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SELECTED ASPECTS OF TRANSPORTATION SYSTEM MODELLING

Abstract: The paper presents selected aspects of the transport services market, particularly issues related to the transportation system modelling. A method for scheduling of regional transport services through formulation of an optimisation problem is discussed.

Keywords: transportation system, transportation services modeling, transportation servicing schedule.

1. Introduction

Transport is the movement of people and/or goods from one geographical location to another. Such movement requires transport services, as companies need to be supplied with materials (or parts or semi-finished products) for production, people need to be carried to work, etc.

The demand for transport services may also result from a dispersed geographical structure of production, transport direction structure or changes in the market supply or product distribution (Ambroziak, Pyza, Ratkiewicz 2005).

In order to achieve their production, production and business or business objectives, companies must assure that the marketed product is delivered to retail outlets and offered to consumers in right quantity and quality, in the right place at the right time, and at acceptable cost (Pyza, Ratkiewicz 2004). Hence, distribution is an intermediate link between production and consumption, and its purpose is to fill gaps between these two spheres.

Efficient functioning of any product distribution system is assured by the conveyance system optimisation (Całczyński 2001).

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2. Characteristics of transportation systems

A conveyance system is defined as a transportation system which performs goods movement tasks through conveyance.

Transport (conveyance) tasks, which are determined by customer needs, are characterised by the three major quantities (Piasecki 1973):

- type and volume of cargoes to be carried,
- cargo movement relation,
- cargo movement deadline.

Depending on time requirements for cargo movement, conveyance systems may be divided into specific-time transportation systems, in which transport tasks need to be performed at specific time instants, and non-specific time transportation systems, in which there are no such specific time requirements (generally, transport tasks need to be performed at any time within a predefined period).

Another division of conveyance systems is a division into regular and irregular transportation systems. Regular conveyance is defined as performance of transport tasks in response to recurring transport demand (e.g. at daily, weekly, monthly or yearly intervals). If no such regular intervals may be identified and transport tasks are performed as the need arises, a conveyance system is referred to as irregular (Piasecki 1971, 1973).

The transport organisation is determined by the schedule of movement of conveyance units. Such a schedule defines when and where the given cargo is to be delivered and by which conveyance unit. Subsequent stages of cargo transport determine the differentiation between a schedule of movement of conveyance units and a schedule of movement of cargoes. Cargo transport may require cooperation of various conveyance systems, which are components of more complex transportation systems. A necessary stage of any cargo transport process is reloading, which is either direct reloading from one means of conveyance onto another or indirect reloading with short-term cargo storage in between.

Some specialised companies, referred to as forwarding agents, have been established to assure the effective organisation of cargo movement and their task is to coordinate the operations of different conveyance systems (Ambroziak, Pyza 2006). Such companies often operate no means of conveyance or warehouses of their own, and simply offer coordination services with respect to different types of transport involved in the transportation process.

Ultimately, cargo movement in time and space results from performance of subsequent movement stages, which are effected by movement of conveyance units (different at each stage) (Pyza 2004).

In such case, the quantity sought is a schedule of movement of conveyance units, while the criterion for its assessment may be the cost of performance of transport tasks.

3. Transportation network identification

A transportation system is a system in which material objects are moved in time and space. Such a system is identified by a structure plus characteristics of structure elements, while the concept of movement is related to conveyance units travelling through a transportation system, and referred to as cargo flows through transportation system elements (Leszczyński 1994).

For the purpose of modelling, a transportation system structure is represented by a graph, which maps the transportation system nodes and the connections between them. Some quantitative characteristics (connection distance, connection capacity, travelling time, etc.) are defined on graph vertices and edges.

Depending on the subject of analysis, graph vertices may be interpreted as railway stations, bus stops, airports (in the case of air transport) etc., whereas graph edges represent immediate connections between the transportation system nodes (represented by graph vertices).

In this approach, a structure of a transportation system is represented by an ordered triple:

$$G = \langle \boldsymbol{I}, \boldsymbol{U}, \hat{\boldsymbol{R}} \rangle \tag{1}$$

where:

$\boldsymbol{I}$ – finite set of transportation network nodes, i.e. $\boldsymbol{I} = \{1, 2, ..., i, ..., j, ..., I\}$,

$\boldsymbol{U}$ – set of transportation network connections, i.e. $\boldsymbol{U} \subset \{u : i, j \in \boldsymbol{I}\}$, in which $\langle i, j \rangle$ is an ordered pair which represents a directed edge between the end vertices,

$\hat{\boldsymbol{R}}$ – ternary relation, $\hat{\boldsymbol{R}} \subset \boldsymbol{I} \times \boldsymbol{U} \times \boldsymbol{I}$, which meets the following conditions:

- for each edge $u \in \boldsymbol{U}$ there exists a pair of vertices $i, j \in \boldsymbol{I}$ such as $\langle i, u, j \rangle \in \hat{\boldsymbol{R}}$,
- if for a certain edge $u \in \boldsymbol{U}$ there exist $\langle i, u, j \rangle \in \hat{\boldsymbol{R}}$ and $\langle v, u, z \rangle \in \hat{\boldsymbol{R}}$, then $i = v$; $j = z$ or $i = z$; $j = v$.

In a number of applications, it is often required to define some quantitative characteristics of the system structure elements. Then, the concept of network (Leszczyński 1994) within the meaning of the graph theory is used for system description.

Such a network is defined as an ordered triple:

$$S = \langle \boldsymbol{G}, \boldsymbol{F}_I, \boldsymbol{F}_U \rangle \tag{2}$$

where:

$$G = \langle \boldsymbol{I}, \boldsymbol{U}, \hat{\boldsymbol{R}} \rangle \tag{3}$$

is a graph, whereas:

$\boldsymbol{F}_I = \{f_1,\ f_2,\ \ldots,\ f_n,\ \ldots,\ f_N\}$ is a set of functions defined on the set $\boldsymbol{I}$ (set of vertices of the graph $\boldsymbol{G}$):

$$f_n : \boldsymbol{I} \longrightarrow \Re^+,$$

$$f_n(i) \in \Re^+, n = 1, 2, ..., N.$$

$\boldsymbol{F}_U = \{g_1,\ g_2,\ ...,\ g_k,\ ...,\ g_K\}$ is a set of functions defined on the set $\boldsymbol{U}$ (set of edges of the graph $\boldsymbol{G}$):

$$g_k : \boldsymbol{I} \times \boldsymbol{I} \longrightarrow \Re^+,$$

$$g_k(i, j) \in \Re^+,\ \ k = 1, 2, ..., K$$

and the quantities $f_n(i)$ and $g_k(i, j)$ have some clearly defined interpretation.

4. Optimisation problem of regional transport service scheduling

Let us consider distribution of goods from a depot to cargo delivery points to assure that the orders related to each point are fully executed. Goods are distributed by means of conveyance. Thus, it is necessary to schedule movement of transport units to minimise the aggregate time of occupation of transport routes by all means of conveyance.

For the purpose of formal description of the issues of transportation system modelling, let as assume that the demand for transport services is communicated at a number of cargo delivery points or cargo shipment points. Let $\boldsymbol{I} = \{1, 2, ..., i, ..., I\}$ be the set of identifiers of cargo delivery/shipment points, where i represents the cargo delivery/shipment point i, while $\boldsymbol{I}$ is the total number of such points.

We assume that the need for transport services is expressed by the volume of cargo to be delivered to or picked up from the point $i \in \boldsymbol{I}$, denoted by Q_i.

We further assume that a transport company which renders transport services has some specific means of conveyance. Let $\boldsymbol{S} = \{1, 2, ..., s, ..., S\}$ be the set of identifiers of means of conveyance, where s represents the means of conveyance, while S is the total number of means of conveyance.

Furthermore, let means of conveyance be characterised by its capacity q_s and mean velocity $V_s = V$. In addition, the opening time $t_B^{'}$ and the closing time $t_B^{''}$ of the depot (distribution centre), from which cargoes are to be carried, and the opening times $t_i^{'}$ and the closing times of individual points $i \in \boldsymbol{I}$ are known.

In addition, means of conveyance require some service time at the depot (loading) and at the delivery point (unloading). Let Δt_B be the loading time at the depot and Δt_i the unloading time at the cargo delivery point $i \in \boldsymbol{I}$. The lengths of direct connections (the distances) between cargo delivery points and the depot are also known and given in the matrix $[d_{ij}]_{(n+1)\times(n+1)}$.

As the velocity of each means of conveyance, $V_s = V$, is known, we can determine the following:

- travelling time of a means of conveyance between the depot and the cargo delivery point i, t_{B_i},
- travelling time of a means of conveyance between the cargo delivery point i and the cargo delivery point $j \in \boldsymbol{I}, t_{B_{ij}}$.

These travelling times are determined from the following formulas:

$$t_{Bi} = \frac{d_{Bi}}{V_s} \text{ and } t_{ij} = \frac{d_{ij}}{V_s} \tag{4}$$

and presented in the matrix $[t_{ij}]_{(n+1)\times(n+1)}$.

4.1. Formulating a Regional Transport Service Optimisation Problem

Find the values of the decision variables:

$$x_{ijk} = \begin{cases} 1, & \text{when the point } p_j \text{ occurs along a route } k \\ & \text{directly after point } p_i \\ 0, & \text{otherwise} \end{cases}$$

to minimise the objective function:

$$F = \sum_{k=1}^{K} \sum_{i=0}^{I} \sum_{j=0}^{I} t_{ij} x_{ijk} \tag{5}$$

subject to the following constraints:

- Each service point i is assigned to exactly one route:

$$\sum_{k=1}^{K} \sum_{i=0}^{I} x_{ijk} = 1 \quad j = 1, 2, ..., I \tag{6}$$

- Each service point j is assigned to exactly one route:

$$\sum_{k=1}^{K} \sum_{j=0}^{I} x_{ijk} = 1 \quad i = 1, 2, ..., I \tag{7}$$

- Route continuity condition:

$$\sum_{i=0}^{I} x_{ipk} - \sum_{j=0}^{I} x_{pjk} = 0 \quad k = 1, 2, ..., K; \quad p = 0, 1, ..., I \tag{8}$$

– Each route includes the depot p_0:

$$\sum_{j=1}^{I} x_{0jk} \leqslant 1 \quad k = 1, 2, ..., K \tag{9}$$

$$\sum_{i=1}^{I} x_{i0k} \leqslant 1 \quad k = 1, 2, ..., K$$

– Limited capacity of a means of conveyance moving along a route:

$$\sum_{i=1}^{I} Q_i \sum_{j=0}^{I} x_{ijk} \leqslant q_s \quad k = 1, 2, ..., K \tag{10}$$

– Limited travelling time for a route:

$$\sum_{i=0}^{I} \sum_{j=0}^{I} t_{ij} x_{ijk} \leqslant r \quad k = 1, 2, ..., K \tag{11}$$

5. Application for the transport route optimisation

The application OPTYMALIZACJA TRAS DOWOZU [TRANSPORT ROUTE OPTIMISATION] can be used for regional transport service scheduling.

The application is an important aid in the decision-making process concerning cargo distribution.

The application consists of three major modules:

– **Data entry module** (Figs 1–2) – procedures for defining the following input data:
  - total number of service points,
  - volume of demand at service points,
  - total number of means of conveyance operated by a transport company plus their characteristics (capacity and mean velocity),
  - depot (distribution centre) opening and closing times,
  - service times at the depot (loading) and service points (unloading),
  - matrix of direct connections between service points and the depot, containing mean travelling times.

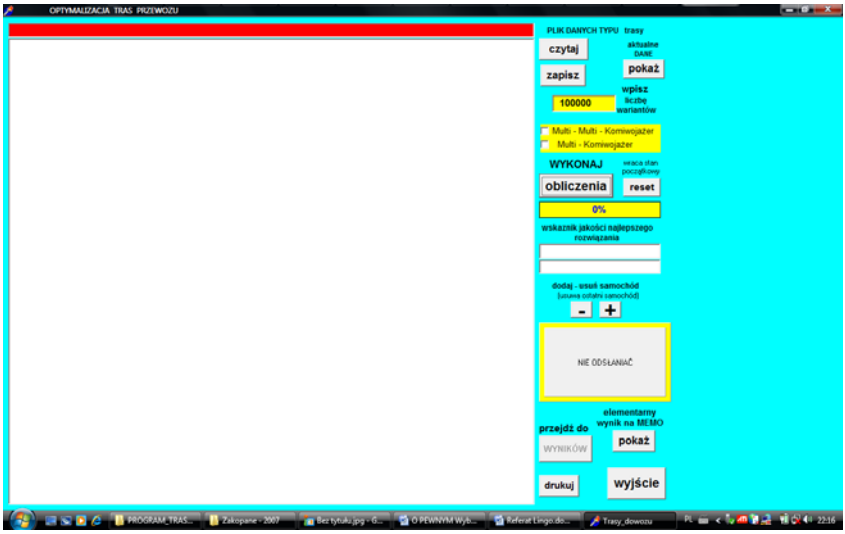

Fig. 1. *TRANSPORT ROUTE OPTIMISATION application: data entry screen*

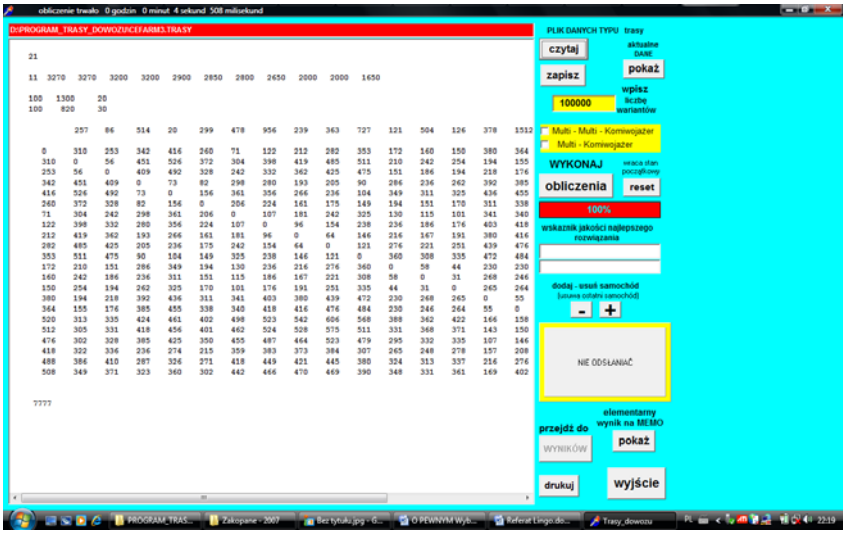

Fig. 2. *TRANSPORT ROUTE OPTIMISATION application: entered data screen*

– **Computational module** – procedures for determining:
  - total number of means of conveyance for the provision of transport services,
  - movement schedules for each means of conveyance used,
  - operating schedules for each means of conveyance used,
  - aggregate time of operation and aggregate unused capacity of means of conveyance as a schedule quality measure,
  - service point operation schedule.

– **Results visualisation module** (Figs 3–6) – procedures for visualisation of results and generation of reports.

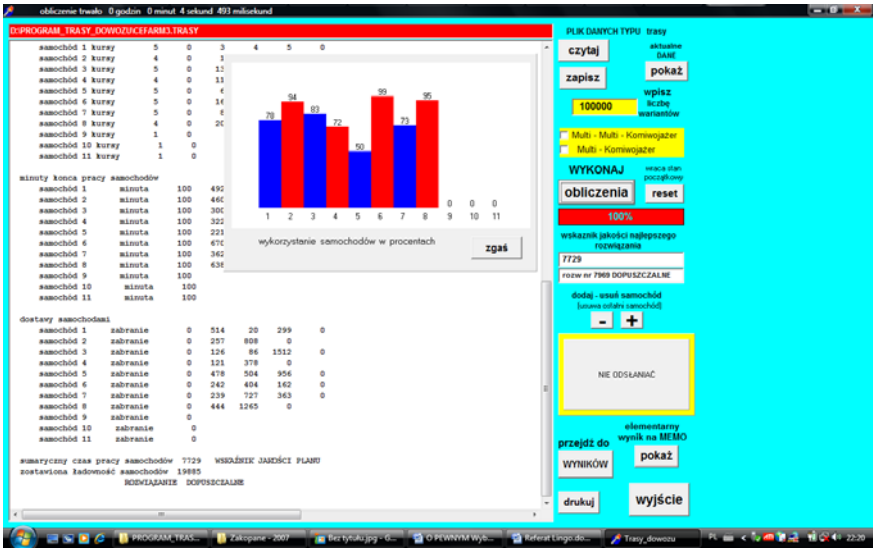

Fig. 3. *TRANSPORT ROUTE OPTIMISATION application: end results screen*

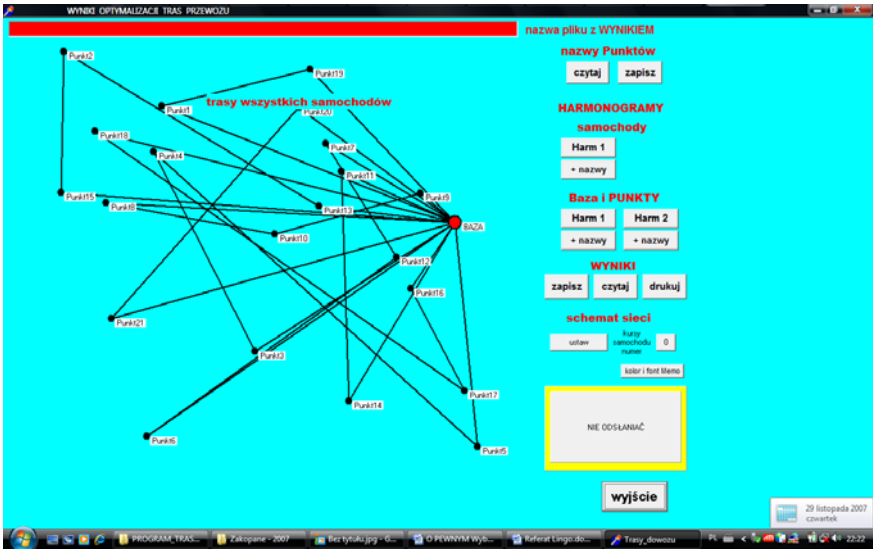

Fig. 4. *TRANSPORT ROUTE OPTIMISATION application: graphical presentation of end results screen*

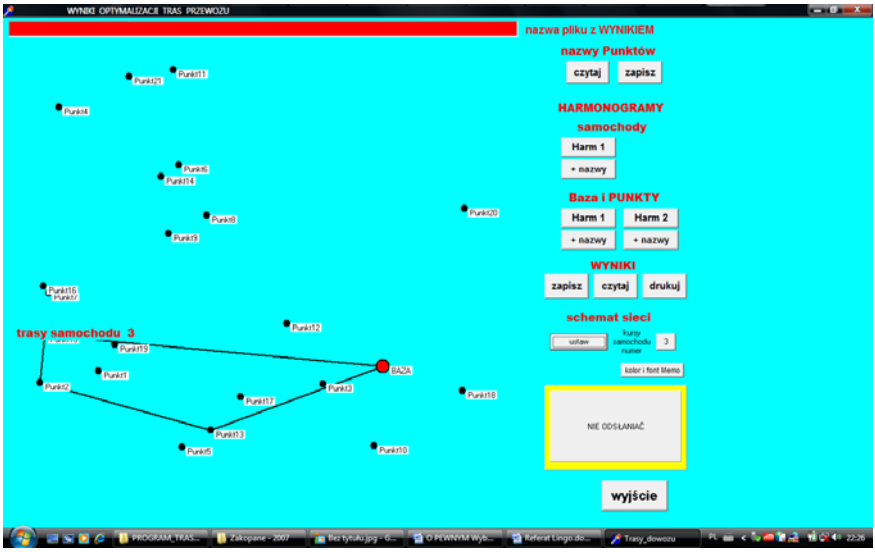

Fig. 5. *TRANSPORT ROUTE OPTIMISATION application: graphical presentation of end results screen (for one route only)*

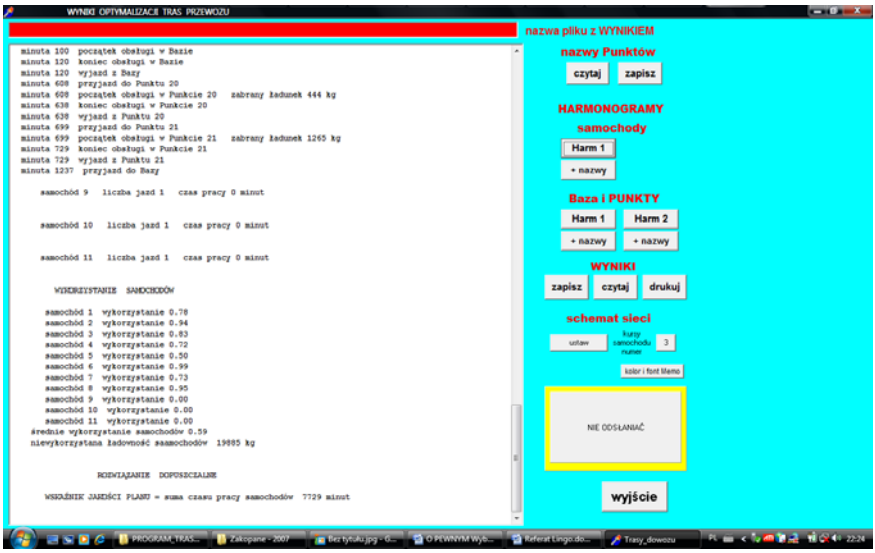

Fig. 6. *TRANSPORT ROUTE OPTIMISATION application: vehicle operation schedule screen*

The optimisation procedure uses the random allocation of resources method supported by an evolutionary method with a limited set of genetic algorithms that enable directed improvement of the outcome. Hence, in theory, the outcome is the best schedule from the set of feasible schedules.

In practice, however, the selected schedule may be considered to be the optimal, as 10 million to 100 million different schedules are reviewed in the process.

6. Conclusions

Transport companies which render conveyance services are an important link in the chain of goods distribution to customers. Customers who formulate demand for transport services differ in their requirements and expectations with respect to service providers. They expect the latter to provide high quality transport services for an acceptable price. Hence, there is a problem of optimising the costs of performance of transport services in the given market conditions. A major factor which affects the transportation cost optimisation is the optimal use of the available means of conveyance, while meeting customer requirements.

The decision making process within a transport company may be supported by the software application OPTYMALIZACJA TRAS DOWOZU (TRANSPORT ROUTE OPTIMISATION), which enables the optimisation of both transport routes and allocation of means of conveyance to transport tasks.

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