing of local energy systems is created in 2006 (by the Electrotechnic Faculty of Rzeszów Polytechnic School) a software tool dedicated to a small municipal client (a house, a block of apartments or a housing district) [2, 3]. The software package considers the costs of supplying energy, emission level, the comfort of the user and the reliability of the system. The tool, although very useful in the case of planning of energy infrastructure development of a district, covers too small an area to be applied in the process of a complex, long-term energy strategy formulation for the whole municipality.

In the recent years many interesting models of local energy systems have also been created in the developing countries. One of them is a linear, optimization model *IRES (Integrated Renewable Energy System)* [1, 6]. It has been created and used in India. The objective function in the model is the cost of meeting local demand for energy (electricity, space heating, heat for preparing meals). *IRES* is dedicated to a village or a group of villages not connected to the power grid or the gas network areas of the third world countries. The model assumes only local, renewable fuels. Although it is a useful tool in the case of peripheral areas of developing countries, *IRES's* algorithms are not adequate to the Central European reality. Another model to optimize development of local energy systems is *DGEP (Distributed Generation Expansion Planning)* [29]. This, created in Iran, tool has a multi-criteria character (a few objective functions – including maximization of costs and minimization of CO<sub>2</sub> emissions). Modeling of local energy systems has also recently become a focus of scientists from China. One of a few examples of this interest may be *ICS-EM (Inexact Community Scale Energy Model)* [4], created by the School of Environment of Beijing Normal University in co-operation with two Canadian Universities (Regina and Waterloo). The tool is a *MIP (mixed integer programming)* optimization model with objective function equal to costs of supplying amounts of energy given in all sub-periods of the given period. The model's advantage is its dynamic character and considering the existing local technologies. However it does not reflect central European user's behavior.

The short review of examples of models of energy systems shows that there are available tools which may support local authorities in the process of strategic planning of local energy infrastructure development. Still, there is a need of creation of an optimizing tool without the following limitations and disadvantages

- limiting the range of the modeled system to the network infrastructure (neglecting for example heat boilers of individual houses),

- considering only one type of final energy,
- universal range of the tool (not only local but also region or a country), which results in not enough detailed representation of local specificity and considering the issue of control and comprehension of local administration (for example locally existing power plants),
- the lack of increasing marginal costs impact of local – renewable fuel, which affects increasing quantities of energy produced,
- cost-benefit analysis seen from private (not social) perspective,
- algorithms not adequate to Central European reality and the Central European user’s behavior.

### **3. A general concept and assumptions of the model**

The following part of this article presents a methodology and mathematical representation of an optimization model of local energy system development which is dedicated to local administration and may be used to formulate plans of supplying local residents in electricity, heat and gas. This non-linear, dynamic model allows us to find the cheapest (socially) way of supplying local residents in the given amount of final energy. The objective function (minimized) is the sum of discounted yearly costs of energy supply, each consisting of the following factors:

- fuel costs,
- variable exploitation costs (beside fuel),
- investment costs,
- fixed exploitation costs (beside depreciation and investments),
- external environmental costs.

Key decision variables are: installed powers of chosen technologies (in each year of the examined period), quantities of energy produced by these technologies to supply the demand of all customer segments and quantities of fuel received from each source. These are the most critical and determining factors of long term strategic energy plans for municipalities so their calculation will make these plans concrete and precise.

Social costs (benefits) differ significantly from private costs (considered usually in energy companies oriented models). While in the case of private perspective we consider costs and benefits of energy supplier, social perspective concentrates on costs and benefits of the whole society, neglecting transfers between different market actors (for instance income from various “green” certificates and subsidies) and in the same time considering all external costs (at present only partially internalized and suffered by energy companies). Such a methodological

approach – so called socio-economic perspective [12] (which is an alternative to a business economic perspective) results in the three following features of the described in the article model:

- considering the external costs of energy production and transmission to the full extent (regardless current regulations),
- neglecting the effects of all (current or future) intervention tools aiming at rectifying market actors behavior (transfers between different actors, neutral from the point of view of the whole society – for instance revenues from the sale of green certificates),
- neglecting profit margins (super normal profits) of local fuel suppliers [15].

Inclusion of the full external costs in the objective function is a very important feature of the proposed model. This approach allows us to find solutions improving social welfare [18]. When external costs are not considered it may lead to sub-optimal (from socio-economic prospective) solutions [12].

Exogenic (given) data which are introduced to the model by the user are the following:

- existing energy infrastructure (capacity of each technology and the expected depreciation of each technology in every year of the examined period),
- demand for electricity and thermal energy (total demand needed for heating, hot-water production, technology purposes and cooking) – expected for all years and all sub-periods of each year,
- potential and costs of local energy resources,
- costs of global energy carriers (electricity from the power grid, coal, etc.),
- technical and economic data of considered technologies,
- compatibility of some fuels with some technologies and of some technologies with some demand sectors.

The existing infrastructure and expected demand for final energy are introduced separately for all sectors of the municipality (declared by the user). Sectors are defined in such a way that:

- each sector is homogenic with respect to the set of technologies which may be used to meet the demand for energy (two customers with the same set of compatible technologies belong to the same sector, two customers with different sets of compatible technologies belong to different sectors),
- sectors are separate and complementary (each customer belongs to one and only one sector).

A very important advantage of the model is the variability of marginal costs of local-renewable fuels. The more local-renewable fuel we use the less attractive sources of this fuel we must exploit. In the traditional approach the user decides which sources are still attractive and which are already unattractive to exploit. Then the total potential of all attractive resources (in the user's opinion) is exogenously given to the model. Also, the unit price (or cost) of the fuel (which actu-

ally increases with the increase of the scale of the local fuel usage) is exogenously given by the user and usually equal to the average price of considered resources. In the presented model we introduce the whole function linking the unit fuel cost (dependent variable) and the fuel quantity (independent variable).

This relation consists of a set of pairs of figures  $\{p_i, k_i\}$ ,  $i = 1, 2, \dots, n$ , where:

$i$  – number of the resource,

$p_i$  – potential of the resource „ $i$ ”,

$k_i$  – unit cost of the fuel from the resource „ $i$ ”,

$n$  – number of locations.

Then the model decides to what point existing renewable fuels should be utilized – not the user in an arbitrary way.

The following assumptions and simplifications have been made:

- unit external costs of energy production and distribution are fixed (not dependent on the quantity of energy produced neither on the technology – for example not related to the height of the stack);
- the quantity of global fuels used by the municipality (for instance gas, coal) is small and does not change unit prices on the global (or national) market;
- new capacity is introduced to the system on the first hour of every year (capacity of each technology during the chosen year is constant);
- investment costs of already installed technologies are neglected (*sunk costs*);
- unit investment costs and unit exploitation costs do not depend on the capacity nor the quantity of energy produced (in reality they usually decrease with the scale);
- economic and technical parameters of each technology are constant during the whole examined period;
- there are two types of sectors:
  - one user – one installation (for example small boilers in individual houses),
  - many energy installations whose outputs are added together – many users profiting from the same distribution network (where outputs of different installations are added together);
- in case of the one installation – one user sector the quantity of energy produced in each technology is proportional to the capacity of this technology; inexistence of this assumption would lead us to the impossible state i.e. meeting the demand of the sector (beside peak hours) only by those technologies whose variable costs are low (impossible because houses equipped in energy technologies with higher variable costs cannot import energy produced elsewhere);
- the whole year is divided into sub-periods reflecting both seasonal and daily variations of demand; for instance four seasons (spring, summer, autumn, and winter) and two daily periods (peak, out of peak) gives us eight sub-periods of the year;

- capacity (installed power) of energy production technologies should be high enough in every year to supply local clients with demanded quantities of energy (in every sub-period of every year);
- in case of houses equipped in boilers supplied with solid fuels (for example wood, coal, etc.) heat for hot water in summer periods may only be produced from electricity or sun energy (assumption reflecting Central European customer behavior – closing solid fuel installations in summer period);
- costs and prices are constant during the whole examined period and equal to prices existing in the first year.

#### 4. Mathematical formulation

Symbols used in the model are presented in the Table 1.

**Table 1**  
Symbols used in the model

symbol	type of represented value	description
$s$	index	sectors of demand
$f$	index	sources of fuel
$t$	index	years
$i$	index	sub-period of the year
$g$	index	technologies considered
$z$	index	pollution type
$S$	set	set of all demand sectors
$F$	set	set of all fuel sources
$T$	set	set of all years
$I$	set	set of all sub-periods of the year
$I_{summer}$	set	sub-set of set I assembling all those and only those sub-periods of the year when heat is produced only for hot water
$I_{winter}$	set	sub-set of set I assembling all those and only those sub-periods of the year when heat is produced both for hot water and for space heating
$G$	set	set of all technologies considered (potentially proper for some sectors of the municipality)

**Table 1 cont.**

$GIE$	set	one element sub-set of set $G$ – hot water electric boilers
$GIS$	set	one element sub-set of set $G$ – hot water solar panels
$G2$	set	sub-set of set $G$ , including gas or solid fuel fired boilers
$G3$	set	sub-set of set $G$ , including solid fuel boilers
$Z$	set	set of all pollutants
$F \times G$	set of pairs	set of all pairs of fuel source – technology
$A$ $A \subset F \times G$	sub-set	sub-set of set $F \times G$ , including all those and only those pairs of fuel sources and technologies which are compatible
$G \times S$	set of pairs	set of all pairs technology - sector of demand
$B \subset G \times S$	sub-set	sub-set of $G \times S$ , including all those and only those pairs of technologies and sectors which are compatible
$E_g$	parameter	efficiency of transforming chemical energy stored in fuel into final energy (given for technology $g$ )
$COGEN_g$	parameter	co-generation coefficient – ratio equal to electrical energy divided by total energy produced (given for technology $g$ )
$MAX\_WORK_{g,i}$	parameter	maximal number of working hours in sub-period $i$ of the year (given for technology $g$ )
$POWER\_RES_{g,s,t}$	parameter	residual power of technology $g$ , in sector $s$ , in year $t$
$C\_INV_g$	parameter	unit investment costs (for technology $g$ )
$C\_FIX\_EL_g$	parameter	unit fixed costs of producing electrical energy (given for technology $g$ )
$C\_VAR\_EL_g$	parameter	unit variable costs of producing electrical energy (given for technology $g$ )
$C\_FIX\_H_g$	parameter	unit fixed costs of producing heat (given for technology $g$ )
$C\_VAR\_H_g$	parameter	unit variable costs of producing heat (given for technology $g$ )
$UNIT\_EMI_{z,g}$	parameter	unit emission of pollution $z$ by technology $g$
$UNIT\_EXT_z$	parameter	unit external cost of pollution $z$

Table 1 cont.

symbol	type of represented value	description
$D\_EN\_EL_{t,i}$	parameter	demand of the municipality for electrical energy in year $t$ , in sub-period $i$ (beside electricity for hot water production which is calculated by the model)
$D\_EN\_TH_{s,t,i}$	parameter	demand for thermal energy (sector $s$ , year $t$ , sub-period $i$ )
$DYSK_t$	parameter	discounting factor in the year $t$
$R_t$	parameter	discounting rate in the year $t$
$CRF_g$	parameter	capital recovery factor for technology $g$
$LT_g$	parameter	life period of technology $g$ (expressed in years)
$PRICE_{ft}$	parameter	unit price of fuel from source $f$ in the year $t$
$POT_{ft}$	parameter	yearly potential (capacity) of source of fuel $f$ in the year $t$
$PRICE\_IMP_t$	parameter	unit cost of electrical energy from power grid (including T&D fees)
$PRICE\_EXP_t$	parameter	unit price received for supplying power grid in electrical energy surplus
$COEF\_SOL\_WIN\_SUM$	parameter	coefficient of the fall of solar panels capacity in winter (ratio equal to capacity in winter / capacity in summer)
$EN\_POWER_s$	parameter	ratio equal to energy produced / power installed (received from historic data)
$cost$	objective function	total social cost of supplying municipality in final energy
$cost\_prod_t$	variable	cost of production of electricity and heat in the year $t$
$cost\_ext_t$	variable	external cost of energy production in the year $t$
$cost\_inv_t$	variable	investment cost in the year $t$
$cost\_fix_t$	variable	fixed costs of energy production in the year $t$
$cost\_fix\_el_t$	variable	fixed costs of electrical energy production in the year $t$
$cost\_fix\_th_t$	variable	fixed costs of thermal energy production in the year $t$

**Table 1 cont.**

$cost\_var_t$	variable	variable costs in the year $t$
$cost\_var\_el_t$	variable	variable costs of electrical energy production in the year $t$
$cost\_var\_th_t$	variable	variable costs of thermal energy production in year the $t$
$cost\_fuel_t$	variable	fuel costs in the year $t$
$expo_{t,i}$	variable	surplus of electrical energy produced in the municipality in the sub-period $i$ of the year $t$ (transferred to the power grid)
$impo_{t,i}$	variable	electrical energy deficit - covered by the power grid in the sub-period $i$ of the year $t$
$b\_exp_t$	variable	benefits from supplying power grid in surplus of energy in the year $t$
$c\_imp_t$	variable	costs of using electrical energy from the power grid in the year $t$
$pow_{g,s,t}$	variable	power (capacity) of technology $g$ , in the sector $s$ , in the year $t$
$new\_pow_{g,s,t}$	variable	power of technology $g$ in the sector $s$ , built between the year 1 and the last year (including those years)
$produ\_en\_el_{g,s,t,i}$	variable	electrical energy produced by technology $g$ , in the sector $s$ , in the year $t$ , in the sub-period $i$
$produ\_en\_th_{g,s,t,i}$	variable	thermal energy produced by technology $g$ , in the sector $s$ , in the year $t$ , in the sub-period $i$
$produ\_tot\_en\_cie_{s,t,i}$	variable	total thermal energy produced in in the sector $s$ , in the year $t$ , in the sub-period $i$ (all technologies)
$q\_fuel_{f,g,t}$	variable	quantity of fuel from the source $f$ to technology $g$ in the year $t$
$emi_{z,g,t}$	variable	emission of pollution $z$ , by technology $g$ in the year $t$
$tot\_power\_th\_g2_{s,t}$	variable	total power of all technologies of gas or solid fuels, in the sector $s$ , in the year $t$
$produ\_g2\_en\_th_{s,t,i}$	variable	production of heat by all technologies of gas or solid fuels, in the sector $s$ , in the year $t$ , in the sub-period $i$
$sol\_tot\_summer_{s,t}$	variable	quantity of heat produced by solar panels in the sector $s$ , in the year $t$

Objective function (minimization) has the following form:

$$cost = \sum_{t \in T} DYSK_t \times (cost\_prod_t + c\_imp_t - b\_exp_t + cost\_ext_t)$$

Yearly costs of production of energy are defined as the sum of investment costs, fixed costs, variable costs and fuel costs, which can be described in the following manner:

$$\forall_{t \in T} cost\_prod_t = cost\_inv_t + cost\_var_t + cost\_fix_t + cost\_fuel_t$$

### Yearly costs of investment

Yearly costs of investment are calculated in the following way:

$$\forall_{t \in T} cost\_inv_t = \sum_{s \in S} \sum_{g \in G} CRF_{g,t} \times C\_INV_g \times new\_pow_{g,s,t}$$

where:

$$\forall_{g \in G, s \in S, t \in T} new\_pow_{g,s,t} = pow_{g,s,t} - POWER\_RES_{g,s,t}$$

and:

$$\forall_{g \in G, t \in T} CRF_{g,t} = \frac{R_t}{1 - (1 + R_t)^{-LTg}}$$

### Yearly variable costs

Yearly variable costs are the sum of yearly, variable electricity production costs and yearly, variable heat production costs. This can be written as follows:

$$\forall_{t \in T} cost\_var_t = cost\_var\_el_t + cost\_var\_th_t$$

Yearly, variable costs of electricity production and yearly, variable costs of heat production are given by the following equations:

$$\forall_{t \in T} cost\_var\_el_t = \sum_{g \in G} \sum_{s \in S} \sum_{s \in I} prod\_en\_el_{g,s,t,i} \times C\_VAR\_EL_g$$

oraz:

$$\forall_{t \in T} cost\_var\_th_t = \sum_{g \in G} \sum_{s \in S} \sum_{s \in I} prod\_en\_th_{g,s,t,i} \times C\_VAR\_TH_g$$

### Yearly fixed costs

Yearly fixed costs are the sum of yearly, fixed electricity production costs and yearly, fixed heat production costs. This can be written as follows:

$$\forall_{t \in T} cost\_fix_t = cost\_fix\_el_t + cost\_fix\_th_t$$

In the same time yearly, fixed costs of electricity production and yearly, fixed costs of heat production are given by the following equations:

$$\forall_{t \in T} \text{cost\_fix\_th}_t = \sum_{g \in G} \sum_{s \in S} C\_FIX\_TH_g \times pow_{g,s,t}$$

oraz:

$$\forall_{t \in T} \text{cost\_fix\_el}_t = \sum_{g \in G} \sum_{s \in S} C\_FIX\_EL_g \times pow_{g,s,t}$$

### Yearly fuel costs

Yearly fuel costs are equal to the sum of products of prices and yearly quantities of each fuel. This can be written as follows:

$$\forall_{t \in T} \text{cost\_fuel}_t = \sum_{g \in G} \sum_{f \in F} q\_fuel_{f,g,t} \times PRICE_{d,t}$$

### Yearly external costs

Yearly external costs are defined in the model as a sum of products of quantities of pollutants which are emitted during the production of energy and unit external costs.

This can be submitted in the following way:

$$\forall_{t \in T} \text{cost\_ext}_t = \sum_{z \in Z} \sum_{g \in G} emi_{z,g,t} \times C\_EXT_z$$

where:

$$\forall_{z \in Z} \forall_{g \in G} \forall_{t \in T} emi_{z,g,t} = \sum_{f \in F} q\_fuel_{f,g,t} \times UNIT\_EMI_{z,g}$$

### Costs of consuming electrical energy

Costs of consuming electrical energy produced outside the municipality and supplied by the power grid, as well as benefits from supplying power grid in electrical energy surpluses (produced in the municipality and not consumed by municipal consumers) can be calculated in the following way:

$$\forall_{t \in T} c\_imp_t = \sum_{i \in I} imp_{t,i} \times PRICE\_IMP_t$$

$$\forall_{t \in T} b\_exp_t = \sum_{i \in I} exp_{t,i} \times PRICE\_EXP_t$$

Constraints of the model are the following:

- yearly production of a fuel – not higher than the potential of the source;
- balances between primary and final energy fluxes;
- balance of flows of electrical energy (production, consumption, transmission to or from the power grid);
- meeting the demand for heat;
- balance between capacities of technologies and energy production by these technologies;
- constant ratio between electrical and thermal energy for each technology (constant co-generation co-efficient);

- zero output in summer, in the case of solid fuel technologies, used by individual, dispersed residents (individual consumers equipped in small solid fuel boilers turn these boilers off in summer period and produce hot water only from the sun energy or electricity);
- zero output in spring, autumn and winter in the case of electrical boilers producing hot water;
- seasonal variation of power of solar panels (for the rest of the technologies installed power is constant throughout the year);
- compatibility of some technologies and some sources of fuel while incompatibility of others;
- compatibility of some technologies and some sectors, while not compatibility of others;
- equality of the two following variables (in the case of sectors with individual, dispersed boilers):
  - share of thermal energy produced by a technology in the total thermal energy production of the sector,
  - share of the installed thermal power of this technology in the total installed power of all technologies installed in the sector.

The equation representing the relation between capacity of fuel sources and fuel production is the following:

$$\forall_{f \in F} \forall_{t \in T} \sum_{g \in G} q_{fuel_{f,g,t}} \leq POT_{d,t}$$

The balance between primary energy of fuels consumed and the energy produced is the following:

$$\forall_{g \in G} \forall_{t \in T} E_g \times \sum_f q_{fuel_{f,g,t}} = \sum_{s \in S} \sum_{i \in I} (prod\_el_{g,s,t,i} + prod\_th_{g,s,t,i})$$

The balance of electrical energy is described by the following formula:

$$\forall_{g \in G} \forall_{s \in S} \forall_{t \in T} \forall_{i \in I} D\_EN\_EL_{g,s,t,i} = prod\_el_{g,s,t,i} + imp_{g,s,t,i} - exp_{g,s,t,i}$$

The condition of satisfying the demand for heat is the following:

$$\forall_{g \in G} \forall_{s \in S} \forall_{t \in T} \forall_{i \in I} \sum_{g \in G} prod\_en\_th_{g,s,t,i} \geq D\_EN\_TH_{s,t,i}$$

The balance of power installed is the following:

$$\begin{aligned} & \forall_{g \in G} \forall_{s \in S} \forall_{t \in T} \forall_{i \in I} \\ & prod\_en\_el_{g,s,t,i} \leq pow_{g,s,t} \times MAX\_WORK_{g,i} \times COGEN_g, \\ & prod\_en\_th_{g,s,t,i} \leq pow_{g,s,t} \times MAX\_WORK_{g,i} \times (1 - COGEN_g) \end{aligned}$$

and

$$\forall_{s \in S} \forall_{t \in T} \left( \sum_g pow_{g,s,t} \times (1 - COGEN_g) \right) \times EN\_POWER_s \leq \sum_i D\_EN\_TH_{s,t,i}$$

The constraint resulting from constant quotient - electrical energy / thermal energy produced by each technology in every period of every year is as follows:

$$\forall_{g \in G} \forall_{s \in S} \forall_{t \in T} \forall_{i \in I} \\ produ\_en\_el_{g,s,t,i} \times (1 - COGEN_g) = produ\_en\_th_{g,s,t,i} \times COGEN_g$$

Production of hot water by small, individual boilers supplied with solid fuels in summer is as follows:

$$\forall_{g \in G3} \forall_{s \in S} \forall_{t \in T} \forall_{i \in Isummer} produ\_en\_th_{g,s,t,i} = 0$$

Production of hot water by electric boilers in autumn, spring and winter is the following:

$$\forall_{g \in G1E} \forall_{s \in S} \forall_{t \in T} \forall_{i \in Iwinter} produ\_en\_th_{g,s,t,i} = 0$$

Decreasing power of solar panels in the winter period is as follows:

$$\forall_{g \in G1S} \forall_{s \in S} \forall_{t \in T} \forall_{i \in Iwinter} \\ produ\_en\_th_{g,s,t,i} = COEF\_SOL\_WIN\_SUM \times MAX\_WORK_{g,i} \\ \times \frac{\sum_{i \in Isummer} produ\_en\_th_{g,s,t,i}}{\sum_{i \in Isummer} MAX\_WORK_{g,i}}$$

Each fuel source in the model is related to some technologies. This relation (compatibility of some fuel sources with some technologies) can be represented by logic or arithmetic relations. In the model arithmetic way is used. The constraint is represented by declaring fuel transfers between these fuel sources and these technologies which are not compatible as equal to zero. It is written as follows:

$$\forall_{t \in T} \forall_{(f,g) \in \{F \times G\} \setminus A} q\_fuel_{f,g,t} = 0$$

Each technology in the model is related to some demand sectors. This relation can be represented by logic or arithmetic relations. In the model arithmetic way is used. The constraint is represented by declaring the power of some technologies in some sectors as equal to zero (when they are not compatible). It is written as follows:

$$\forall_{t \in T} \forall_{(g,s) \in \{G \times S\} \setminus B} power_{g,s,t} = 0$$

Proportionality of thermal energy produced by a technology to the contribution of the power of this technology to the total installed power of the sector is described below. It concerns sectors of consumers with individual boilers and only those periods when thermal energy is used both for heating and for hot water production (not summer). The constraint has the following form:

$$\forall_{g \in G2} \forall_{s \in S} \forall_{t \in T} \forall_{i \in Iwinter}$$

$$\frac{\text{produ\_en\_th}_{g,s,t,i}}{\sum_g \text{produ\_en\_th}_{g,s,t,i}} = \frac{\text{pow}_{g,s,t}}{\sum_g \text{pow}_{g,s,t}} \times (1 - \text{COGEN}_g)$$

## 5. Conclusions

Existing models for the optimization of development of local energy systems have numerous disadvantages which limit their usefulness in the process of designing a strategy of meeting local energy needs by local administration (which is required by Polish Energy Law). In this paper a methodology of construction of a dynamic, non-linear model for optimization of local energy system is presented. The methodology leads us to the construction of a mathematical model which may be a good starting point to develop a simple software package well suited for local authorities. The model is a tool which combines various types of final energy and consumers - both connected and not connected to the power grid or the district heating system). The model considers local conditions in a very detailed way (local resources, residual infrastructure, local - sectorial demand for energy). The decision variables reflect only those factors which may be influenced by local administration, while all factors beyond the local authorities are given (exogenic). The model reflects both private and external costs and is designed to find socially (not privately) optimal technologies and fuels (for each sector of the given municipality). It corresponds with the Central European environment (customer behavior, grid infrastructure, etc.). The tool, when combined with a proper software (for example *General Algebraic Modeling System*), may constitute a simple and user-friendly package dedicated to municipal administration and can be helpful in the process of preparing plans for supplying a local community in final energy. It will be able to find and show exactly the optimal evolution of the existing (residual) energy infrastructure. The results of the calculations executed by such a mathematical software tool will include the power and energy production of all considered technologies (year by year), optimal fuels and yearly emissions of pollutants.

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