Ramdan Ahmed*, Stefan Miska*, Włodzimierz Miska**

FRICTION PRESSURE LOSS DETERMINATION OF YIELD POWER LAW FLUID IN ECCENTRIC ANNULAR LAMINAR FLOW***

1. MATHEMATICAL MODEL

In general, the wall shear rate in pipe and slit flows can be expressed as

$$\dot{\gamma}_w = a\tau_w \frac{d\left(\frac{8U}{D_h}\right)}{d\tau_w} + b\left(\frac{8U}{D_h}\right) = \left[\frac{a}{N} + b\right]\left(\frac{8U}{D_h}\right) \tag{1}$$

where a and b are duct geometric parameters and N is a generalized flow behavior index defined as below

$$N = \frac{d(\ln \tau_w)}{d\left(\ln \frac{8U}{D_h}\right)} \tag{2}$$

It is known that the parameters a and b are: $a = 0.25$ and $b = 0.75$ for pipes and $a = 0.5$, $b = 1$ for slits (narrow slots). The hydraulic diameter, D_h, is simply equal to pipe diameter, D, for pipe and $D_h = D_o - D_i$ for slits flow. We propose that a similar equation can be written for an eccentric annular flow, if instead of τ_w we would use the average shear stress $\bar{\tau}_w$. Hence we write

$$\bar{\dot{\gamma}}_w = a\bar{\tau}_w \frac{d\left(\frac{8U}{D_h}\right)}{d\bar{\tau}_w} + b\left(\frac{8U}{D_h}\right) = \left[\frac{a}{N} + b\right]\left(\frac{8U}{D_h}\right) \tag{3}$$

* University of Tulsa
** Wydział Wiertnictwa, Nafty i Gazu AGH, Kraków
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In Eqn (3), the generalized shear rate is defined in terms of generalized flow behavior index, nominal Newtonian wall shear rate and geometric parameters *a* and *b*. It has been shown that for the eccentric annulus the geometric parameters (*a* and *b*) can be calculated as follows:

$$a = a_0 e^3 + a_1 e^2 + a_2 e + a_3 \tag{4}$$

$$b = \alpha_0 e^3 + \alpha_1 e^2 + \alpha_2 e + \alpha_3 \tag{5}$$

where:

$e = E/(D_o - D_i)$ – the dimensionless eccentricity,

$a_0, a_1, a_2, a_3, \alpha_0, \alpha_1, \alpha_2, \alpha_3$ – coefficients dependent upon the diameter ratio $\kappa = D_i/D_o$ as presented in Table 1.

Table 1

Equation to Calculate Coefficients for Geometric Parameters

$a_0 = -2.8711\kappa^2 - 0.1029\kappa + 2.6581$	$\alpha_0 = 3.0422\kappa^2 + 2.4094\kappa - 3.1931$
$a_1 = 2.8156\kappa^2 + 3.6114\kappa - 4.9072$	$\alpha_1 = -2.7817\kappa^2 - 7.9865\kappa + 5.8970$
$a_2 = 0.7444\kappa^2 - 4.8048\kappa + 2.2764$	$\alpha_2 = -0.3406\kappa^2 + 6.0164\kappa - 3.3614$
$a_3 = -0.3939\kappa^2 + 0.7211\kappa + 0.1503$	$\alpha_3 = 0.2500\kappa^2 - 0.5780\kappa + 1.3591$

Analysis of the above equations clearly indicates that if annular diameters (D_o and D_i) and eccentricity are given, one can determine the parameters *a* and *b* from Eqn (4) and Eqn (5), respectively. To calculate the generalized flow behavior index, *N*, we propose to use the pipe flow equation in which a pipe diameter is replaced with the hydraulic diameter. Hence, we write

$$\frac{8U}{D_h} = \frac{(\tau_{w,p} - \tau_y)^{(1+1/m)}}{K^{1/m}\tau_{w,p}^3}\left(\frac{4m}{3m+1}\right)\left[\tau_{w,p}^2 + \frac{2m}{1+2m}\tau_y\tau_{w,p} + \frac{2m^2}{(1+m)(1+2m)}\tau_y^2\right] \tag{6}$$

where $\tau_{w,p}$ is the pipe wall shear stress that corresponds to the nominal Newtonian shear rate of $8U/D_h$. Once $\tau_{w,p}$ is determined numerically, the generalized flow behavior index can be calculated from Eqn (6)

$$N = \frac{mC_c}{3m(1 - C_c) + 1} \tag{7}$$

where

$$C_c = (1 - x)\left[\frac{2m^2}{(1+2m)(1+m)}x^2 + \frac{2m}{1+2m}x + 1\right] \tag{8}$$

and

$$x = \frac{\tau_y}{\tau_{w,p}} \tag{9}$$

Upon calculating geometric parameters *a* and *b* and the generalized flow behavior index *N*, the average wall shear rate can be determined from Eqn (3) and the corresponding shear stress can be determined from the constitutive equation as follows

$$\bar{\tau}_w = \tau_y + K\bar{\gamma}_w^m \tag{10}$$

and, finally frictional pressure gradient as

$$\frac{\Delta P}{\Delta L} = \frac{4\bar{\tau}_w}{D_h} \tag{11}$$

To confirm that flow is indeed laminar, we calculate the Reynolds number for eccentric annulus as below

$$\mathrm{Re} = \frac{8\rho U^2}{\bar{\tau}_w} \tag{12}$$

2. EXPERIMENTAL INVESTIGATION

Extensive pipe and annular flow experiments with polymeric fluids were conducted using the dynamic testing facility of the Tulsa University Drilling Research projects. During the experiments, flow rate was varied from 0.024 gpm to 21.91 gpm [0.09 l/min to 82.82 l/min]. Test temperatures and pressures were ranging from 82°F to 113°F [27.78°C to 45°C] and from 20.94 psi to 193.2 psi [144.37 kPa to 1332 kPa].

3. EXPERIMENTAL SETUP

Annular test sections with four different diameter ratios (0.27, 0.36, 0.49 and 0.76) were considered in the investigation.

The test sections were rearranged in two different ways:

1) three-pipe (0.50″, 0.82″ and 1.38″) and one-annulus configuration;
2) three-annuli (Annulus #1, #2 and #4) and one-pipe (0.5″) configuration.

In three-pipe and one-annulus configuration, pipe flow curves were used to verify the absence of wall slip. Eccentricities of the annular test sections were varied from 0% to 100%, where 0% refers to a concentric drill pipe. Polymeric fluids were prepared by varying concentrations of Xanthan Gum (XCD) and Polyanionic Cellulose (PAC) in the system.

A progressive cavity pump (Moyno) was used to circulate the test fluid through the loop. Test fluids are prepared in a PVC tank that has a high-speed stirrer. Hot water is provided from a water boiler to facilitate the mixing of polymers. Each test section is about 18 ft [5.5 m] in length. Differential pressure transmitters and flow meters are installed to measure the frictional pressure loss and volumetric flow rate.

The loop is equipment with computer based data acquisition system and measuring instruments that are necessary for conducting rheology and hydraulic investigations.

4. SYSTEM CHARACTERIZATION EXPERIMENTS

Firstly a series of experiments were carried out with Power-Law fluids to test the system and determine the actual eccentricity. Eccentricities of concentric and partially eccentric annuli were determined by matching experimentally determined and numerically obtained values for a Power-Law fluid. Three test fluids (XCD-PAC3, XCD-PAC6 and XCD-PAC7) that best fit power-law model were prepared and used for determining the actual eccentricities of the annular test sections.

5. EXPERIMENTAL RESULTS

The first set of experiments that include six tests (from Test #1 to Test #6) was performed with three-pipe and one-annulus configuration. Results presented in Figure 1, which shows the log-log plot of the average wall shear stresses as a function of $8U/D_h$ in pipe and fully eccentric Annulus #3 for test fluid XCD3.

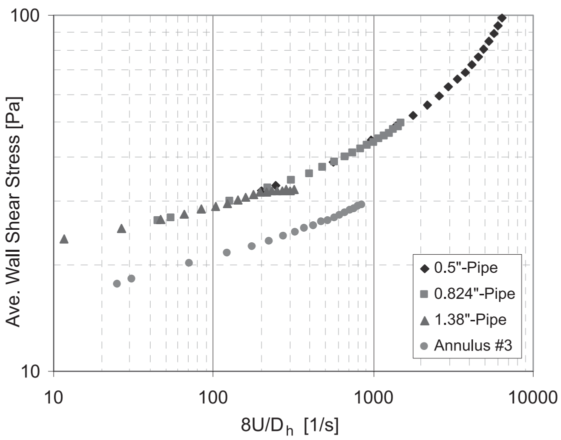

Fig. 1. Average wall shear stress *vs*. $8U/D_h$ for XCD3 in Pipe Viscometers and Fully Eccentric Annulus #3

Examination of Figure 1 reveals that the flow curves of this fluid, obtained from different pipes approximately lie on a single curve (viscometric flow curve) indicating the absence of wall-slip. Similar results were obtained for other fluids that were tested using three-pipe and one-annulus configuration. For the Annulus #3, the curve of average shear stress is approximately parallel with the viscometric flow curves. Results for other test fluids also indicate similar patterns of the average shear stress *vs*. $8U/D_h$.

The second set of experiments that include nine tests (excluding power-law fluid tests conducted for system characterization) was conducted with three-annulus and one-pipe configuration. Test fluids with similar compositions as those tested in the first set of experiments were prepared.

6. COMPARISON BETWEEN MODEL PREDICTIONS AND MEASUREMENTS

Extensive model evaluation has been conducted by comparing measured annular pressure losses against the model predictions. For annulus #1 and #2, measurements that were obtained at low flow rates (i.e. less 1 gpm) are not considered in the evaluation because at low flow rates model predictions requires extrapolation of viscometric flow curve to estimate the generalized flow behavior index.

Figures 2 and 3 are samples of model predicted and measured pressure losses.

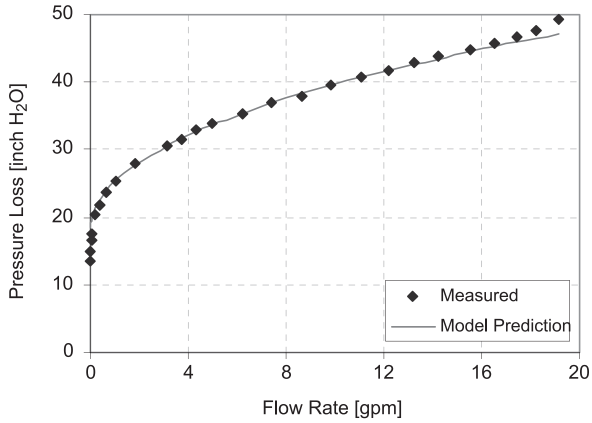

Fig. 2. Measured and Predicted Pressure Losses *vs*. Flow Rate for XCD5 in Fully Eccentric Annulus Section #1

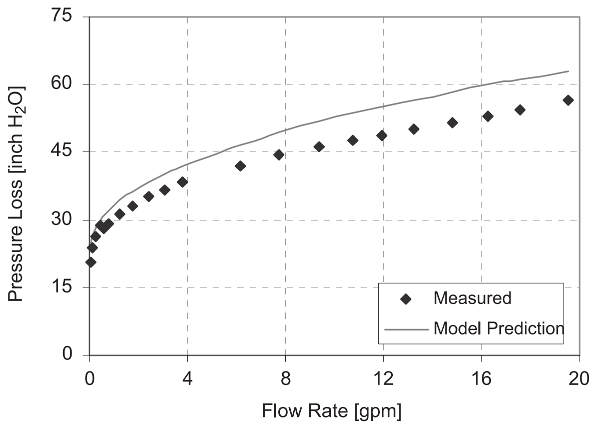

Fig. 3. Measured and Predicted Pressure Losses *vs*. Flow Rate for XCD9 in 75% Eccentric Annulus Section #1

All model predictions show relatively higher discrepancies at low flow rates. However, in most cases, model predictions and measurements are in a reasonably good agreement; the average maximum differences are less than 10% and frequently less than 5%.

7. CONCLUDING REMARKS

A mathematical model for predicting friction pressure losses in laminar flow of Yield Power-Law fluid in eccentric annuli is presented and verified by experiments. The fluids rheological properties were determined in pipe viscometer and subsequently used for predictions of pressure losses as a function of flow rate. While the discrepancies between predictions and measurements are relatively high at low shear rates a satisfactory agreement is obtained for medium and higher shear rates, however still in the laminar flow conditions. The maximum error in predictions is in the range of 15% indicating that the proposed model can be effectively used for practical design applications.

NOMENCLATURE

- a – geometric constant
- b – geometric constant
- C_c – dimensionless parameter
- D – pipe diameter
- D_h – hydraulic diameter
- e – dimensionless eccentricity
- E – offset distance between centers
- f – friction factor
- K – fluid's consistency index
- L – pipe length
- m – YPL fluid behavior index
- n – power-law fluid behavior index
- N – generalized flow behavior index
- P – pressure
- Q – volumetric flow rate
- Re – Reynolds number
- U – mean flow velocity
- x – ratio of yield stress to wall shear stress
- $\dot{\gamma}$ – shear rate
- $\bar{\dot{\gamma}}$ – average shear rate
- κ – diameters ratio
- τ – shear stress
- $\bar{\tau}$ – average shear stress
- $\tau_{w,p}$ – wall shear stress in equivalent pipe

Subscripts

i – inside
o – outside
y – yield
w – wall

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