AGH Journal of Mining and Geoengineering • Vol. 36 • No. 2 • 2012

*Roman Kinash *, Oksana Kinash **

AN ESTIMATION OF REINFORCED CONCRETE BEAMS RELIABILITY BY THE METHOD OF HISTOGRAM

1. Introduction

For the analyzing of mechanical characteristics and geometrical sizes of building structures as a rule input (primary) data from measuments are not enough for precise calculations of reliability. Especially in the case of measuring individual objects. But in this case, one can approximately evaluate its reliability [2].

More frequently one needs to evaluate the reliability of the individual object at a known or partly known division of loading with the help of measuring devices one could detect the parameters of construction — materials' strength, geometrical sizes, not etc always accurately, but with limits of interval, expressed with measurement errors nothing is known. About distribution characteristics in the middle of this interval, as a rule, because only a few super precise measuring devices are supplied of the certificate with indicating the distribution of error. Their application in construction is limited, theresore one must be satisfied with the value of relative (or absolute) error, the value of which in general could depend on the value of measured figures.

So, the interval plays the role of input data, where a measured figure is found with a certain percent of accuracy. Obviously to work with the interval, it is reasonable to use designed methods of interval analasis. Having calculated the bearing ability of a construction (in interval presentation) it is possible to compare it with the loading and obtain an approximate estimation of the reliability of the construction [3–5].

When classical methods of mathematical statistics or the Monte Carlo method are used for calculations, it is considered that the functions of changeable parameters of distribution as well as their characteristics are precisely known. Such calculations are often done with data regarding the distribution function, which has been determined with an inadequate reliability level

* AGH University of Science and Technology, Faculty of Mining and Geoengineering, Krakow

resulting from the insufficient input of experimental material or because the approximation of the distribution function was not performed using the best equation. Conclusions about reliability in particular carried out on the basis of in precise formulas are not good at all, this being especially true regarding “the tails” of distributions which mainly contribute to “the number of nines” [10].

Recent developments in mathematics allow the gap between classical methods and the Monte Carlo method to be worked out, and the interval analysis considered earlier in [6], calkulated, by “a transition link” which will be suitable for calculations in such cases where the amount of experimental data is insufficient for an estimation of the distribution function and is evidently excessive for a simple estimation of the interval.

In [1] the method of direct calculations with the use of histograms, is shown a method being based on an aprobated interval algorithms, was offered. The essence of the method is as follows: — it is assumed that inside each bar of the histogram, the distribution of the quantity is uniform; — the width and the number of bars are not regulated and the total area beneath the histogram diagram is always equal to one.

2. Comparing of interval and histogram methods with Monte Carlo method

For the illustration of the application of designed algorythms [7, 8], it was considered as a solution of tasks, which comes to the final formula as well as those, which require the detection of the roots of the system of linear algebra equations (SLAE).

Let’s study a calculation of reinforced concrete structures sound in the roofs of building, which could be done using algorithms worked out previously by interval and histogram methods [2, 4, 5]. The resulting control will be done by comparing these calculations with the Monte-Carlo method (MCM). Let’s choose the normative data for calculations carried out for 50 and 100 years.

TABLE 1
Parameters of snow loads

Locality	**Repeated ness not more often than once in, [Pa]**		**Real observations max., [Pa]**
	50 years	**100 years**	
Ternopil	1362	1516	1054
Kherson	1116	1245	1235
Cherkasy	1420	1580	1900
Poltava	1409	1568	1196

Data given in table 1 carried out using forecasting for 50 and 100 years mean the probability of over loadings are 2% and 1% accordingly if there is no the global climate change.

The calculations were made for two types of metal-concrete roof beam BC6-4AIII (type 1) and BC6-2 AIII (type 2) on series 1.420, which were tested in real conditions to determine the changeability of their physical-mechanical characteristics and geometrical sizes (fig.1).

TABLE 2
Normative values of loading

Beam's type	1	2
Own weight, [kH/m]	1,9	1,9
Roof's weight without snow, [kN/m]	24,0	21,0
Variation, [%]	2	2

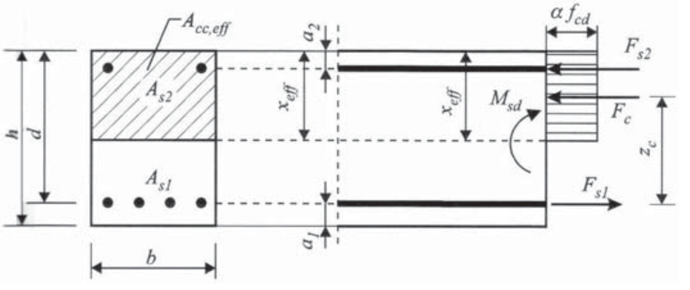

Fig. 1. Calculating scheme of beam

Tasks regarding the reliability of roof constructions are solved by comparing the maximum bending moment caused by external loading, which includes the weight of the beam, roof plates and the isolation and weight of snow with the bending moment in middle section, where the maximum moment in the beam is achieved by taking into account the changeability of its physical-mechanical parameters. As a theoretical probability of steady work, it was set to the attitude of the number of beams, which had not been broken, to the general quantity of numeric experiments. It was shown that before the distribution of physical-mechanical parameters the beam is normal, but with the distribution of annual maximums of snow loading, it is described by Gumbel's curve of the 1st order. Degradation of materials in due course and climate's changes were not taking into account.

The moment caused by an outside loading is given by formula:

$$M = \frac{ql^2}{8} \tag{1}$$

where:

q — is summarizing the distributed loading onto the beam caused by the weight of the roof, while with its own weight and with weight of snow, all summand are of a static nature,

l — and the beam's length, is of a statistical nature.

The moment, that characterize the wearing ability of beam, is set by the formula [9]

$$M_{Sd} \le x_{eff}\, b\left(d - \frac{1}{2}x_{eff}\right) a f_{cd} + A_{s2}\left(d - a_2\right) f_{yd} \qquad (2)$$

where:

f_y — is the calculated resistance of the armature upon the sprain,
f_{yd} — the calculated resistance of the armature upon the compression,
f_{cd} — the calculated resistance of the concrete at compressing,
A_{s2} — a square of a transverse section of the armature,
b — the width of the transverse section of concrete,
d — the working height of the construction's section,
a_1 — the thickness of the lower protecting layer of concrete,
a_2 — the thickness of upper protecting layer of the concrete (fig.1).

TABLE 3
Normative values parameters

Parameters	**Average values of parameters**		**C_{var}, [%]**
	beam's type 1	**beam's type 2**	
d, [cm]	47	47	1.5
f_y, [MPa]	365	365	6
F_{s1}, [cm^2]	12.32	9.42	5
f_{yd}, [MPa]	365	365	6
F_{s1}, [cm^2]	2	2	1.5
b, [cm]	20	20	5
f_{cd}, [MPa]	30	25	12
a_2, [cm]	4	4	16.7

When's solving this task using the integral method. It becomes necessary to note, that the interval method is intended for use with a shortage of data and the impossibility of detecting different kinds of divigion and when the results of only few experiments are known. Comparing results in this case with the MCM method is not quite correct, because mang of the experiments of MCM are much more precise.

Let's assume, that we do not know the distribution of variables, but only their minimum and maximum values, which will be taken as interval limits.

Let's assume, that the experimental data goes through the limits of "three sigma", i.e. cover 97% of sample. For the values of snow loading and let's take its aktual observed annual maximums and minimums:

The software package for solving this task is described in previous works [2, 4, 5]. Let's solve this task using the method of histogram calculations and the let's compare the received results. For calculations using formula (2) with histogram calculations it is necessary to make only one iteration, with the on-line program on Pascal language accounting, using virtual histogram operations.

TABLE 4

Normative values of loading

Populated area	**Really observed annual, [Pa]**	
	minimum	**maximum**
Ternopil	100	1054
Kherson	50	1235
Cherkasy	100	1900
Poltava	50	1196

In this program all variables are declared as a histogram type PGist, and, obviously, when using Pascal this could not be done. With the influence of histograms pre-processor on the program we had a new program, which was difficult to read, but done using Pascal with a provision for the laws of histogram operations.

Now let's compare the results obtained by the Monte-Carlo method as well as the interval and histogram methods. The results, obtained by solving the task using the Monte-Carlo method with histogram (columns), and also by the histogram method — with a range (line). Let's mark with interval limits found by the interval method. Calculations of MCM were carried out for 1000 iterations due to the efficiency of the different methods compared here. There is no sense increasing the accuracy of MCM, as far as the accuracy of calculations begins to exceed the accuracy of raw data. The height of the interval was taken absolutely conditionally, and its square on determination also have to be equal 1 (fig. 2, 3, 4 and 5).

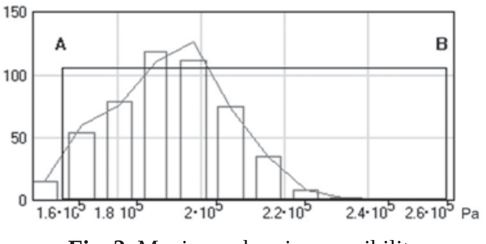

Fig. 2. Maximum bearing possibility of the beam type 1

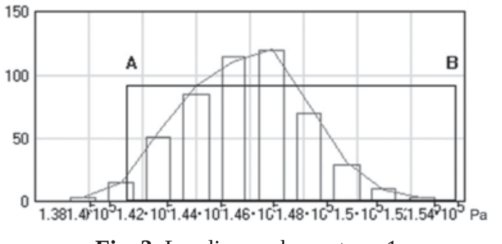

Fig. 3. Loading on beam type 1

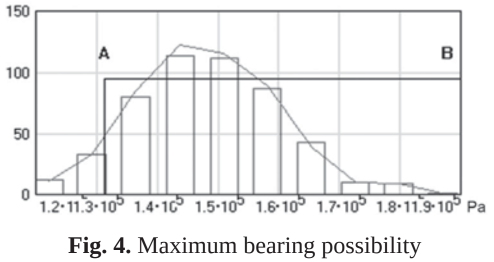

Fig. 4. Maximum bearing possibility of the beam type 2

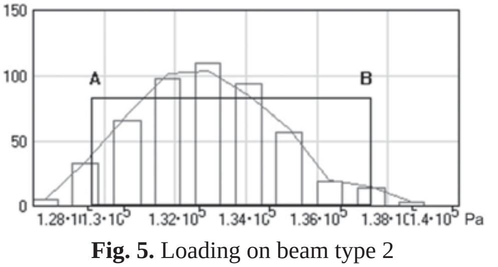

Fig. 5. Loading on beam type 2

As seen from the figures, calculations of the Monte Carlo method which used 1000 iterations and calculations by histogram methods, using the only direct calculation (that practically leads to several expressions, keeping the rules of histogram mathematics) give practically the same results. At the same time interval calculations, which could be acceptable as a last resort, give some value-added lower limit. Such a result is the consequence not of a disadvantage of the interval method of calculation, but due to primary conditions, when a minimum of snow loading was taken as 50–100 Pa, whereas a fact minimum could be equal to 0. On the contrary the upper limit is also a maximum because the most often observed value during the entire period of obseration was chosen as maximum, which is a very rare fact. Obviously MCM and calculations of histogram method don't have such disadvantages because they operate with a kind of distribution, and take into account the frequency of certain values of parameters appearing. Determining the reliability of known loading combinations and the wearing ability of different combinations of their presentation (histogram-interval, interval-interval, number-interval etc.) was considering in previous works [4, 5].

3. Using the results of histogram calculations for estimation the reliability of structures

For comparing obtained data on analogy to the interval reserve index introduced earlier [3] let's use the histogram reserve index.

The histogram reserve index will be named as the relation of the bearing ability of the construction element compared to the loading on it, when both of these values are presented in form of histogram.

The designed pre-processor for the calculations with the histogram presentation of variables allows one to conduct such operations easily. In the majority of practical situations, the existing number of experimental data is not enough for establishing this kind of distribution, but for the differentiation of two similar kinds of distribution it's necessary to have dozens of thousands of examples of experimental data, which is an unreal practically. So in practice, instead of approximating the real histogram by theoretical distribution and then using the parameters of this distribution in calculations, it is reasonable to directly use the original histogram.

Practically introduced histogram reserve index coincides with the classic conception of the reliability function and later on it'll be shown, that using it one can do arithmetic operations, with the usual variable for determining the reliability (histogram reserve index) of construction through the reliability of its elements.

Let's calculate using the histogram reserve index for two types of beam of which the bearing ability and loading was found before (fig. 6, 7):

$$K = \frac{P}{F} \tag{3}$$

where:

P — loading,

F — bearing ability,

K — reserve index).

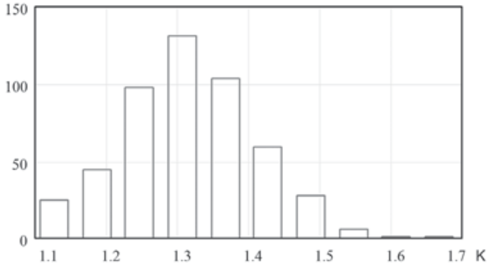

Fig. 6. Histogram presentation of reserve index for beam type 1

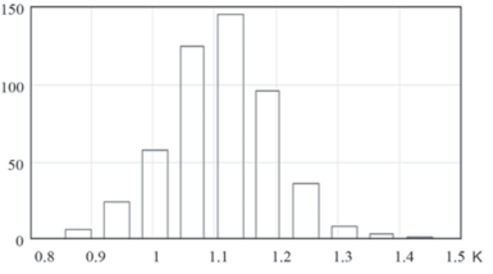

Fig. 7. Histogram presentation of reserve index for beam type 2

The advantage of histogram reserve index over the classic metod for determinating reliability by a number is that in a case when loading is much lower, then the bearing ability doesn't at any time exceed its smallest values in the calculations and reliability is obtained automatically and is equal to 1. However this value tells nothing about the degree of exceeding the strength which could be removed from the point of view of the economy of the material.

Let's evaluate possible variations in the reserve index for different cases of mutual positioning of loading histograms and the bearing ability as it was done for the interval reserve. For determining the searching value, let's use the dividing function of histogram. The samples of histogram reserve indexes for different cases are given below (fig. 8, 9 and 10):

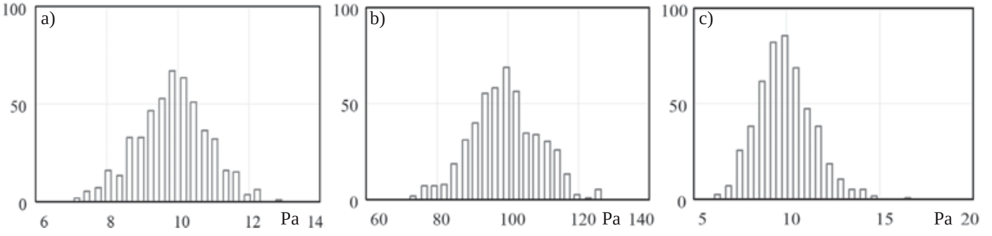

Fig. 8. Variant of positioning 1: a) loading, b) bearing ability, c) reserve index

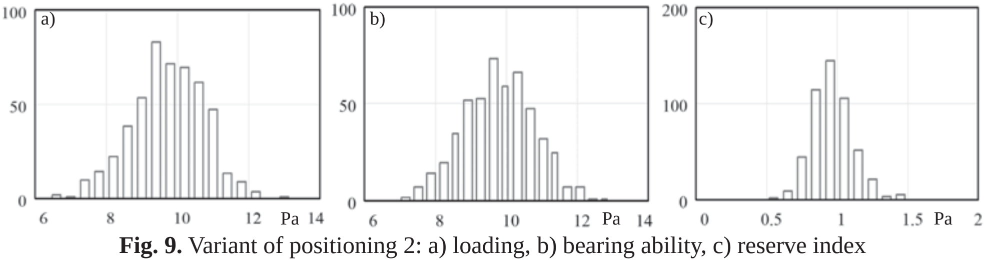

Fig. 9. Variant of positioning 2: a) loading, b) bearing ability, c) reserve index

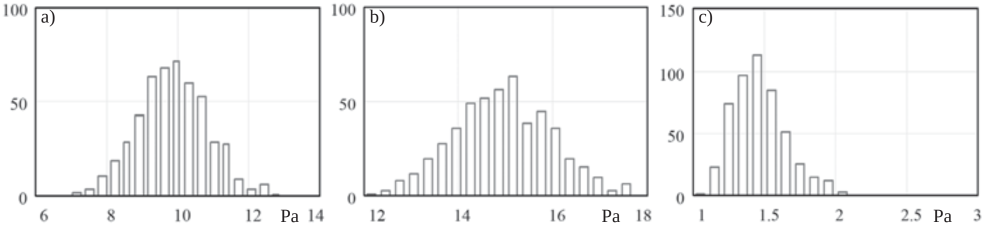

Fig. 10. Variant of positioning 3: a) loading, b) bearing ability, c) reserve index

Variant 1 demonstrates the variation of reserve index in terms, when traditional methods of calculation give a reliability equal to 1.

Variant 3 shows how sharply the ability of obtaining the high reserve level is decreased though both the loading and bearing ability have normal distribution laws which is interesting.

4. Summary

Having analyzed the results received for solving the reliability tasks, which were set by final formulas, one can't give preference to this or that method of calculation. It is necessary to separately study the method of interval calculations which can be used for searching the limits of values of calculated results only when a minimal quantity of probable experimental data (by the way this method has its advantages when there is a necessity to solve by-side tasks as far as the algorithms of solving the system of linear arithmetic equations are designed). Among the MCM and histogram operations which are similar in accuracy, it's necessary to choose a method that allows for its realization in the shortest terms taking into account the time of writing and calculating the program. When writing the program package for frequent use, preference should be given to the histogram operation method.

As it is seen from the diagrams, depending on the distance of positioning on the diagram, the loading from bearing ability and the kind of distribution could be as normal as not to be the such. The application of the method of histogram calculations change from the interval reserve index which give comprehensive information about the calculative resource of a construction (without changes of its physical-mechanical properties through the time). The application of the method of histogram calculations allows one to receive results with a precise level, co-measured with a precise level of raw data without any extra operations as it is used in the MCM (there is no necessity to carry the kind of distribution, generated from casual variables with this distribution, or making a large number of iterations). The only serious problem, which is necessary to solve in the near future is the design of number algorithms for solving the system of linear arithmetic equations in systems with histogram indexes. Most importantly the design for calculating algorithms with depending variables and improving the interface of the software program directed on its simplification could also be introduced.

This research work was performed as a part of the statutory investigations of Technical University of Mining and Metallurgy in Cracow no. 11.11.100.197, financed by the Polish Committee for Scientific Research.

REFERENCES

[1] Berleant D.: *Automatically Verified Reasoning with Both Intervals and Probability Density Functions*. Interval Computational, no 2, 1993. p. 48–68.

[2] Кінаш Р.І.: *Інтервальна оцінка показників надійності будівельних конструкцій*. Матеріали до 37-го Міжнародного семінару з моделювання і оптимізації композитів «Моделювання в матеріаловеденні». Одеса: Астропремт, 1998. с. 151–152.

[3] Кінаш Р.І.: *Застосування інтервального коефіцієнта запасу міцності для оцінки надійності*. Матеріали до 37-го Міжнародного семінару з моделювання і оптимізації композитів «Моделювання в матеріаловеденні». Одеса: Астропремт, 1998 . с. 153–154.

[4] Kinash R.: *Estimation of Reliability of Building Structures by Means of the Method of Interval Calculation*. Materiaøy XLIV Konferencji naukowej. Tom 3. Krynica 1998, Polska. p. 101–107.

[5] Kinash R.: *Creating Software for Reliability Calculation of Reinforced Concrete Structures*. Proceedings of the 13th FIP Congress 1998, Amsterdam, vol. 2. p. 823–827.

[6] Kinash R.: *Application of Interval and Histogram Methods to the Task of Reliability of Building Structures*. Proceedings of the 14th FIP Congress 1999, Praha, vol.2. p. 535–536.

[7] Kinash R.I.: *Interval-analysis-based Methods of Calculation of Building Structures Reliability*. Górnictwo i Geoinżynieria. Kwartalnik AGH. 2003. r. 27 z. 3–4. p. 359–364.

[8] Kinash R.I.: *Concept of Safety Factor’s Histogram in the Reliability Calculations of Reinforced Concrete Structures*. Górnictwo i Geoinżynieria. Kwartalnik AGH. 2003. r. 27 z. 3–4. p. 349–357.

[9] Łapko A.: *Projektowanie konstrukcji żelbetowych*. Warszawa, Arkady 2001.

[10] Perelmuter A.B.: *Selected Problems of Reliability and Safety of Building Constructions* [in Russia]. Kiev: UkrNII-Proektstalkonstruktsiya, 2000.