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DISSIPATION OF WAVES ENERGY IN CABLES BY OPTIMAL DAMPER FORCE

SUMMARY

In this paper the problem of optimal damping force necessary to suppress the wave traveling along the cable is considered. The energy dissipated by the damper force is assumed as the objective function of the considered problem. Optimal concentrated force and optimal distributed force of damper are determined. Equations were solved using the methods of Laplace transform and numerical integration. Results are illustrated by numerical examples.

Keywords: *cable vibration, optimal damping*

OPTYMALNE ROZPRASZANIE ENERGII FAL BIEGNĄCYCH W LINACH

W pracy przedstawiono teoretyczne rozważania dotyczące optymalnego tłumika eliminującego fale biegnące wzdłuż liny. Jako funkcję celu przyjęto energię rozpraszaną przez tłumik. Wyznaczono optymalne siły tłumika w przypadku oddziaływania na linę siłą skupioną oraz obciążeniem ciągłym. Równania rozwiązano metodami analitycznymi, wykorzystując przekształcenie Laplace'a, oraz metodami numerycznymi. Wyniki rozważań zostały przedstawione w postaci wykresów.

Słowa kluczowe: *drgania lin, optymalne tłumienie*

1. INTRODUCTION

Cables are very often used in many engineering structures placed in the natural environment. They are very efficient in cable-stayed bridges and mast supporting. Above all cables are used in overhead transmission lines [8].

A number of cables used in overhead transmission lines and cable-stayed bridges have exhibited large vibrations. They are the main reason of many fatigue failures.

Vibrations and wave motion are usually the result of vortex shedding, galloping, rivulet formation and buffeting. Introduction of exterior cable surface modification can eliminate most of the rain and wind vibration problems. However, longer cables are still vulnerable to the aerodynamic disturbances due to their low intrinsic damping.

The types of oscillation depend on: blow-up angle of wind, initial angle, turbulence intensity, surface roughness, diversities of the wind field, cable length, mass damping ratio. The loading process within a span is often only imprecisely known through interpretation of displacement response.

Various countermeasures are proposed to protect cables against vibrations and wave motion. They can be arranged into following groups of methods:

- The methods of modifying of the cables surface by using the elements preventing from aerodynamic forces induced by the air flow.
- The methods of the wave energy dissipation by increasing the cable internal damping or by using the special dampers, damping loops and spacers with the dissipating elements. Results of researches are presented in [2–4]. Prototypes of the new dampers to be applied mainly in cable-stayed bridges and overhead transmission lines are being tested in laboratories.

The forced motion of the cables is usually analysed as a superposition of the modes [1]. This approach is very efficient for describing the response of short cables. For very long cables and high frequencies the wavelength is small relative to the cable length and the time that the waves take to pass along the cable is relatively long. In this case the motion of the cable (especially the transient response) is better and more naturally described using the superposition law of travelling waves [5–7, 9]. In some works the equations of motion are solved using numerical methods.

In this study the problem of optimal damping force necessary to suppress the wave travelling along the cable is considered. The effectiveness of external damper is investigated by analytical and numerical methods. The concentrated force and distributed force are taken into considerations. The energy dissipated by damper force is assumed as the objective function.

2. EQUATIONS OF TRAVELING WAVES

According to the superposition law, the resultant wave traveling along the cable is the sum of the original wave, the reflected and transmitted waves. The original wave arises as a result of the environment interaction while the reflected and transmitted waves arise due to the force in constraints. The damper attached to the cable is also the constraint for the traveling wave. The damper force can be modeled as the concentrated force or the force distributed on the short segment of the cable.

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2.1. Equations of waves induced by the concentrated force

The concentrated force applied to the cable induces two waves traveling in opposite directions with the same velocity. This waves and the coordinate system are presented schematically in Figure 1.

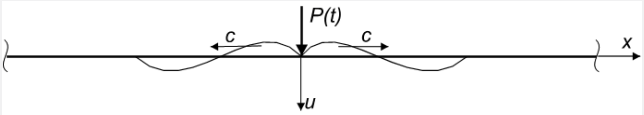

Fig. 1. Concentrated force as a source of waves

The cable motion induced by the concentrated force $P(t)$ can be described by the following differential equation

$$\mu \frac{\partial^2 u}{\partial t^2} - T \frac{\partial^2 u}{\partial x^2} = P(t)\delta(x) \tag{1}$$

with additional condition at the point $x = 0$

$$u(0^-, t) = u(0^+, t) \tag{2}$$

In equation (1) μ is a linear mass density and T is a tension force.

Equation (1) was solved using the Laplace transform method. Finally the displacement $u(x, t)$ can be written as

$$u(x,t) = u\left(0, t - \frac{|x|}{c}\right) \tag{3}$$

The above expression describes two waves traveling with velocity c on both sides of the force $P(t)$. These waves transfer the displacement $u(0, t)$ along the cable. The relationship between the displacement $u(0, t)$ and the force $P(t)$ is described by the following equation

$$\frac{du(0,t)}{dt} = \frac{c}{2T} P(t) \tag{4}$$

Using equations (3) and (4) we can solve the wave propagation problem for arbitrary force $P(t)$.

2.2. Equations of waves induced by the distributed force applied to the short segment of the cable

In the case of distributed force applied to the short segment of the cable, the problem of waves generating is more complicated. The distributed force induces waves traveling in opposite directions, outside the segment, as well as standing waves inside the load segment (Fig. 2).

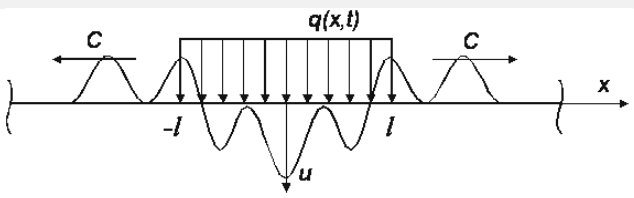

Fig. 2. Distributed force as a source of waves

The equation of the cable motion with excitation force applied to the short segment, placed symmetrically about the origin of coordinate system, can be written as

$$\mu \frac{\partial^2 u}{\partial t^2} - T \frac{\partial^2 u}{\partial x^2} = \tilde{q}(x,t) \tag{5}$$

where:

$$\tilde{q}(x,t) = \begin{cases} q(x,t) & |x| \le l \\ 0 & |x| > l \end{cases} \tag{6}$$

When the width of the active segment is small in relation to the wave length the following form of distributed force can be considered

$$q(x,t) = q_0 \cdot f(t) \tag{7}$$

In this case the distributed force is a product of the unknown function of time and the uniform function of space coordinate. Taking into account this form of load, equation (5) may be solved using the Laplace transform method. The waves traveling to the left and to the right outside the load segment are described by the following formula

$$u(x,t) = \begin{cases} u\left(l, t - \dfrac{x-l}{c}\right) & |x| \ge l \\ u\left(-l, t + \dfrac{xl}{c}\right) & |x| \le -l \end{cases} \tag{8}$$

The relationship between displacements $u(l, t) = u(-l, t)$ and the function $f(t)$ has the form of differential equation with time delay

$$\frac{d^2 u(l,t)}{dt^2} = \frac{q_0}{2\mu}\left[f(t) - f\left(t - \frac{2l}{c}\right)\right] \tag{9}$$

Using equations (8) and (9) we can determine the waves traveling outside the load segment for arbitrary function $f(t)$.

The general solution of equation (5) can be written in the following integral form

$$u(x,t) = \frac{1}{2c\mu} \int_0^t \int_{x-c(t-\tau)}^{x+c(t-\tau)} \tilde{q}(\xi,\tau)\, d\xi\, d\tau \tag{10}$$

This form is useful to simulate the phenomenon of the waves travelling along a cable for any function $\tilde{q}(x,t)$. The solution of equation (5) at point x and at time t depends only on values of the function $\tilde{q}$ in the triangle Δ as shown in Figure 3.

From the general solution (10) one can derive the expression for the cable velocity which is suitable in numerical calculations of energies of traveling waves.

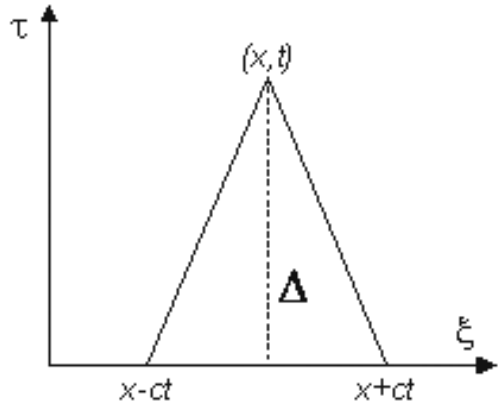

Fig. 3. Area of integration

3. OPTIMAL DAMPER FORCE

In order to suppress the original wave traveling along the cable, the damper force should be controlled optimally. The energy dissipated by the damper force can be assumed as the objective function of the considered problem. The complete dissipate of the wave is impossible, because the damper force is a source of the secondary waves. The original wave and secondary waves traveling in the same direction can partly cancel each other but the remaining secondary waves traveling in the opposite direction. It is apparent that the optimal force exists both for the concentrated and for the distributed damper force.

3.1. Optimal concentrated damper force

During the motion of the original wave, the damper exerts the resisting force applied to the cable. This force is a source of two secondary waves (Fig. 1). The secondary wave that travels in opposite direction to the original wave is called the reflected wave. The superposition of the other secondary wave and the original wave is called the transmitted wave.

The displacement of the point at which the damper is attached is a sum of the displacement associated with the original wave $u_0(0, t)$ and the displacement associated with the wave $u(0, t)$ induced by the damper force. The power of damper force $P(t)$ can be expressed as

$$W = \left(\frac{du_0(0,t)}{dt} + \frac{du(0,t)}{dt} \right) \cdot P(t) \tag{11}$$

If the above quantity is negative, the wave energy is dissipated. Substituting the relation (4) into equation (11) we can determine the objective function $|W(P)|$ in a relatively simple form. It is easy to show that the relationship between the original wave and the optimal damper force takes the following form

$$P_{opt} = -\frac{T}{c} \frac{du_0(0,t)}{dt} \tag{12}$$

The energy dissipated by the optimal damper force is equal to the half of original wave energy.

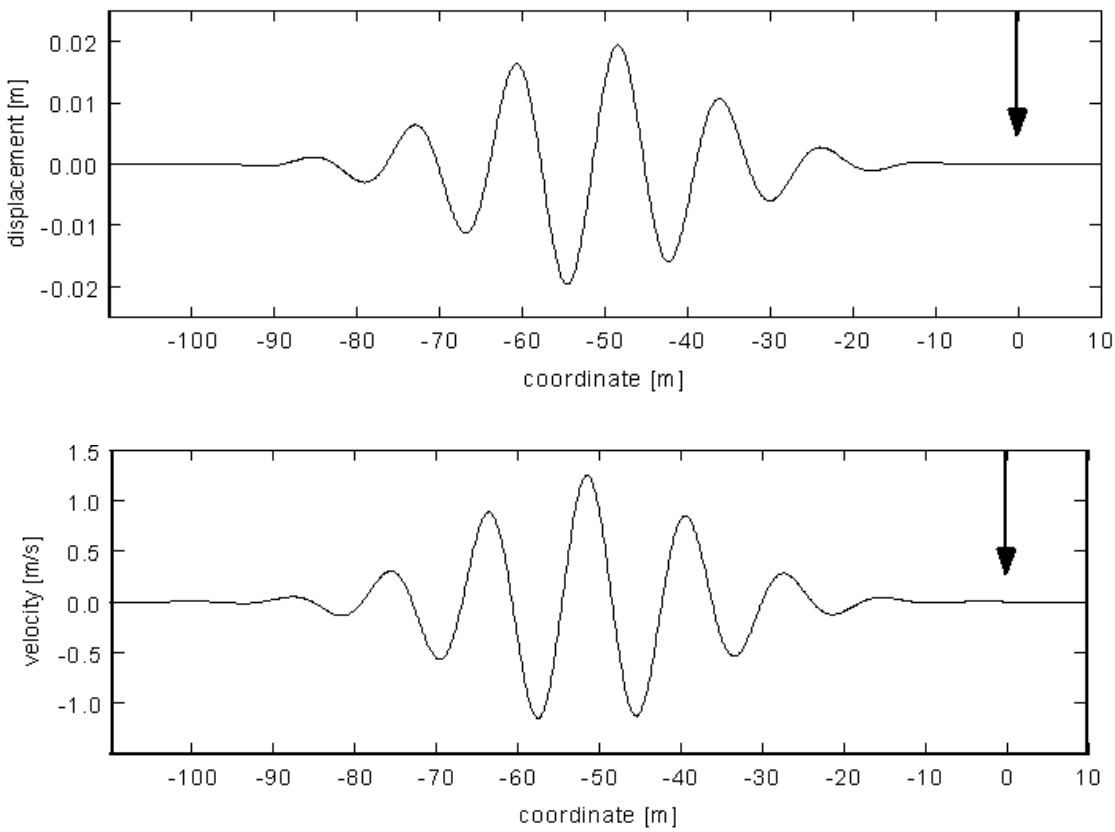

Fig. 4. Original wave before the damper

The original wave used in numerical simulations was assumed in the form of packet wave, described by

$$u_0(x,t) = \Psi_0 \exp\left(-\frac{(x-ct)^2}{4\sigma^2}\right)\sin(k_0(x-ct)) \tag{13}$$

A lot of disturbances observed in cables can be modeled using this type of wave with amplitude modulation. The original wave propagating in the positive x-direction is presented in Figure 4 at the moment before it reaches the point where the damping force is placed. The location of damping force is marked with the vertical arrow. The transmitted wave and the reflected wave for the optimal damper force are presented in Figure 5.

3.2. Optimal distributed damper force

Taking into account the relationship (12) between the original wave and the optimal damper force the similar strategy for the distributed damper force have been proposed. The damping force is proportional to the component of cable velocity resulting from the motion of the original wave with appropriate switching that is realized by the Heaviside function

$$q(x,t) = -\alpha \frac{\partial u_0(x,t)}{\partial t} H(ct-x) \tag{14}$$

Figures 6, 7 and 8 present the velocity associated with the motion of the original wave for different damping force coefficient α.

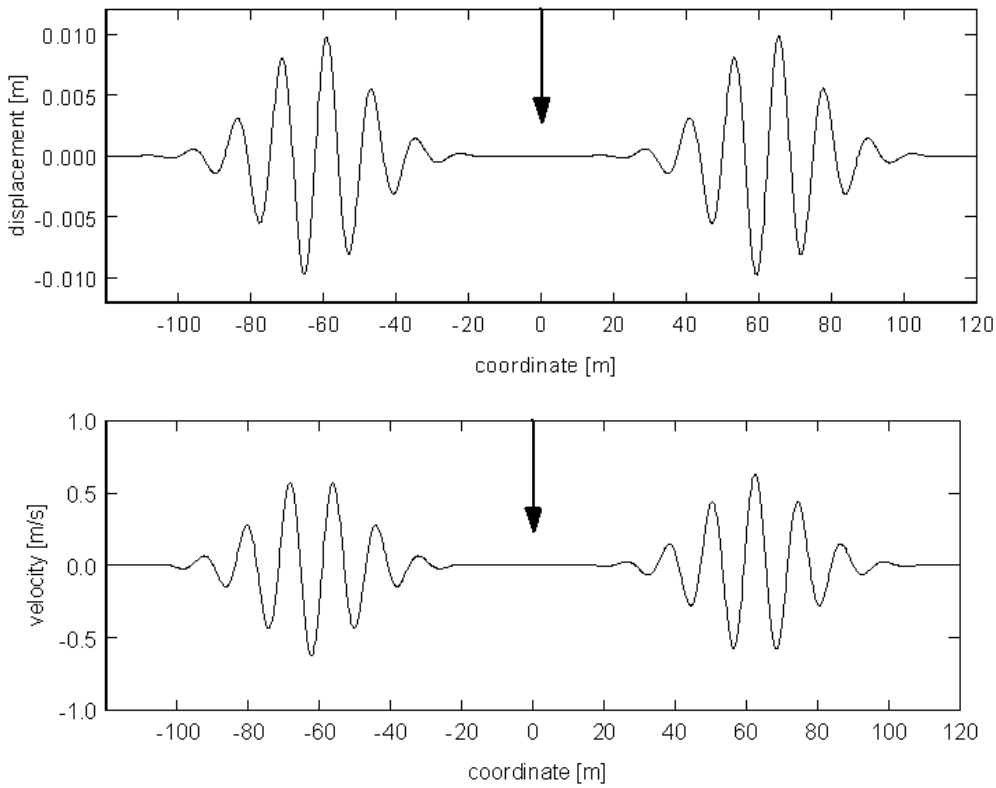

Fig. 5. Transmitted and reflected waves

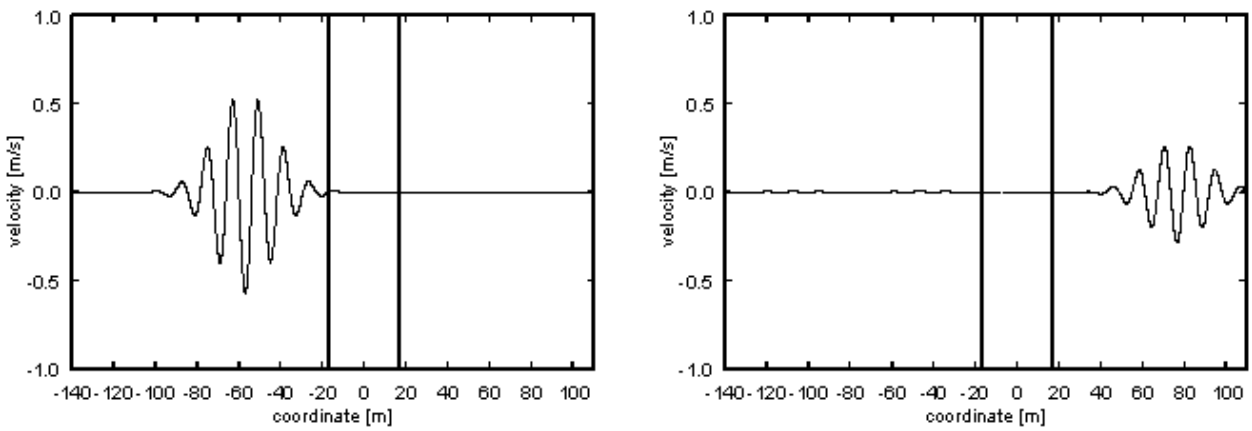

Fig. 6. Schematic graphs of waves travelling through the active segment $\left(2l/\lambda = 2.60,\ \alpha/\mu = 4.0\ \mathrm{s}^{-1}\right)$

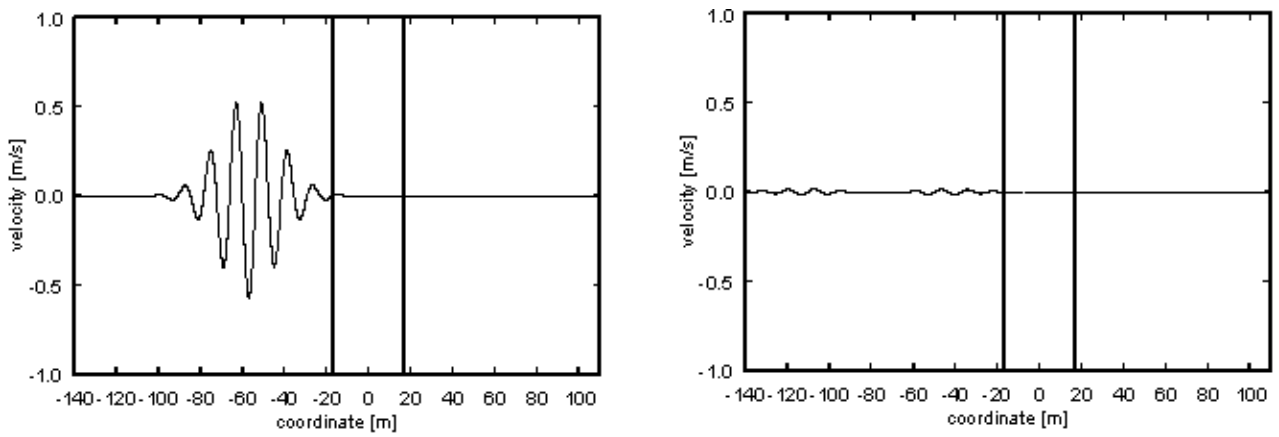

Fig. 7. Schematic graphs of waves travelling through the active segment $\left(2l/\lambda = 2.60,\ \alpha/\mu = 8.0\ \mathrm{s}^{-1}\right)$

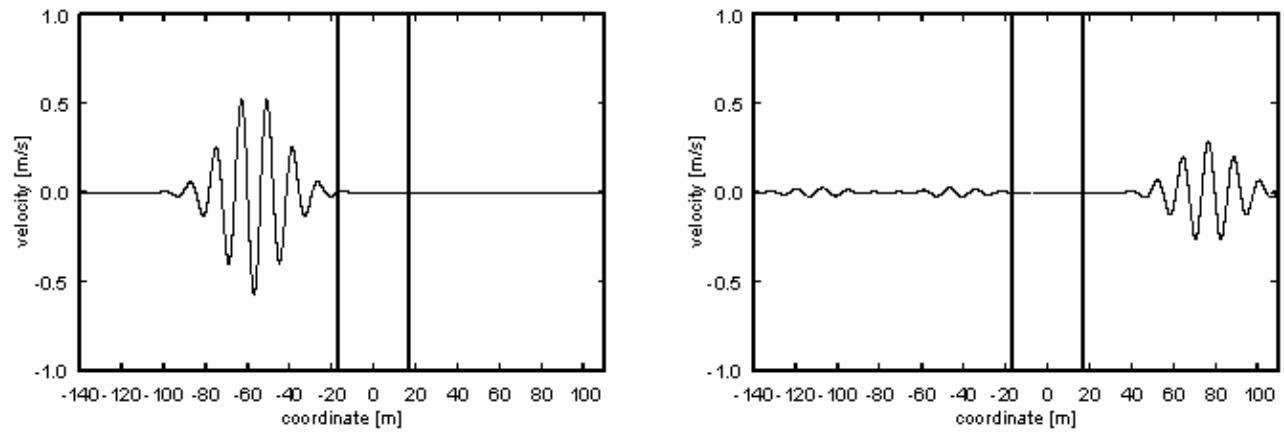

Fig. 8. Schematic graphs of waves travelling through the active segment $\left(2l/\lambda = 2.60,\ \alpha/\mu = 12.0\ \mathrm{s}^{-1}\right)$

The aim of calculations was to find the optimal value of α/μ which maximizes the energy dissipated in the active segment. We can determine this energy by calculation the work done by distributed force applying in active segment. On the other hand we can use the principle of energy balance to determine the energy dissipated in the active segment. The second method is more convenient in numerical calculation. The sought energy can be calculated from

$$E_D = E_S - (E_R + E_T) \tag{15}$$

where

$$E_S = \int_{t_{S1}}^{t_{S2}} \sqrt{T\mu}\left(\frac{\partial u_0(-l,t)}{\partial t}\right)^2 dt \tag{16}$$

is the energy of the original wave

$$E_R = \int_{t_{R1}}^{t_{R2}} \sqrt{T\mu}\left(\frac{\partial u(-l,t)}{\partial t}\right)^2 dt \tag{17}$$

is the energy of the reflected wave, and

$$E_T = \int_{t_{T1}}^{t_{T2}} \sqrt{T\mu}\left(\frac{\partial u(l,t)}{\partial t} + \frac{\partial u_0(l,t)}{\partial t}\right)^2 dt \tag{18}$$

is the energy of the transmitted wave.

Time intervals $[t_{S1}, t_{S2}]$, $[t_{R1}, t_{R2}]$, $[t_{T1}, t_{T2}]$ refer to dislocations of appropriate waves.

The ratio of energy dissipated in the active segment to the energy of original wave given by

$$\eta = \frac{E_D}{E_S} \tag{19}$$

is called the dissipation efficiency ratio. For optimal distributed force this ratio takes on maximum value.

Figures 9, 10 and 11 illustrate the dissipation efficiency ratio versus the relative damping force coefficient for different segment widths. As shown, the dissipation efficiency ratio as a function of damping coefficient has maximum value for all segment widths.

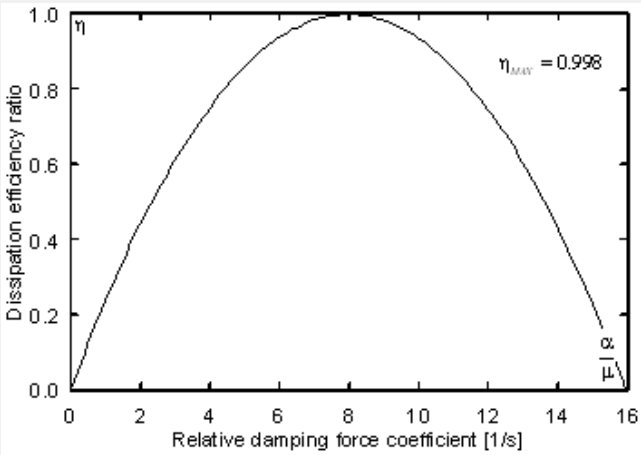

Fig. 9. Dissipation efficiency ratio *vs*. relative damping force coefficient $\left(2l/\lambda = 2.60\right)$

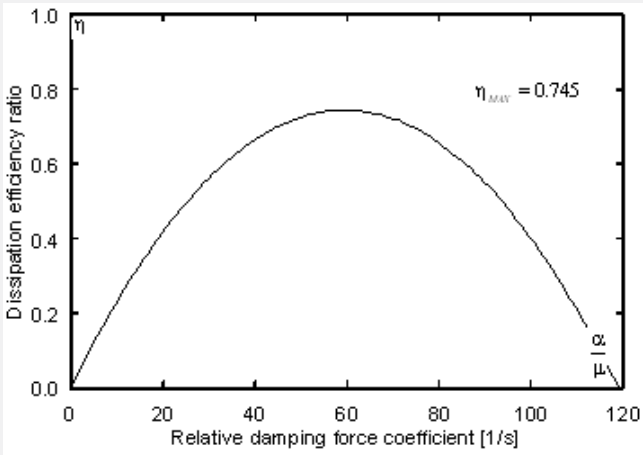

Fig. 10. Dissipation efficiency ratio *vs*. relative damping force coefficient $\left(2l/\lambda = 0.26\right)$

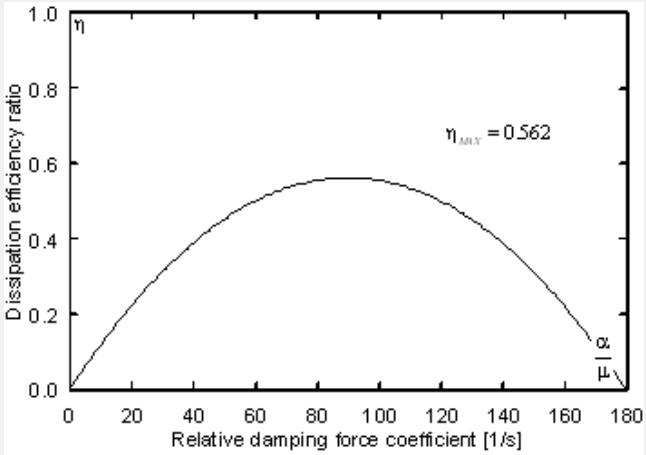

Fig. 11. Dissipation efficiency ratio *vs*. relative damping force coefficient $\left(2l/\lambda = 0.13\right)$

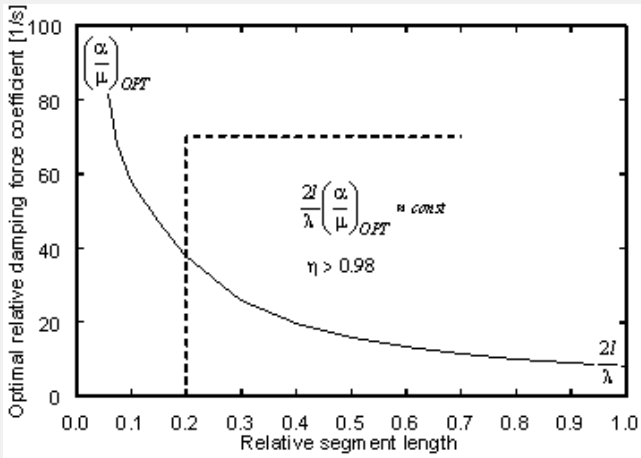

Fig. 12. Optimal relative damping force coefficient *vs*. active segment length

When the segment width decreases the maximal efficiency ratio decreases and approaches 0.5, as for optimal concentrated force.

Taking into account the results of calculations of maximal efficiency ratio done so far we can perform the summarizing plot presented in Figure 12. The plot shows the optimal relative damping force coefficient versus the damping segment length. When the segment is longer than 0.2λ, the optimal damping coefficient is almost reciprocal to the segment length and the optimal distributed force dissipates over 98% of energy of the original wave.

4. CONCLUSIONS

Optimal damper forces based on the distributed parameters model are intuitive and physically motivated. But distributed parameter methods require more difficult calculations.

In the case of the concentrated force the optimization problem can be solved exactly. The optimal force takes place in the opposite direction to this component of cable velocity vector that is associated with the original wave. The force magnitude is proportional to the magnitude of velocity component. The constant of proportionality is equal to the ratio of the cable tension to the wave velocity. The maximum value of the dissipated energy is equal to the half of the original wave energy.

In the case of the distributed load calculations are more complex. The optimal force can be obtained using numerical methods – almost the whole energy of the original wave can be dissipated. It is possible to adjust the damper parameters so the damper may be efficient in a wide range of wave lengths.

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