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AGGREGATE PLANNING IN MANUFACTURING COMPANY – LINEAR PROGRAMMING APPROACH

Abstract: The authors take into consideration the process of translating demand forecasts (or sale plans) into production plan. In the first part of the paper the characteristics and parameters of a aggregate planning are summarized. The main part contains the proposal of integer programming model which supports aggregate planning decisions. In the proposed model production quantities, timing of inputs, inventory levels and workloads are considered. The last part of the paper contains the presentation of computational result obtained by using the introduced model.

Keywords: supply chain, operational research, production capacity.

1. Introduction

Sales and operations planning is getting more and more popular with enterprises which cooperate in chains of supply. According to AMR Research [3, p.17] report, in 1998–2004, companies in the USA spent almost 12 billion dollars on software which enhance supply chain planning. Increased interest in S&OP procedure is caused by an aspiration to broaden the scope of analysis from one enterprise to several enterprises, where each of them is a part of a common supply chain. Planning a business activity of a few independent companies, on the operational level, is not possible by means of the same tools which are successful in planning a business activity of a single company. These difficulties result in the fact that a bigger impact is put on planning on the tactic level, which, although less detailed, allows to include in the analysis a bigger functional range (e.g. shared sales, production and purchase planning in the chain of supply).

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2. S&OP procedure

S&OP procedure was described many times by researchers and practitioners in various publications. L. Lapide'a [3, p.17] is considered a precursor of this approach. He published, in The Journal of Business Forecasting (2004–2005), a series of articles concerning the abovementioned procedure. In his first article, the author enumerates 12 main factors of success while using S&OP procedure. Two of these factors stress the importance of applying, in the process, operational research and information technology.

The first of them is the usage of integrated supply-demand planning technology. The tools to forecast and control demand, which are in use nowadays, are not the standard element of computer systems ERP class. This situation is caused by the variety of approaches which are available. These tools are very often a set of different spreadsheets, which were created by people directly involved in forecasting procedure. These spreadsheets are not integrated with one another (different spreadsheets for different groups of products), and they do not also include the information contained in the computer systems of supply plans (purchase plans, the actual state of the warehouse, or production capacity). In order to carry out the S&OP procedure correctly all the elements of planning system should be integrated, both, on the supply and demand side.

Considering external inputs in the S&OP procedure is the second important factor. The usage of additional data from the external systems such as VMI, CPFR, or the information from the points of sales (POS), enables us to plan thoroughly the operation of a company and synchronize the supply and demand operations.

3. The scope of procedure's support

The implementation of S&OP procedure is a complicated business task, which requires some work on the three levels: organization, interpersonal and factual. Apart from work with the participants of the procedure, on the schedule and course of meetings, simultaneously the work is done in the field of computer systems. The aim is to provide the participants with the latest data and equip them with some appropriate tools allowing to make quick decisions. Fields, where operational research can be applied, are determined by the specificity of the supply chain including cost parameters.

In the following part of this paper two models of mixed integer programming (MIP) are presented. They were established and implemented in the supply chain consisting of production and trade company. Ensuring the fluent flow of information about the plans and sales, and quick determining the level of utilization of production capacity in specific stages of production plan were the main goals of their implementation. The procedure, used before the introduction of S&OP model, which try to determine the capacity by calculating the overall number of manufactured products even when the assortment was low, gave the incorrect results. These actions led to a situation in which the stock, from which the sales was carried out, was increased,

even when the planned production was not carried out. The implementation of operational research allowed to reduce the times of changeovers which in the modeled production process can last longer than the production time itself.

4. Production process

The production process carried out by the analyzed enterprise is a discreet two stage process. In the first stage half-finished products are manufactured in the sets where the size of every of these products is fixed. One set of the half-finished products is manufactured during one or more operations carried out in a sequence. One complete process of manufacturing a half-finished product has to be carried out by means of one of the available machines. Finished products are manufactured in the second stage. They consist of exactly one half-finished product, produced in the first stage, and some wrapping materials. This process is also carried out in series of different size, at the same time one series of finished products can only consist of just one series of half-finished products. Analogically, one complete process of manufacturing a finished product has to be carried out by means of one of the available machines.

In the production process two types of changeovers can be distinguished: short and long changeover. Short changeover is carried out when on the machine, in turn, the same half-finished products or finished products, made of the same half-finished products not including wrapping materials, are manufactured. Long changeover is required when, on the machine, in turn, different half-finished products or finished products, made of different half-finished products, are manufactured. Changeover times are different for different machines and they are not determined by half-finished products or finished products.

5. Operational research models

The basic problem in the process of verification the production capacity level, in the described example, is an appropriate ascribing of manufactured series of half-finished products and finished products to machines in the whole planning procedure. The number of products and the variety of technological routs are the reason for the fact that people who planned the process needed even 2 weeks to carry it out. That's why in the first stage of the work a operational research model has been suggested. This model allows to ascribe the production series to the machines in a way that the production capacity is only slightly exceeded. The model, presented in the next part of this study, is used to analyze one planning period. Its application to the whole planning horizon is possible, but the model should be used separately for every planning period. This form of model is not a classical formulation which enhance planning. In models like this, the planning process is carried out for the whole planning horizon, and one of the basic variables included in the model are stocks on hand in the warehouse. Evaluating the production capacity was the main goal of putting into operation the model in the analyzed example. The size of production

and appropriate levels of stock on hand are chosen by employees of the company. In the model written by the formulas (1) to (5) two groups of decision-making variables x_{ijk} and z_{max} were used. The first group are binary variables which task is to ascribe operation j manufacturing a series of half-finished and finished products i on the machine k. The variable has the value 1 if an according operation is carried out on the machine, or 0 otherwise. The list of series i is generated, on the basis of production plans and the size of series of given half-finished products, before the operational research process starts. Another group is a variable z_{max} which can have any real values. For the positive values it is interpreted as the highest exceeding of machine's capacity, whereas negative values correspond to the lowest reserve of the machine's capacity.

$$z_{\max} \to \min \tag{1}$$

$$\sum_{k \in \left\{k : c_{jks_i} > 0\right\}} x_{ijk} = 1 \qquad \forall i, j \tag{2}$$

$$z_{\max} + D_k - \sum_{i,j} (c_{jks_i} + h_k^A) x_{ijk} \geqslant 0 \qquad \forall k \tag{3}$$

$$x_{ijk} \in \{0, 1\} \quad \forall i, j, k \tag{4}$$

$$z_{\max} \in R \tag{5}$$

The target function in the form of the formula (1) ensures that in the operational research procedure there will be found a solution which ascribe manufacturing operations to the machines in a way that will guarantee the highest minimal exceeding of the capacity or the lowest maximum reserve of the capacity. The limitation (2) assures that every operation in the production series is carried out only once on a given machine. The symbol s_i denotes the index of the product manufactured in series i, while c_{jks_i} denotes the time of the manufacturing operation j production of the half-finished or finished product s_i on the machine k. In the situation when half-finished or finished product cannot be manufactured on a given machine, the parameter c_{jks_i} has got value 0. Suggested form of limitation (2) assures that on the left side of the limitation the variables, which match the possible operations to be carried out – number of the operation, a machine and a product, are only taken into consideration.

The limitation (3) joins the utilization of production capacity with the variable z_{max}. The available working time of the machine in the period when the plan is carried out was denoted by the symbol D_k whereas h_k^A stands for the short changeover of the machine k. Limitations (4) and (5) are bank limitations for the decisional-making variables. As it can be observed, in the suggested model, only times of short changeovers were considered. Initial verification of this model has some positive results. After generating the task which contained the real number of production series, it was possible to solve it in a short period of time (less than 1 second) by means of a simple software like *lp solve*. Moreover, the analysis carried out by planners proved the greater accuracy than, used before, total number of manufactured products. As a part of further research a second version of the model was established. This for-

mulation, apart from the assurance that the plan will be fulfilled (if it's possible), minimizes the times of changeovers, including long and short changeovers. In the model denoted by the formulas (6)–(12) three groups of decision-making variables x_{ijk}, z_{max} and y_{pjk} were used. The first two have got the same interpretation as in the earlier formulation, whereas the variable denoted as y_{pjk} is a binary variable which has got the value 1 if the operation j of manufacturing the product p is carried out on the machine k. As opposed to the variable x_{ijk} which corresponds to ascribing the operation of a specific production series to the machine, the variable y_{pjk} allows us to tell if, at least, one operation j of a production series of half-finished or finished product p is manufactured on the machine k. Carrying out the operation of a production series of half-finished or finished product requires to perform a long changeover.

$$M z_{\max} + \sum_{p,k,j \in \left\{j : c_{jkp} > 0\right\}} (h_k^B - h_k^A) y_{pjk} \to \min \tag{6}$$

$$\sum_{k \in \left\{k : c_{jks_i} > 0\right\}} x_{ijk} = 1 \qquad \forall i, j \tag{7}$$

$$z_{\max} + D_k - \sum_{i,j} (c_{jks_i} + h_k^A) x_{ijk} - \sum_{p,j \in \left\{j : c_{jkp} > 0\right\}} (h_k^B - h_k^A) y_{pjk} \geqslant 0 \geqslant 0 \qquad \forall k \tag{8}$$

$$\sum_{i \in \left\{i : s_i = p\right\}} x_{ijk} \leqslant M y_{pjk} \qquad \forall p, k, j \tag{9}$$

$$x_{ijk} \in \{0, 1\} \qquad \forall i, j, k \tag{10}$$

$$z_{\max} \geqslant 0 \tag{11}$$

$$y_{pjk} \in \{0, 1\} \qquad \forall p, k, j \tag{12}$$

In the target function denoted by the formula (6) the variables, which are supposed to guarantee the accomplishment of production processes in the scope of available production capacity (variable z_{max}) and minimal times of changeovers (variable y_{pjk}), were taken into account. Symbol M stands for a big positive value which was introduced to the model in order to raise the importance of plan realization. The decision-maker requires first to accomplish the production process in the available production capacity. The second objective is to ascribe the series to the machines which will guarantee the minimal overall changeover time. The lack of multiplier M next to the variable z_{max} would cause that every operation of manufacturing half-finished and finished product is carried out on exactly one single machine (guarantee minimal changeover times). This would result in exceeding the production capacity as the overall times of all long changeovers are much longer than the available working time of the machine in the period. Second element of the sum in the formula (6) takes into account only the differences between the times of long and short changeovers for the operation of manufacturing the series of half-finished or finished product. This notation is established because of the observation that the times of long changeovers, on the machines used alternatively, are always much longer than the times of short changeovers. Knowing, that for every operation of production series

carried out on a given machine, at least one short changeover should be performed, it was assumed that main differences in changeover times will result in the number of long changeovers. Limitation (7), analogically as in the previous model, guarantees that every operation in the production series will be carried out exactly once on a given machine. Symbols s_i, c_{jks_i} denote the same values as in the first version of the model. Limitation (8) links the usage of production capacity with the variable z_{max}. Symbol D_k stands for the available machine working time in the period when the plan is carried out. Symbols h_k^A and h_k^B stand for the short and long times of changeovers of the machine k, accordingly. In comparison to the limitation (3) in the first model, the utilization of the production capacity was increased by the sum of difference between short and long changeovers, since the short changeover times are already considered for every production series manufactured on a given machine. Limitation (9) ensures the link between the decision-making variable y_{pjk} and the decision-making variable x_{ijk}. The fact that carrying out, at least, one operation of manufacturing the series of half-finished or finished product on a given machine causes the situation where the variable y_{pjk} has got the value 1. Limitations (10), (11) and (12) are border limitations. As opposed to the first model, the variable z_{max} can have only values higher than zero, since, when the changeover times are included, it is limited only to guarantee the plan accomplishment. The limitation (12) was added because of the use of a new group of variables in the model.

For a problem, which was described in such a way, it was possible to solve the real task. The problem included around 200 machines and 750 half-finished and finished products. In order to solve the problem, the library of the operational research program CoinMP was used. The library, which was run on the computer equipped with Intel Core2 Duo 2GHz processor and computer system Win XP, can provide the solution in the time of around 1s. The use of the simple operational research system *lp solve* which was responsible for providing a solution to a previous task, in this instance did not guarantee a solution, even in the time of 30 minutes. From the factual point of view the high precision of obtained solutions was confirmed by planners. Whereas the value of the indicator OEE (overall equipment effectiveness), which is used to measure the effectiveness of equipment, was raised from the level of 20%, which was achieved before the implementation of presented model, to the level of 38% after its implementation. At present, the system has worked in the company for over half a year and is used after every change of production plan in the planning period. The time needed for finding the solution is usually 2 seconds.

6. Conclusion

In this paper, an operational research model, which was used to support S&OP procedure in the chain of supply consisting of a trade and production company, was presented. The implementation of the model required many actions such as preparation of detailed information about production process or production capacity and changeover times of the machines. Moreover, the model had to include some information about scheduled production since operations of approved production orders

had to be excluded from the operational research process. Obtained values of OEE indicator, after introduction of S&OP procedure, as well as the assessment made by people who were taking part in the procedure, confirmed the usefulness of the model. Gathering all the information concerning production process(material specification, production times, technological routes, the use of machines, changeover times) in the computer system was another notable effect of implementing the model. Before the implementation of the procedure, all abovementioned data was only known to production workers, which made the planning process difficult and limited the works which aimed at improving production capacity. A very important quality of the models is the fact that, in the operational research process, they take into consideration some situations where it is not possible to accomplish the production because of exceeding of production capacity. In the simplest formulations the exceeding of production capacity results in an empty set of acceptable solutions. This fact, from the point of view of a decision-maker, does not introduce any information. Presented formulations determine the assignment of manufacturing operation to the machine, even when the production capacity is exceeded. In such a situation the biggest exceeding of production capacity on a single machine is minimized. While, in remaining situations (when it is possible to carry the production out in the scope of the available production capacity) the operational research process provides solutions where the biggest reserve of time on a single machine is maximum (model I) or the overall time of changeovers is minimum (model II).

As a part of further research on enhancement the S&OP procedure, it is important to develop presented formulations so as to include stocks on hand and suggest the size of production in a given period. In the approach, presented in this paper, the workers of the company decide on the size of production in individual months. They take into consideration the rotation level and they analyze the utilization of production capacity. This approach will result in fact that the operational research will be carried out only once for the whole planning horizon and the complexity of calculation will rise.

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