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## APPLICATION OF RESPONSE SPECTRUM METHOD FOR DYNAMIC ANALYSIS OF SPATIAL EARTH STRUCTURES

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### 1. Introduction

Calculations of dynamic response of structures to kinematic excitation are carried out on the basis of Time History Analysis (THA) by numerical integration of equations of motion or on the basis of the quasi-static method — Response Spectrum Analysis (RSA). The classic version of the RSA method leads to a conservative assessment of the dynamic response of structure to kinematic excitation in comparison with the THA method: displacements, stresses and strains obtained from RSA are greater than that obtained from THA. Hence, there is no danger of underestimation of the dynamic response of a structure.

In a typical analysis of the dynamic response of a structure to kinematic excitation, spatial variation of ground motion is commonly neglected. In practice it means that calculations of the dynamic response of a structure to seismic shock are based on the assumption that vibrations at every point of the ground beneath the structure are identical, so the excitation is uniform. However, the influence of the spatial variety of excitation on the dynamic response of large structures may be significant. The observations of earthquakes allowed formulation conclusions concerning the reasons of the occurrence of non-uniform excitation. The following three phenomena are responsible for this effect [7, 11]:

- 1) wave passage effect (difference in time when the wave reaches various points of the structure foundation),
- 2) incoherence effect (loss of coherence resulting from wave reflection and refraction in foundation ground),
- 3) local soil effect (difference in ground conditions in particular points of subsoil beneath a structure).

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Many researches state that on the assumption of non-uniform kinematic excitation the dynamic response is smaller in comparison with the response to uniform excitation. The cause of the dynamic response decreases in this case as do the reduction of the average amplitudes of kinematic excitation. On the other hand, some authors mention that quasi-static effects resulting from the differences in phases and amplitudes of kinematic excitation beneath foundations of structure may lead to an increased global response. The final response of a large structure to non-uniform kinematic excitation depends on mechanical properties and dynamic characteristics of a structure.

In the classic Response Spectrum Analysis equivalent seismic forces are derived on the assumption of uniform kinematic excitation. It may occur that the application of this method does not lead to a conservative assessment of dynamic response for large-dimensional of structure exposed to non-uniform kinematic excitation [8]. The consideration of non-uniformity of the kinematic excitation is especially important for earth dams and embankments. These structures are exposed to spatially different vibrations, since their dimensions are comparable with wavelength of earthquake ground motion [3, 5, 7, 12, 15, 16].

The paper presents the results of the dynamic analysis of a tailings dam to a mining tremor and an earth dam to a seismic shock. Numerical calculation of equations of motion was used on the assumption of uniform and non-uniform excitation. For comparison the response spectrum method was also applied.

## 2. Models of kinematic excitation

The equation of motion of a general multi-degree of freedom structure under kinematic non-uniform excitations could be presented by the following formula [4]:

$$[M_{ss}] \cdot \{\ddot{u}_s^d\} + [C_{ss}] \cdot \{\dot{u}_s^d\} + [K_{ss}] \cdot \{u_s^d\} = (-[M_{ss}] \cdot [R] - [M_{ss}]) \cdot \{\ddot{u}_g\} \quad (1)$$

where:

$s, g$  — degrees of freedom of a structure and ground,

$[M], [C], [K]$  — mass, damping, stiffness matrix respectively,

$\{\ddot{u}_s^d\}, \{\dot{u}_s^d\}, \{u_s^d\}$  — vectors of dynamic components of total accelerations, velocities and displacements of the structure,

$\{\ddot{u}_g\}$  — vector of accelerations of the ground motion,

$[R] = -[K_{ss}^{-1}] \cdot [K_{sg}]$  — transform matrix.

The individual components of the ground accelerations vector  $\{\ddot{u}_g\}$  in equation (1), represent time histories of ground accelerations at particular supports of a structure. These time histories are usually recorded in three directions at one control point and a model non-uniform kinematic excitation is assumed to obtain ground motion at every support.

In this paper, a model of non-uniform kinematic excitation which takes into consideration the wave passage effect was applied for dynamic analysis. The wave passage effect

causes differences in phases of ground vibration under the structure resulting from a finite velocity of wave propagation in the ground. In the model of non-uniform excitation it is assumed that subsequent points of the ground in the direction of wave propagation repeat the same movement with a certain time delay dependent on the wave velocity. Neither loss of coherence of the signal nor possible changes of vibration amplitudes with regard to different local ground conditions were taken into consideration.

For the calculations of the dynamic response of the selected structures, i.e. the earth dam subjected to a seismic shock and the tailings dam exposed to a mining tremor the ABAQUS program was used. Three methods of calculations were applied:

- 1) Time History Analysis with a model of uniform kinematic excitation (*THA\_U*),
- 2) Time History Analysis with a model of non-uniform kinematic excitation (*THA\_N*),
- 3) Response Spectrum Analysis (*RSA*).

The time history analyses were carried out with the Hilber-Hughes-Taylor time integration algorithm provided in the ABAQUS software for a direct step-by-step solution. A time increment 0.01 was applied in the numerical integration of equation of motion. A small value of artificial damping (0.05) was introduced into the system to ensure numerical stability. To examine the dependence of the dynamic response on the earth structures to kinematic excitation the analyses of maximal shear stresses  $\tau$  were performed.

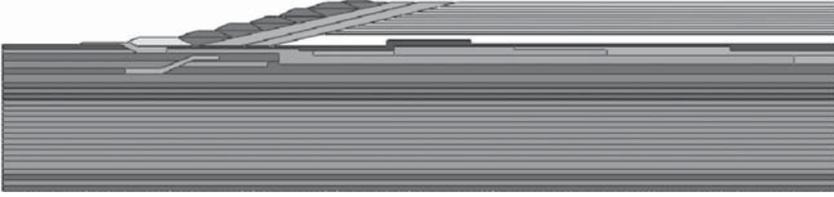
### 3. Dynamic response of tailings dam to mining shock

#### 3.1. FE model of tailings dam

A detailed analysis of the dynamic response was also performed for a tailings dam of a diameter of 5.5 km and a height of maximum 45 m. The length of the dam was 600 m, the total height — 140 m. The geometry of the dam and material data of soils necessary for creating the numerical model of the dam were adopted on the basis of the technical report [6].

The geometry, the different types of deposit materials and subsoil under the dam are shown in figure 1. The points selected for further dynamic analysis were located in the different zones of the structure built of materials which differ in condensation ratio. Point SP was located at the starter dike built of borrow materials, mainly sands (Fig. 1 — yellow colour). Points lettered *SI–S6* were located at dikes erected subsequently and composed of compacted tailings (Fig. 1 — red colour). The dikes have poor seismic resistance since the upstream method of dam construction was used. Points lettered *US1–US6* were located at consolidated zones of tailings beneath the dikes (see Fig. 1 — grey colour). Points *FS1–FS6* were situated in the zone of the impoundment, which consist of compacted fine sand fraction of tailings (Fig. 1 — orange colour).

A two dimensional numerical FE model of the tailings dam was taken into consideration. The dam body was discretized using quadrilateral plain strain elements. Infinite viscous boundary elements were used along the interface between the dam and the valley to avoid spurious reflections of the seismic wave at the artificial boundaries. Further details concerning the numerical model of the dam were presented in [1].



**Fig. 1.** Two dimensional model of the tailings dam

The Mohr–Coulomb constitutive model of soil with a non-associated plastic flow was applied in calculations. It was assumed that the shear modulus of all parts of the tailings dam varied with the height of the dam and depended on the average effective stresses and void ratio of soil. Similar models of dams for dynamic calculations were presented in recent investigations [2, 3, 16].

The shear modulus was evaluated on the basis on an empirical formulae for evaluation of soil properties for use in earthquake response analysis [9]. These equations, based on field tests, enabled to determine shear modulus for different soils in case of small strains under dynamic loading. The shear modulus of the clayey soils was determined from the equation:

$$G_0 = 3270 \cdot \frac{(2.97 - e)^2}{1 + e} \cdot (\sigma'_0)^{0.5} \quad (2)$$

where:

- $G_0$  — shear modulus,
- $\sigma'_0$  — average effective stress,
- $e$  — void ratio.

The shear modulus of all parts which were built of sands and gravel were calculated from the formula:

$$G_0 = 7230 \cdot \frac{(2.97 - e)^2}{1 + e} \cdot (\sigma'_0)^{0.38} \quad (3)$$

The elasticity modulus was calculated according to the equation:

$$E = G_0 \cdot 2 \cdot (1 + \nu) \quad (4)$$

In the dynamic analysis damping matrix was assumed as a linear combination of mass matrix and stiffness matrix. For evaluation of the Rayleigh damping coefficients  $\alpha$  and  $\beta$  were determined from [4]:

$$2 \cdot \xi_1 = \frac{\alpha}{2\pi \cdot f_1} + \beta \cdot 2\pi \cdot f_1 \quad 2 \cdot \xi_2 = \frac{\alpha}{2\pi \cdot f_2} + \beta \cdot 2\pi \cdot f_2 \quad (5)$$

The first natural frequency of the tailings dam (0.93 Hz) was assumed as  $f_1$  in equations (5). Frequency  $f_2$  which appears in formulae (5) was specified as 5 Hz. This is a dominant frequency of mining shock which was taken into consideration as the kinematic excitation of the tailings dam (Fig. 2). The damping ratios  $\xi_1, \xi_2$  for frequencies  $f_1$  and  $f_2$  respectively, according to the results of experiment were assumed on the level of 7%. In view of the foregoing, Rayleigh damping coefficients obtained from the formulae (5) were as follows:  $\alpha = 0.72, \beta = 0.005$ .

### 3.2. Data of mining shock

The dynamic analysis of the dam was carried out for a strong mining shock registered at the base of the dam in horizontal and vertical direction by a system of continuous monitoring [10, 14]. Figures 2 and 3 show the input accelerations in horizontal and vertical direction, respectively, as well as the acceleration response spectra obtained on the basis of the time histories.

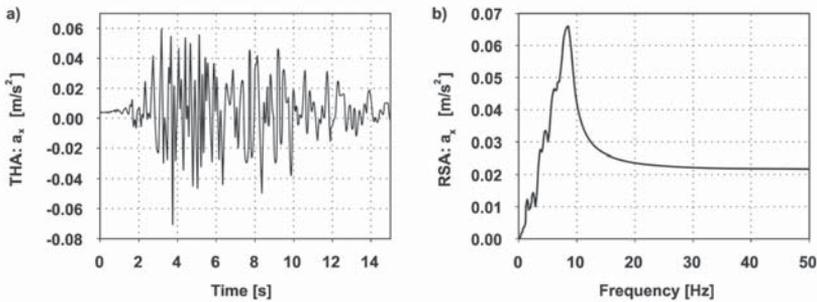


Fig. 2. Acceleration in horizontal direction: a) time history, b) response spectrum

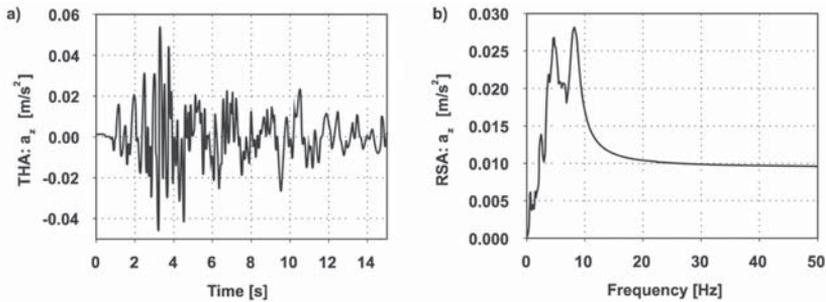


Fig. 3. Acceleration in vertical direction: a) time history, b) response spectrum

### 3.3. Dynamic responses of the tailings dam obtained for different calculation methods

Figures 4 and 5 show the comparison of maximal shear stresses obtained at points  $S3$  and  $FS1$  (see Fig. 1) from different methods of calculations: (a) uniform excitation ( $THA_U$ ) — continuous line, (b) non-uniform excitation ( $THA_N$ ) with the shear wave velocity in the bedrock 1000 m/s — dashed line, (c) response spectrum method ( $RSA$ ) — dotted line.

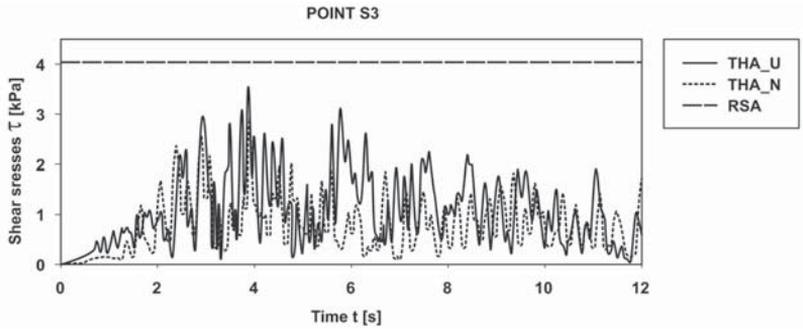


Fig. 4. Maximal shear stresses at point *S3* located in middle dike built of compacted tailings

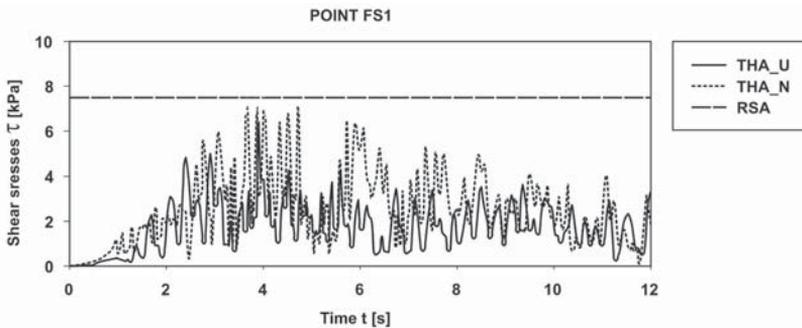


Fig. 5. Maximal shear stresses at point *FS1* located under the impoundment built of fine sand

It could be observed that at the point *S3* (as well as at other points located in the dikes *S1–S6* and beneath *US1–US6*) the shear stresses obtained on the assumption of both uniform (*THA\_U*) and non-uniform kinematic excitation (*THA\_N*) were smaller than the shear stresses obtained from response spectrum method (*RSA*). The same effect appeared at the point *FS1* and at all points situated under the beach zone of the impoundment (*FS1–FS2*). It results from the reduction in the average values of kinematic excitation amplitudes. Quasi-static effects do not play a significant role in the dynamic response of the tailings dam. Stresses depend less on the geometrical changes of the structure occurring in consequence of non-uniform subsoil motion than on the effects of inertia. In this case the response spectrum method leads to conservative assessment — there is no danger of underestimation of the dynamic response of a structure when is applied.

## 4. Dynamic response of earth dam to seismic shock

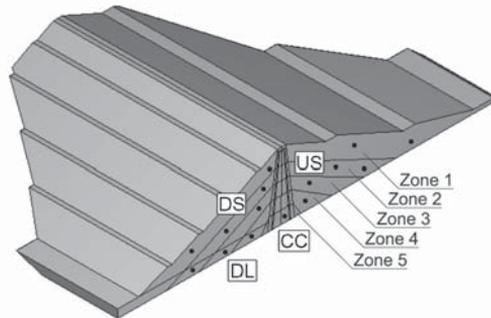
### 4.1. FE model of earth dam

A detailed analysis of the dynamic response was also performed for the Czorsztyn-Niedzica earth dam with a clay core. The dam was subjected to a seismic shock which occurred in 2004 in Podhale region [7]. The main dimensions of the dam were as follows: the length of the crown — 404 m, the height — 56 m, the width of the crown — 7 m, the width at the

base — 256 m. The dam was founded directly on the bedrock. The dam shells, upstream and downstream were composed of sands and gravels, the core of the dam was built of clay, the drainage layer consisted of coarse gravels. The geometry and the properties of the dam material were assumed on the basis of the technical reports [13].

In the dynamic analysis of the earth dam the Mohr-Coulomb constitutive model of soil with a non-associated plastic flow was also assumed. The shear modulus of all parts of the dam  $G_0$  varied with the height. It depended on the average effective stresses  $\sigma'_0$  and void ratio of soil  $e$  according to the formula (2) for the clayey core of the dam and according to the formula (3) for the shell, the filter layers and the drainage layer made of sands and gravels.

For calculations of the dynamic response of the earth dam to seismic shock the ABAQUS program was also used. A three dimensional model of the dam with points selected for further dynamic analysis is shown in figure 6. The dam was divided into five zones, each having a constant value of stress-dependent elastic modulus. Points lettered *US* and *DS* were located in the shell of the dam, upstream and downstream, respectively. Points lettered *CC* were located in the clayey core of the dam. Points located in the drainage layer were lettered *DL*. In the FE model the dam body was discretized using about 8000 first-order tetrahedral and hexahedral elements. The cooperation of the dam and the water in the reservoir was taken into account by applying the Westergaard formula for added mass per unit area of the upstream wall.



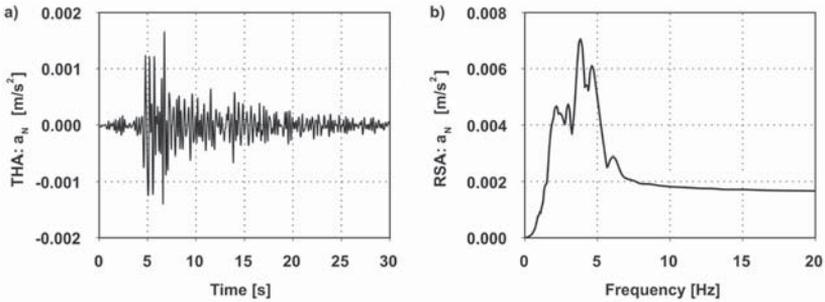
**Fig. 6.** Fragment of three dimensional model of the earth dam

In the dynamic analysis the Rayleigh damping was assumed. The first natural frequency of the dam (2.1 Hz) was assumed as  $f_1$  in equations (5). Frequency  $f_2$  was specified as 5 Hz — a dominant frequency of seismic shock which was taken into consideration as the kinematic excitation of the dam (Fig. 7). The damping ratios  $\xi_1, \xi_2$  for frequencies  $f_1$  and  $f_2$  respectively, according to the results of *in situ* experiments [7] were adopted on the level of 6%. Rayleigh damping coefficients obtained from the formulae (5) were as follows:  $\alpha = 1.1, \beta = 0.003$ .

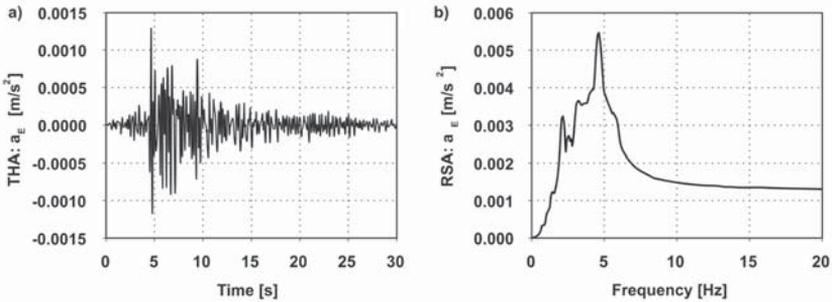
#### 4.2. Data of a seismic shock

The seismic shocks which occurred in Podhale region in November and December 2004 were one of the strongest seismic phenomena ever recorded in Poland. A detailed analysis of

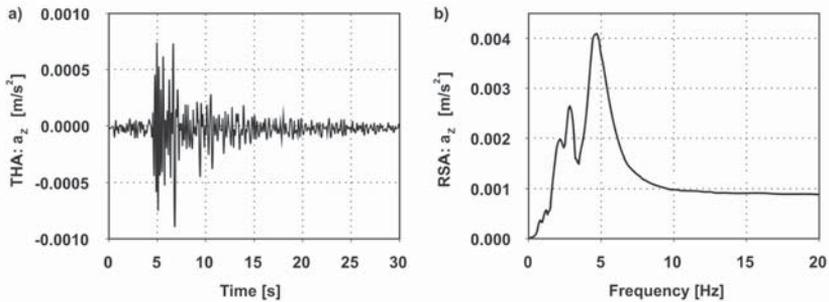
the dynamic response of the earth dam to non-uniform kinematic excitation was carried out for a seismic event of the Richter magnitude 3.7. It took place on the 2<sup>nd</sup> December 2004 and was registered by the local Seismologic Station [7]. The epicentre of the earthquake was located in Czarny Dunajec at a distance of 33 km from the dam, so the dam was affected by the shock. It was assumed that the shock wave propagated along the longitudinal axis of the dam. This assumption resulted from the position of the structure in relation to the shock epicentre. The epicentre was situated directly to the west of the dam and the longitudinal axis of the dam indicates exactly the west-east direction. Figures 7, 8 and 9 show the ground accelerations in three directions as well as the acceleration response spectra obtained on the basis of the time histories.



**Fig. 7.** Bedrock acceleration in horizontal WE direction: a) time history, b) response spectrum



**Fig. 8.** Bedrock acceleration in horizontal NS direction: a) time history, b) response spectrum



**Fig. 9.** Bedrock acceleration in vertical Z direction: a) time history, b) response spectrum

### 4.3. Dynamic responses of the earth dam obtained from different calculation methods

Figures 10, 11 and 12 show the comparison of maximal shear stresses obtained at the points located in the upstream shell (*US*), the drainage layer (*DL*) and at the bottom of the clayey core (*CC*). The shear stresses were obtained from different types of dynamic analysis: (a) uniform excitation (*THA\_U*) — continuous line, (b) non-uniform excitation (*THA\_N*) with the wave velocity in the bedrock 3000 m/s — dashed line, (c) response spectrum method (*RSA*) — dotted line.

It could be observed that at the point *US5* (Fig. 10) located in the upstream shell shear stresses obtained on the assumption of both uniform (*THA\_U*) and non-uniform kinematic excitation (*THA\_N*) are smaller than shear stresses obtained from response spectrum method (*RSA*). The same effect occurred at all points located in the upstream shell (*US*), the downstream shell (*DS*) and at the upper part of the core (*CC*).

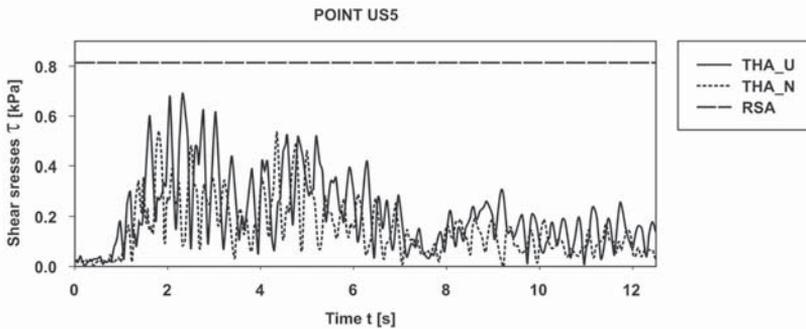


Fig. 10. Maximal shear stresses at point *US5* located in the upstream shell

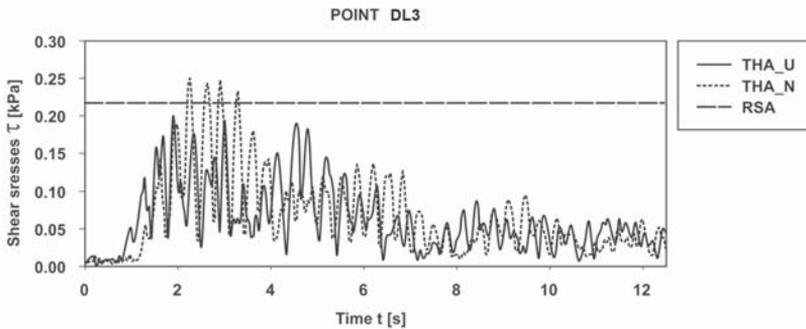
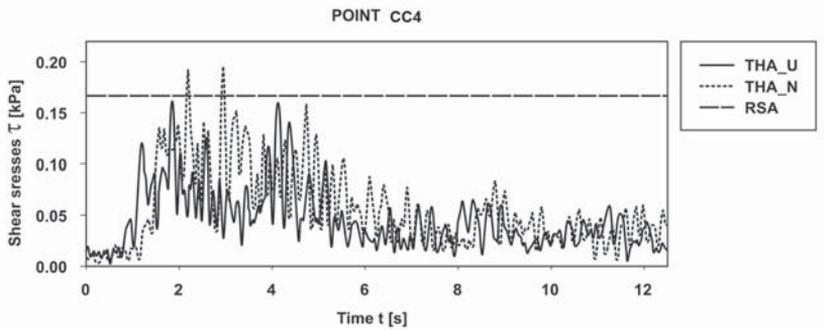


Fig. 11. Maximal shear stresses at point *DL3* located in drainage layer

For the point *DL3* (see Fig. 11) and for all points located in the drainage layer (*DL*) the increase in the maximal shear stresses occurs on the assumption of non-uniform kinematic excitation (*THA\_N*) with respect to the assumption of identical ground motion (*THA\_U*) and with respect to stresses obtained from quasi-static method (*RSA*). This is due to quasi-static



**Fig. 12.** Maximal shear stresses at point *CC4* located at the bottom of the clayey core

effects which result from changes of the bedrock geometry during the seismic shock and affect mainly the lowest part of the dam adjacent to the bedrock. The same effect could be seen at the point *CC4* which located at the bottom of the core (Fig. 12).

## 5. Summary

The following conclusions as well as some general remarks for engineering practice may be drawn from the analyses provided:

- 1) The assumption that non-uniform kinematic excitation causes the decrease in dynamic response at all analyzed points of the tailings dam with respect to the assumption of uniform kinematic excitation. The same effect is observed in the upper parts of the earth dam, i.e. the clay core and the shells both downstream and upstream. So called quasi-static effects are negligibly small. The dynamic response depends less on geometrical changes of the structure than on effects of inertia. The dynamic responses obtained from the response spectrum method are greater than the responses resulted from time history analysis. Hence, the application of the response spectrum method leads to conservative results.
- 2) In the drainage layer of the earth dam and in the lowest part of the core the increase in the maximal shear stresses occurs on the assumption of non-uniform kinematic excitation with respect to the assumption of identical ground motion. The zone is affected by the quasi-static effects resulting from changes of the bedrock geometry during the seismic shock. In these cases ignoring the wave passage effect may cause underestimation of dynamic response of this part of the dam. The shear stresses obtained from response spectrum analysis are smaller than those obtained for time history analysis with the assumption of non-uniform kinematic excitation. Hence, the application of the response spectrum method may lead to non-conservative results.
- 3) The presented results indicate that the response spectrum method does not always lead to conservative assessment of dynamic response of spatial earth structures subjected to kinematic excitation. Application of response spectrum method for spatial earth structures may result in inexpedient underestimation of the dynamic response of a structure.

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