MECHANICS
Vol. 27 No. 1 2008

Marian JABŁOŃSKI*, Agnieszka OZGA**

STATISTICAL CHARACTERISTICS OF VIBRATIONS OF A STRING FORCED BY STOCHASTIC FORCES***

SUMMARY

Our theoretical study aims at finding the statistical parameters characterizing the vibrations of a string without damping and forced by stochastic impulses when the places and moments of actions are random. We derive the dependence of these parameters from the parameters of the string as well as from the stochastic distributions of the impulse magnitude, and of the place of the action. We also carry out a numerical simulation verifying the derived mathematical model and interpret the differences between the results obtained in the simulation and the mathematical calculations. This study is the third stage of research aimed at designing and calibrating a probe that will facilitate measuring of parameters determining the quality of a technological process.

Keywords: *string, stochastic impulses, statistic parameters characterizing the vibrations, expectation, variance*

STATYSTYCZNE CECHY DRGAŃ STRUNY WYMUSZONYCH STOCHASTYCZNYMI SIŁAMI

Celem badań teoretycznych w tej pracy jest znalezienie parametrów charakteryzujących drgania struny bez tłumienia i będącej pod wpływem działania sił impulsowych, dla których miejsce i moment zadziałania są przypadkowe. Zostanie wyprowadzona zależność tych parametrów od parametrów struny oraz zarówno od stochastycznego rozkładu wielkości impulsu, jak i rozkładu miejsca zadziałania siły. Przeprowadzono również symulację numeryczną weryfikującą wyprowadzony model matematyczny oraz zinterpretowano różnice pomiędzy wynikami otrzymanymi z symulacji a obliczeniami teoretycznymi. Badania te są trzecim etapem prac mających na celu zaprojektowanie i skalibrowanie sondy, która umożliwi dokonywanie pomiarów parametrów określających jakość procesu technologicznego.

Słowa kluczowe: *struna, stochastyczne siły impulsowe, statystyczne cechy drgań, nadzieja matematyczna, wariancja*

1. INTRODUCTION

The work is one of a series inspired by attempts at constructing a measuring device that would control the composition of the medium in a dust pipeline. The difficulties that arose then in connection with statistical interpretation forced us to search for a mathematical model that would explain its causes.

The developed model described below is based on the theorem proved in (Jabłoński and Ozga 2006). This theorem allows for calculation of the basic statistical parameters like mathematical expectation and variance that characterize the movement of continuous and discrete systems stimulated by stochastic impulses. In this theorem we assume that the probability that an impulse will occur in a short time interval is proportional to its duration and the moments of impulse occurrences; their magnitude and places of occurrence are probabilistically independent. These assumptions seem quite natural in regard to the actual working conditions of the above mentioned measuring device.

In our previous studies (Jabłoński and Ozga 2006a, 2006b, 2006c) we dealt with the statistical characteristics of action of discrete systems stimulated by stochastic impulses. The first partial mathematical results regarding vibration of oscillators forced by stochastic impulses can be found in the following works (Campbell 1909a, 1909b; Hurwitz and Kac 1944; Roberts 1972, 1973; Rowland 1936). In the study (Roberts and Spanes 1986), we can find the results that generalize the findings published in our previous works. The works (Khintchine 1938; Rice 1944; Roberts 1965a, b; Roberts 1972) consider vibrations of oscillators forced by stochastic impulses and suggest their possible technological applications. These results will help solve the problem of stochastic response of systems to the action of stochastic impulses. As regards other works discussing similar problems connected with the movement of oscillators forced by stochastic impulses, partial mathematical results can be found in (Campbell 1909a, b; Hurwitz and Kac 1944; Khintchine 1938), while the results generalizing the outcome of the previous studies are discussed in (Takác 1994). Monographs (Kasprzyk 1994; Plucińska and Pluciński 2000; Sobczyk 1996) introduce the foundations of the considered problems. Works (Iwankiewicz and Nielsen 1996; Iwankiewicz 2003, 2002) include certain results concerning nonlinear systems subjected to stochastic forces that might not act in a continuous way, systems which are solved by stochastic equations with Ito integral. The methods of investigating the stochastic stability of systems are described in (Tylikowski 1991).

* Faculty of Mathematics and Computer Science, Jagiellonian University, Krakow, Poland
** Department of Mechanics and Vibroacustic, AGH University of Science and Technology, Krakow, Poland
*** The work has been carried as a part of the research project 11.11.130.859 supported by the AGH University of Science and Technology, Poland

In the present work we will discuss a model that allows for calculation of statistical properties of a one dimensional continuous system without damping, whose vibrations are forced by stochastic impulses.

We will carry out numerical simulation of the motion of a string, compare the results of the simulation with theoretical calculations and interpret the differences that may occur at various parameters of the string and of the distributions of stochastic forces. Finally, on the basis of the acquired results, we will suggest the ways of reasoning about the statistical properties of forces influencing the system, depending on the statistical characteristic of the data obtained from the measurements of the systems.

2. THEORICAL BACKGROUND

Now, for the convenience of the reader we quote the theorem from (Jabłoński and Ozga 2006a).

Let $g_i : [0,\infty) \to \mathrm{R}$, $i = 1,2,3, ..., m$ be a sequence of continuous functions, A be a bounded connected Borel subset of R^p for some $p \in N$, h_i: A $\to$R, $i = 1, 2, 3, ..., m$ be a sequence of bounded and continuous functions, $\{\tau_i\}_{i=1}^{\infty}$ – a sequence of independent identically distributed (i.i.d.) random variables with exponential distribution $F(x) = 1 - \exp(-\lambda x)$ for $x > 0$ and $F(x) = 0$ for $x < 0$, $\{\eta_i\}_{i=1}^{\infty}$ – a sequence of i.i.d. random variables with finite expectation, $\{\zeta_i\}_{i=1}^{\infty}$ – a sequence of i.i.d. random variables with values in the set A and finally let $\{\alpha_i\}_{i=1}^{\infty}$ be a sequence of real numbers. Let us put

$$t_0 = 0,\ t_i = \sum_{j=1}^{i} \tau_j,\ \ i = 1,2,3,\ldots$$

and

$$\xi(t) = \sum_{n=1}^{m} \alpha_n \sum_{0<t_j<t} \eta_j h_n(\zeta_j) g_n(t - t_j) \tag{1}$$

Denote by ϕ_ζ and ϕ_η distributions of and respectively. Let $A_i \subset \mathrm{C}$ and $B_i \subset [0,\infty)$ $i = 1, 2, \ldots, m$ be Borel sets and $k(i,j)$, for every fixed i, be an increasing sequence of all natural numbers such that

$$\chi_{A_i}(\eta_{k(i,j)}) \chi_{B_i}(\xi_{k(i,j)}) = 1 \tag{2}$$

where $\chi_A(x) = 1$ if $x \in A$ and $\chi_A(x) = 0$ if $x \notin A$.

Write $t_j^i = t_{k(i,j)}$ and $\tau_j^i = t_j^i - t_{j-1}^i$.

We will say that $\xi(t)$ is decomposable if for every $n \in \mathrm{N}$, all Borel sets $A_i \subset A$ and $B_i \subset [0,\infty)$, $i = 1, 2, \ldots n$ such that $A_i \times B_i$ are mutually disjoint

$$\bigcup_{i=1}^{\tau} A_i \times B_i = A \times B \tag{3}$$

τ_j^i are i.i.d. random variables with exponential distribution for $x>0$ and

$$\begin{aligned} &F(x) = 1 - \exp(-\lambda \Phi_\xi(A_i) \Phi_\eta(B_i)) \\ &\text{for } x > 0 \text{ and } F(x) = 0 \text{ for } x < 0 \end{aligned} \tag{4}$$

$$\xi_i(t) = \sum_{n=1}^{m} \alpha_n \sum_{0<t_i^j<t} \eta_{k(i,j)} h_n(\zeta_{k(i,j)}) g_n(t - t_j^i) \tag{5}$$

are independent and

$$\xi(t) = \sum_{i=1}^{n} \xi_i \tag{6}$$

From the technical point of view, decomposability of the process $\xi(t)$ means that we can divide the acting stochastic forces in any way, and the space onto which they acted (for a string, a membrane etc) can be divided into any areas. If we consider the processes corresponding with the acting forces, let us say group number i acting on the area number j, we will receive a series of processes. As regards these processes we assume that they are independent.

3. THEOREM 1

If the above defined process $\xi(t)$ is decomposable, then:

1) characteristic function of $\xi(t)$ is given by

$$\begin{aligned} &\varphi(s) = \\ &= \exp\left(\lambda t \left(\int_A \int_0^\infty \int_0^1 \exp\left(is \sum_{n=1}^{m} \alpha_n y h_n(z) g(ut)\right) du \phi_\eta(dy) \phi_\zeta(dz) - 1\right)\right) \end{aligned} \tag{7}$$

2) the expectation of $\xi(t)$ is

$$\begin{aligned} &E(\xi(t)) = \frac{\varphi'(0)}{i} = \\ &= \lambda t E(\eta) \sum_{n=1}^{m} \alpha_n E(h_n(\zeta)) \int_0^1 g_n(tu) du \end{aligned} \tag{8}$$

3) the variance of $\xi(t)$ is

$$\begin{aligned} &D^2(\xi(t)) = E(\xi^2(t)) - E^2(\xi(t)) = \\ &= \frac{1}{i^2}(\varphi''(0) - (\varphi')^2(0)) = \\ &= \lambda t E(\eta^2) \sum_{n=1}^{m} \sum_{j=1}^{m} \alpha_n \alpha_j E(h_n(\zeta) h_j(\zeta)) \int_0^1 g_n(tu) g_j(tu) du \end{aligned} \tag{9}$$

4. APPLICATIONS

We shall apply the theorem presented above to a one dimensional continuous system without damping, i.e., to a string for which the equation describing vibrations induced by forces is as follows:

$$\frac{\partial^2 u(x,t)}{dt^2} = a^2 \frac{\partial^2 u(x,t)}{dx^2} + f(x,t) \tag{10}$$

The boundary and initial conditions are as follows:

$$u(0,t) = 0 \quad \text{and} \quad u(l,t) = 0 \tag{11}$$

$$u(x,0) = 0 \quad \text{and} \quad \frac{\partial u(x,0)}{\partial t} = 0 \tag{12}$$

for $t \geq 0$, $x \in [0,l]$.

If $f(x,t)$ is given by

$$f(x,t) = \sum_{i=1}^{\infty} \eta_i \delta_{t_i \varsigma_i} \tag{13}$$

where $\delta_{t_i\varsigma_i}$ – are Dirac distributions at t_i and ς_i, then the solution of (10), (11) (12) takes the following form

$$u(x,t)=\sum_{n=1}^{\infty}T_n(t)\sin\frac{n\pi x}{l} \tag{14}$$

where

$$T_n(t)=\frac{2}{an\pi}\sum_{0<t_i<t}\eta_i\sin\frac{\pi n\varsigma_i}{l}\sin\left(\frac{an\pi}{l}(t-t_i)\right) \tag{15}$$

Let us notice that the derivation of function (14) is discontinuous at t_i and, consequently, the second derivative of this function does not exist in the classical sense. Fortunately, function (14) can be considered as a solution of (10), (11), (12) in the distribution sense and it is sufficient for our purposes.

If η_i and ζ_i, $i = 1, 2, ...$ are two sequences of independent and identically distributed random variables with distributions Φ_n and Φ_ς respectively and $\tau_i = t_i - t_{i-1}$, $i = 1, 2, ...$, are also independent and identically distributed random variables with exponential distribution ($F(u)=1-\exp(-\lambda u)$ (for $u>0$ and for some λ and $F(u) = 0$ for $u<0$) then for any fixed x

$$\xi_m(t,x)=u_m(x,t)=\sum_{n=1}^{m}T_n(t)\sin\frac{n\pi x}{l} \tag{16}$$

is a stochastic process satisfying assumptions of theorem 1 with any m and $h_i = \sin(i\pi x/l)$, $i = 1, ..., m$. Because of conditions of Theorem 1 we can't assume $m = \infty$. Instead of that we can take limit of $u_m(x, t)$ as m tends to infinity to take required results.

Applying this theorem we get the following formulas for the expectation and variance of $u_m(x, t)$:

$$\begin{aligned}
&E(\xi_m(t,x))=\\
&=\lambda E(\eta)\sum_{n=1}^{m}\frac{2}{an\pi}\sin\frac{\pi nx}{l}E\left(\sin\left(\frac{\pi n\varsigma}{l}\right)\right)\int_0^t\sin\frac{an\pi}{l}v\,dv= \\
&=\lambda E(\eta)\sum_{n=1}^{m}\frac{2l}{(an\pi)^2}\sin\frac{\pi nx}{l}E\left(\sin\left(\frac{\pi n\varsigma}{l}\right)\right)\left(1-\cos\frac{an\pi}{l}t\right)
\end{aligned} \tag{17}$$

$$\begin{aligned}
&D^2(\xi_m(x,t))=\frac{4}{(a\pi)^2}\lambda E(\eta^2)\times\\
&\times\sum_{n=1}^{m}\sum_{j=1}^{m}\frac{\sin(\pi nx/l)\sin(\pi jx/l)}{nj}E\left(\sin\frac{\pi n\varsigma}{l}\sin\frac{\pi j\varsigma}{l}\right)\times\\
&\times\int_0^t\sin\left(\frac{van\pi}{l}\right)\sin\left(\frac{vaj\pi}{l}\right)dv=\frac{4}{(a\pi)^2}\lambda E(\eta^2)\times\\
&\times\sum_{n\neq 1}^{m}\frac{\sin(\pi nx/l)\sin(\pi jx/l)}{nj}E\left(\sin\frac{\pi n\varsigma}{l}\sin\frac{\pi j\varsigma}{l}\right)\times\\
&\times\left(\frac{l\sin(\pi a(n-j)t/l)}{2a(n-j)\pi}-\frac{l\sin(\pi a(n+j)t/l)}{2a(n+j)\pi}\right)+\\
&+\frac{4}{(a\pi)^2}\lambda E(\eta^2)\times\\
&\times\sum_{n=1}^{m}\frac{\sin^2(\pi nx/l)}{n^2}E\left(\sin^2\frac{\pi n\varsigma}{l}\right)\times\left(\frac{t}{2}-\frac{l\sin(2an\pi t/l)}{4an\pi}\right)
\end{aligned} \tag{18}$$

5. NUMERICAL SIMULATION

To check numerically that theoretical formulas are correct we need a statistical sample T. To get one with n elements we repeat the following procedure n times. First we choose randomly τ_i in accordance with the exponential distribution. Remember that $t_m=\sum_{i=1}^{m}\tau_i$. Then, we choose randomly the values of η_i (magnitude of the force with which particles strike the string) with discrete distribution, and finally, we select randomly ς_i (the point of the action on the string). We substitute these data into (16), (17) and thus we obtain an element of our statistical sample. Elements of the sample are denoted by $u_m^k(x,t)$. Taking

$$\widetilde{E}(u_m(x,t))=\sum_{k=1}^{n}\frac{u_m^k(x,t)}{n} \tag{19}$$

where n is the number of elements of the sample we obtain an estimator of $E(u_m(x,t))$. Taking

$$\widetilde{D}^2(u_m(x,t))=\sum_{k=1}^{n}\frac{(u_m^k(x,t)-E(u_m(x,t)))^2}{n} \tag{20}$$

we obtain an estimator of $D^2(u_m(x,t))$.

We will consider a number of cases, depending on the distribution of striking force values, the distribution of striking points and the parameter a of the string.

5.1. The first stage of simulation

The simulation will be divided into three stages. At the first stage we will check the correctness of the formula (16) for the expected value $E(u_m(x,t))$ for any natural m we will find what values are achieved by the amplitudes of mathematical expectation for particular components in the formula (16) for different distributions and various parameters a; we will assess the number of elements of the statistical sample for which estimators are close to theoretical assessments.

From the linear character of mathematical expectation it follows that

$$E(u_m(x,t))=\sum_{i=1}^{m}E\left(T_i(t)\sin\frac{i\pi x}{l}\right) \tag{21}$$

Figure 1 shows theoretical mathematical expectations of harmonic components for the expectation $E(u_m(x,t))$ as well as estimators of harmonic components $\widetilde{E}(u_m(x,t))$ of $E(u_m)x,t))$ which are found in the formula (17) with $m = 5$, $a = 20$, $x = l/2$ as well as $x = l/4$, distribution of the variable η such that $\eta\in\{728,214\}$, $P(\eta=728)=2/3$, $P(\eta=214)=1/3$, a uniform and continuous distribution of the variable ς and for a 1 million sample. In all computations we assume discrete distributions of η. Random variable η can assume any other distribution. We consider such distribution because many computations are simple and , moreover is easy to interpretation the result of computation.

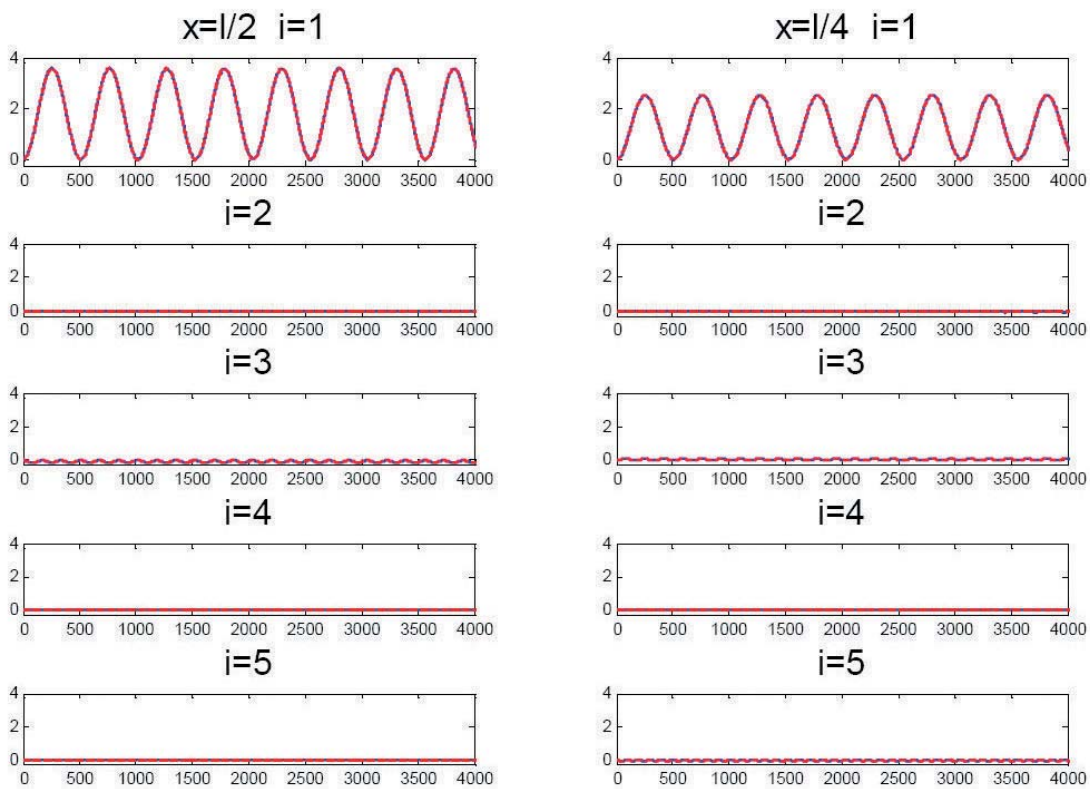

Fig 1. Expectations of harmonic constituents for the expectation $E(u_m(x,T))$ and $\widetilde{E}(u_m(x,t))$

Remark 1

First of all, let us notice that the estimator $\widetilde{E}(u_m(x,t))$ is so close to $E(u_m(x,t))$ that the differences are not visible on the figure.

Secondly, simple integration shows that $E\left(\sin\left(\frac{\pi n \varsigma}{l}\right)\right)=0$ for any even number n. Therefore any even component of the sum (17) equals zero. This is confirmed by the simulation (see Fig. 1). The same procedure shows that the odd components of the sum (17) rapidly tend to zero. Simulation confirms this, see (Fig. 1). It is also clear that the differences between theoretical computation and simulation are negligible. Hence, it follows that high frequencies can be neglected in practice. Because of this in the further simulations we will assume $m = 10$. Moreover, in the investigations of vibrations of a string it is enough to focus on the vibrations of the point $x = l/2$.

Figure 2 presents estimators

$$\widetilde{E}_n(u_m(x,t))=\frac{1}{n}\sum_{i=1}^{n} u_m{}^i(x,t) \tag{22}$$

for $m = 10$, $\lambda = 10$, $a = 2$, $a = 20$, $a = 200$, $x = l/2$, statistical samples of 1 thousand, 10 thousand, 100 thousand and 1 million, distributions identical as in the previous remarks.

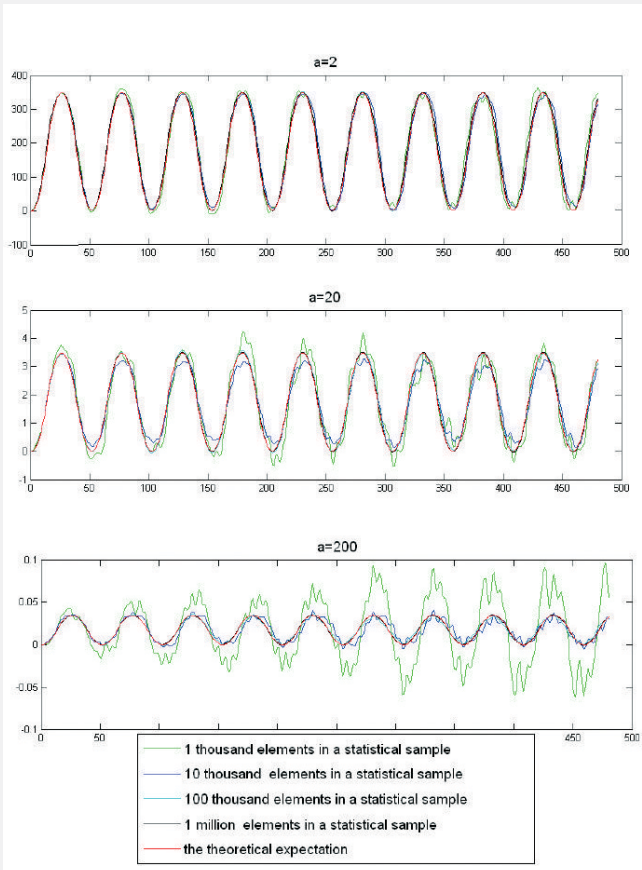

Fig. 2. Estimators of the expectation and the theoretical expectation

Remark 2

Figure 2 shows that the number of a statistical sample equal to 1 million is sufficient for a in the range from 2 to 200. We can notice, however, that for high values of a, the size of the statistical sample must be bigger to achieve a similar proportional precision of the results of simulation. It is worth mentioning that in the case of a string samples must be at least ten times as large as in the case of an oscillator if the assessments of mathematical expectations of these systems are to be executed with similar precision (Jabłoński and Ozga 2006a, b, 2006c). This follows from the fact that a string is a mechanical system having one dimension more than an oscillator.

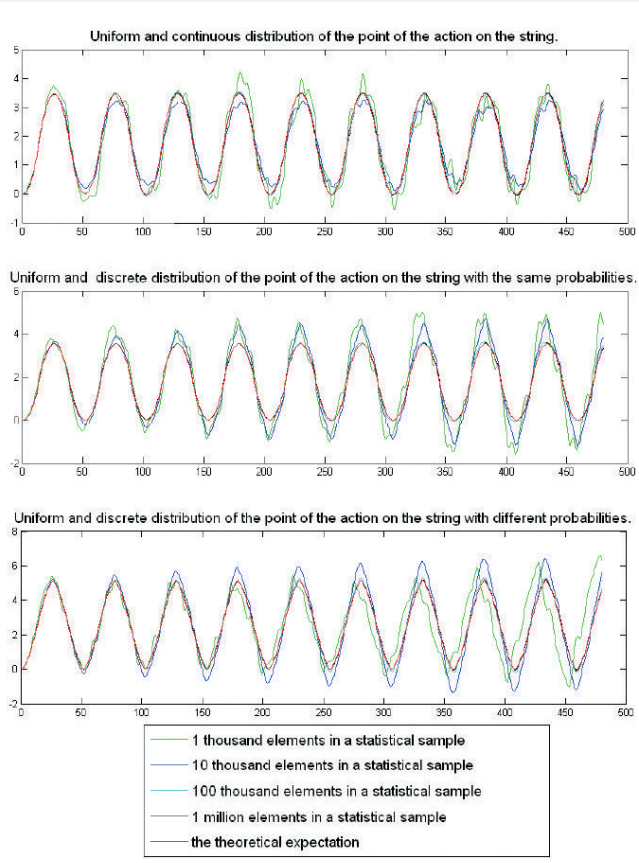

Fig. 3. Estimators of the expectation and the theoretical expectation

Figure 3 presents estimators $\widetilde{E}_n(u_m(x,t)) = \frac{1}{n}\sum_{i=1}^{n} u_m^i(x,t)$ for $m = 10$, $\lambda = 10$, $a = 20$, $x = l/2$, statistical samples of 1 thousand, 10 thousand, 100 thousand and 1 million elements, distribution of the variable identical as in the previous remarks and three different distributions of the variable uniform and continuous uniform and discrete determined on the points from the set $A = \{0.1, 0.3, 0.5, 0.7, 0.9\}$ ($P(\zeta = x_i) = 1/5$ for $x_i \in A$):

– discrete determined on points from the set $A = \{0.1, 0.3, 0.5, 0.7, 0.9\}$ with probabilities: $P(\zeta = 0.1) = 0.05$, $P(\zeta = 0.3) = 0.2$, $P(\zeta = 0.5) = 0.5$, $P(\zeta = 0.7) = 0.2$, $P(\zeta = 0.9) = 0.05$.

Remark 3

Figure 3 shows that the change of distribution of the variable ζ influences the mean values of expectation.

5.2. The second stage of simulation

The second stage of simulation was to test correctness of the formula (18) for variance $D^2(u_m(x,t))$ as well as assessment of the size of a statistical sample for which the estimator of variance

$$\widetilde{D}_n^{\,2}(u_m(x,t)) = \frac{1}{n}\sum_{i=1}^{n}(u_m^{\,i}(x,t) - E(u_m(x,t)))^2 \quad (23)$$

is close to theoretical calculations.

Figure 4 shows estimators

$$\widetilde{D}_n^{\,2}(u_m(x,t)) = \frac{1}{n}\sum_{i=1}^{n}(u^i{}_m(x,t) - E(u_m(x,t)))^2$$

for $m = 10$, $\lambda = 10$, $a = 2$, $a = 20$, $a = 200$, $x = l/2$, statistical samples equal to 1 thousand, 10 thousand, 100 thousand and one million, for distributions as in remark 1.

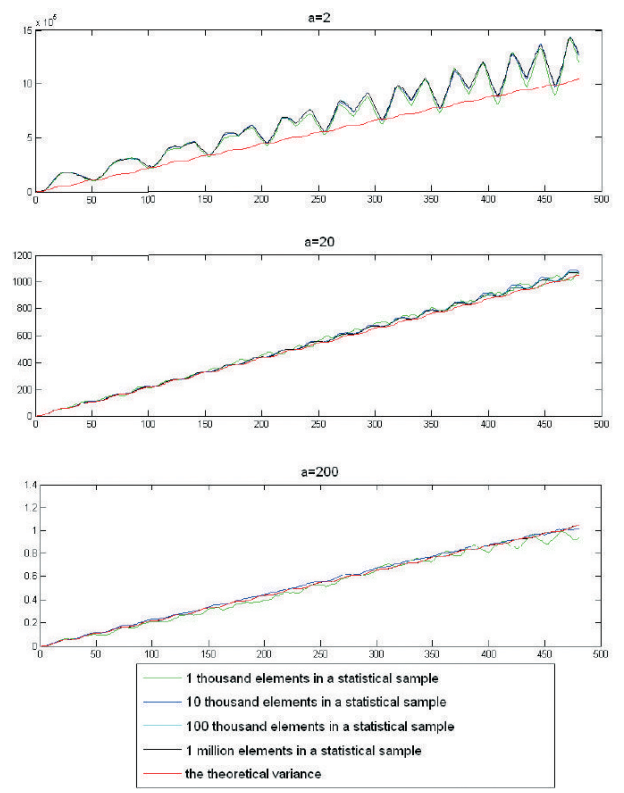

Fig. 4. Estimators of the variance and the theoretical variance

Remark 4

From figure 4 it follows that with the growth of a the estimators of variance get closer to theoretical values and the statistical sample of 1 million elements becomes sufficient for $a = 20$ and for $a = 200$.The behavior of the estimator for variance for $a = 2$ issues from the fairly intuitive fact that such

a string has much higher amplitudes for the same impulses acting at it and from the errors that are unavoidable in calculations (the scale of diagrams should be taken into account here).

Figure 5 shows estimators

$$\widetilde{D}_n^{\,2}(u_m(x,t)) = \frac{1}{n}\sum_{i=1}^{n}(u^i{}_m(x,t) - E(u_m(x,t)))^2$$

for $m = 10$, $\lambda = 10$, $a = 20$, $x = l/2$, statistical samples of 1 thousand, 10 thousand, 100 thousand and 1 million elements respectively, distribution of the variable η as presented in the previous remarks, and three different distributions of the variable ς as in remark 3.

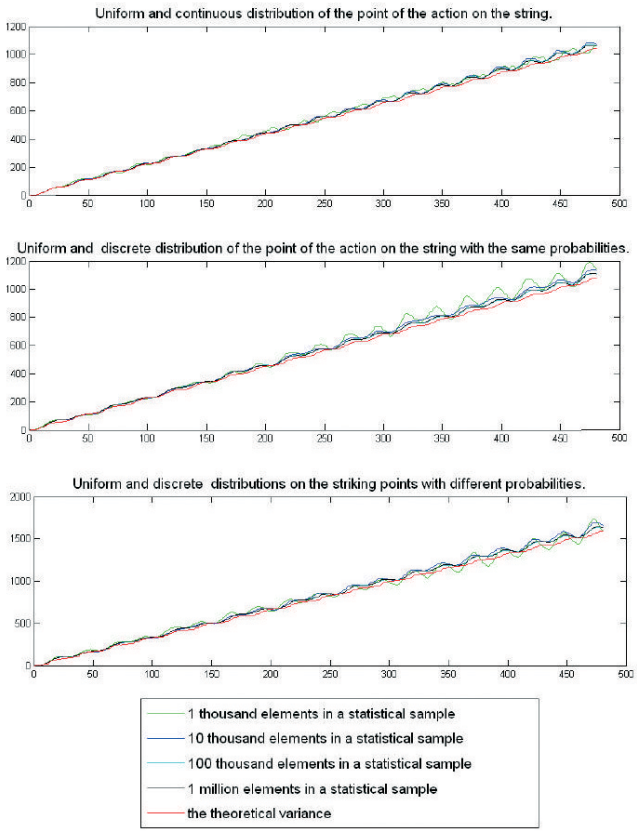

Fig. 5. Estimators of the variance and the theoretical variance

Remark 5

In accordance with the law of large numbers, $\widetilde{E}_n(u_m(x,t))$ and $\widetilde{D}_n^{\,2}(u_m(x,t))$ are convergent respectively with the theoretical expected value $E(u_m(x,t))$ and the theoretical variance $D^2(u_m(x,t))$, when n approaches infinity. Therefore, if we take a large number of such random courses, let us say, 1,000,000, calculate their sum and then divide them by their number, we will always obtain a result close to the diagram presented in Figures 2 and 3 respectively. The differences between theoretical results and simulations are small and do not exceed acceptable statistical deviations. The differences growing with time between $\widetilde{E}_n(u_m(x,t))$ and the theoretical expectation $E(u_m(x,t))$ as well as those between $\widetilde{D}_n^2(u_m(x,t))$ and the theoretical variance $D^2(u_m(x,t))$, which can be observed in all figures, issue from the fact that $u_m(x,t)$ is the sum of increasing number of components, which are random variables.

5.3. The third stage of simulation

The third stage of simulation was aimed at visualisation of the dependence between the distribution of the sizes of particles with the same multiplication product of mean value and the strike rate λ, but with different variances for $m = 10$, $a = 20$, $x = l/2$, statistical samples of 1 million, uniform continuous distribution of the variable ς and distributions η given below.

$\eta \in \{728, 214\}$ and $P(\eta = 728) = 2/3, P(\eta = 214) = 1/3;$

$\eta \in \{728, 42.66\}$ and $P(\eta = 728) = 3/4, P(\eta = 42.66) = 1/4;$

$\eta \in \{728, 385.33\}$ and

$P(\eta = 728) = 1/2, P(\eta = 385.33) = 1/2;$

$\eta \in \{352, 240, 120, 33\}$ and $P(\eta = 352) = 2/3,$

$P(\eta = 240) = 1/9, P(\eta = 120) = 1/9\, P(\eta = 33) = 1/9,$

$\eta \in \{330, 217, 120, 33\}$ and $P(\eta = 330) = 3/4,$

$P(\eta = 217) = 1/12, P(\eta = 120) = 1/12\, P(\eta = 33) = 1/12.$

It is easy to calculate that in the first three cases $E(\eta) = 556.67$, in the last two cases $E(\eta) = 278.33$ and $E(\eta^2)$ is equal to 368588.00, 397943.11, 339231.60, 90723.67, 86889.83 respectively. The values of λ are assumed in such a way that $\lambda E(\eta) = 5566.7$ that is $\lambda = 10$ in the first three cases and $\lambda = 20$ in the last two cases ($\lambda E(\eta)$ represents the mass of the medium flowing through the pipe in the unit of the time).The results of simulation are presented in Figure 6.

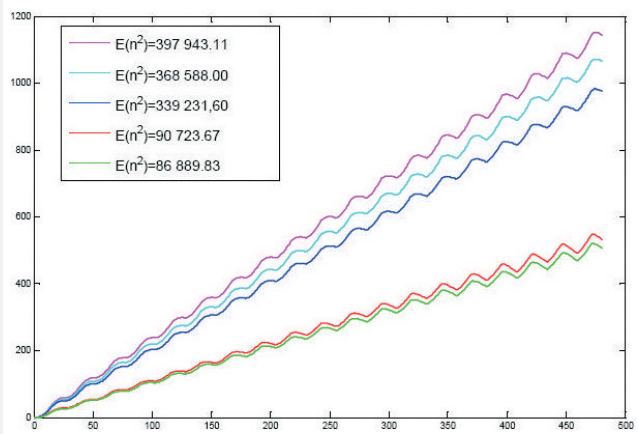

Fig. 6. The variance for five different distributions η

6. CONCLUSIONS

The occurrence of too large particles stimulating motion of the measuring device, with too high probability at the same mean value of transported mass in a unit of time is an un-

wanted effect. Calculation of variance allows for detection of this phenomenon. The diagrams of variance estimators presented above (Fig. 4) suggest how we should conclude about the distribution of stochastic impulses forcing a string from the values of the variance estimators of the process $u(x,t)$ given by (14). As we see, greater impulses imply faster rise of variance in time.

If λ increases, then, at constant flow $\lambda E(\eta)=const$, the lesser the $D^2(u(x,t))$, the closer the size of a falling particle to the mean value and the lesser the probability of a large particle strike. The mean value of the distribution of particle sizes multiplied by the mean strike rate is constant, at least in certain time intervals. The value of variance that we will be able to measure with the methods similar to those used in simulation will inform us about irregularities in the technological process.

Obtaining the results requires further studies of more complex mechanical systems like a string with damping, a membrane or a hollow beam.

REFERENCES

Billingsley P. 1987: *Prawdopodobieństwo i miara*. PWN, Warszawa.

Campbell N. 1909a: *The study of discontinuous phencmena*. Proceedings of the Cambridge Philosophical Society, 15.

Campbell N. 1909b: *Discontinuities in light emission*. Proc. Cambr. Phil Soc., 15.

Hurwitz H., Kac M. 1944: *Statistical analysis of certain types of random functions*. Annals of Math. Stat., 15.

Iwankiewicz R., Nielsen S.R.K. 1996: *Dynamic response of non-linear systems to renewal impulses by path integration*. J. Sound Vib. Mech.

Iwankiewicz R. 2003: *Dynamic systems under random impulses driven by a generalized Erlang renewal process*. In: Proceedings of the 10th IFIP, WIG 7.5 Working Conference on Reliability and Optimization of Structural Systems, 25–27 March 2002, Kansai University, Osaka, Japan.

Iwankiewicz R. 2002: *Dynamic response of non-linear systems to random trains of non-overlapping pulses*. Meccanica, 37.

Jabłoński M., Ozga A. 2006a: *On statistical parameters characterizing vibrations of oscillators without damping and forced by stochastic impulses*. Mechanika.

Jabłoński M., Ozga A. 2006b: *Parameters characterising vibrations of oscilators with damping forced by stochastic impulses*. Archives of Acoustics, 31.

Jabłoński M., Ozga A. 2006c: *Parametry charakteryzujące wymuszone stochastycznie drgania układu o jednym stopniu swobody z tłumieniem nadkrytycznym, podkrytycznym i krytycznym*. WIBROTECH.

Kasprzyk S. 1994: *Dynamika układów ciągłych*. Kraków.

Khintchine A. 1938: *Theorie des abklingenden Spontaneffektes*. Bull. Acad. Sci URSS, Ser. Math., 3.

Plucińska A., Pluciński E. 2000: *Probabilistyka*. Wydawnictwa Naukowo--Techniczne, Warszawa.

Rice S. 1944: *Mathematical analysis of random noise I*. Bell. System Technical Journal, 23.

Roberts J.B. 1965a: *On the Harmonic Analysis of Evolutionary Random Vibrations*. J. Sound Vibr.,1965

Roberts J.B. 1965b: *The Response of Linear Vibratory Systems to Random Impulses*. J. Sound Vibr., 2.

Roberts J.B.1972: *System Response to Random Impulses*. J. Sound Vibr., 24.

Roberts J.B. 1973: *Distribution of the Response of Linear Systems to Piosson Distributed Random Pulses*. J. Sound Vibr., 28.

Roberts J.B., Spanos P.D. 1986: *Stochastic Averaging: an Approximate Method for Solving Random Vibration Problems*. Int. J. Non-Linear Mech., 21 (2).

Rowland E.N. 1936: *The theory of mean square variation of a function formed by adding; known functions with random phases and* applic*ations to the theories of shot effect and of light*. Proc. Cambr. Phil. Soc., 32.

Sobczyk K. 1996: *Stochastyczne równania różniczkowe*. Wydawnictwa Naukowo-Techniczne, Warszawa.

Takác L. 1994: *On secondary processes generated by a Poisson process and their applications in physics*. Acta Math. Acad. Sci. Hungar. 5, 1954.

Gawęcki A. 1994: *Mechanics of slackened systems*. Computer Assisted Mech. Engrg. Sci., 1:3–25.

Tylikowski A. 1991: *Stochastyczna stateczność układów ciągłych*. PWN, Warszawa.