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MODELLING OF PDC DRILL BIT WHIRLING

1. INTRODUCTION

Today, as a rule, a drill bits and other kind of drilling instrument are designed taking into account intercommunications between structural elements, durability descriptions of material and etc, but, as a basic application of drill bit is destruction of rock, at this work another way is utilized – from properties of rock to the conditions of destructions by concrete drill bit.

To the development of drill bit mathematical modeling historically there were two basic approaches.

Majority of the developed models are based on the statistical analyzing of the drilling results, the best mathematical approximation determination with a lot of different empiric coefficients. Structural and operating properties of the drill bits were also taken into account by the coefficients. Models were based on experimental information, and value of the rate of penetration was determined by the empiric dependences. As a rule such way of modeling gives results similar to the practical information, but is not suitable for serious scientific research.

Besides, the theoretical models were the developed too; they described important dependences but were bulky enough and not always similar to the practical data [1].

The second way of modeling which is especially theoretical to my opinion is more perspective, because it may be the basis for drill bit design improvement. The main idea (and the main disadvantage to my mind) of a lot of models were assumption that cutting forces, which generates at face reaction during drilling process are the same (or almost the same) for every PDC drill bit cutting chisel. Such assumption causes the substantial limits to the model ability to represent adequately the process well drilling. But, Behr S.H., Warren T.M., Sinor L.A., Brett J.F. in [2] presented description of model which reproduced forces, and partly took into account the of possible casual motion of bit, caused by the disbalance of forces which are very important for PDC drill bits.

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Then in 1986 J. Brett described the PDC drill whirling, this process was considered as absolutely negative phenomenon, which just sometimes can occur. One of the main causes of whirling was considered the rock mass heterogeneity, unstable value of drill face reaction, etc. But, rock mass are always heterogeneous, stress redistribution among drill bit cutters are always not fixed, so it worse to consider that bit whirling is not just an accompanying process of drilling, but its integral part. That why it worse to get whirling into account and to admit that we can just to minimize whirling (or its numerical estimation-misbalance), but it impossible to avoid it at all.

2. PDC DRILL BIT MOVEMENT DESCRIPTION

Generally, for PDC drill bit as a solid of 2 degrees of freedom can be used system of LaGrange equations

$$\begin{cases} \dfrac{d}{dt}\left[\dfrac{\partial T}{\partial q_1^*}\right] - \dfrac{\partial T}{\partial q_1^*} = Q_1 \\ \dfrac{d}{dt}\left[\dfrac{\partial T}{\partial q_2^*}\right] - \dfrac{\partial T}{\partial q_2^*} = Q_2 \end{cases} \tag{1}$$

where:

T – kinetic energy of PDC bit,
q_i^* – generalized coordinates,
Q_1 – generalized forces sum (generalized force of face reaction),
$Q_2 = \sum F_i \cdot r_i$ – generalized resistance moment,
F_i – generalized force of face reaction for every cutter,
r_i – cutter installation radius.

As generalized coordinates can be used:

$q_1^* = \Delta Z(t)$ – dill bit centre of mass movement down to the hole, as time function;

$q_2^* = \theta(t)$ – drill bit pivot turn angle, as time function.

Besides, first derivative of function $\Delta Z(t)$ defines the pintle speed of drill bit movement

$$\frac{dq_1^*}{dt} = V_Z \tag{2}$$

Theoretically, PDC drill bit position can defined by the Lagrange equations solution, but such way is not approachable for real conditions, because of unstable value of force of drilling face.

So, the problem lies in the kinetic energy of PDC bit determination.

For flexure of well tubes bend part determination use deflection curve equation:

$$u(x) = u_0 + \Theta_0 x + \frac{1}{EJ}\left(M_0 \cdot \frac{l^2}{2} + Q_0 \cdot \frac{l^3}{3} + \sum_0^l q_i \cdot \frac{x^4}{4!} \right) \tag{3}$$

where:

$u(x)$	–	flexure value for analyzed section,
u_0, Θ_0	–	zero value of flexure and turning angle correspondingly,
E	–	coefficient of elasticity for drill bit material,
J	–	second moment of area (moment of inertia) analyzed section,
q	–	value of drilling face,
M_0, Q_0	–	values of moment and force at drill bit console jam correspondingly.

Max value of this flexure is at the drilling face, so such section speed at the horizontal plane is

$$u'(t) = \frac{du(x)}{dt} = \frac{du(x)_{\max}}{dt} = \frac{d\left(u_0 + \Theta_0 x + \frac{1}{EJ}\left(M_0 \cdot \frac{l^2}{2} + Q_0 \cdot \frac{l^3}{3} + \int_0^{l_1} q_i(x) \cdot \frac{x^4}{4!} dx \right)\right)}{dt} \tag{4}$$

So, kinetic energy of *dx* length elementary unit of well tubes (or drill bit – it depends on analyzed section) is

$$dT = du(t) \cdot \frac{\gamma \cdot S \cdot dx}{2g} = \frac{\gamma \cdot S \cdot dx}{2g} \cdot \frac{d\left(u_0 + \Theta_0 x + \frac{1}{EJ}\left(M_0 \cdot \frac{l^2}{2} + Q_0 \cdot \frac{l^3}{3} + \int_0^{l_1} q_i(x) \cdot \frac{x^4}{4!} dx \right)\right)}{dt} \tag{5}$$

where:

γ	–	conditional density of drill bit material,
S	–	contact area of borehole and drill bit cutters foe analyzed section.

Kinetic energy for drill bit is determined by last formula integration

$$T = \int_0^l \frac{\gamma \cdot S \cdot dx}{2g} d\left(u_0 + \Theta_0 x + \frac{1}{EJ}\left(M_0 \cdot \frac{l^2}{2} + Q_0 \cdot \frac{l^3}{3} + \int_0^{l_1} q_i(x) \cdot \frac{x^4}{4!} dx \right)\right) \tag{6}$$

Length [0, l] is drill bit length.

Due to PDC drill bit whirling and non- convergence of axes of drill bit and hole, Lagrange equations for real conditions are

$$\begin{cases} \dfrac{d}{dt}\left[\dfrac{\int \dfrac{\gamma\cdot S\cdot dx}{2g} d\left(u_0+\Theta_0 x+\dfrac{1}{EJ}\left(M_0\cdot\dfrac{l^2}{2}+Q_0\cdot\dfrac{l^3}{3}+\int\limits_0^{l_1} q_i(x)\cdot\dfrac{x^4}{4!}dx\right)\right)}{\partial z(t)}\right]- \\ \dfrac{\partial\int \dfrac{\gamma\cdot S\cdot dx}{2g} d\left(u_0+\Theta_0 x+\dfrac{1}{EJ}\left(M_0\cdot\dfrac{l^2}{2}+Q_0\cdot\dfrac{l^3}{3}+\int\limits_0^{l_1} q_i(x)\cdot\dfrac{x^4}{4!}dx\right)\right)}{\partial z(t)}=\sum Q_i \\ \dfrac{d}{dt}\left[\dfrac{\partial\int \dfrac{\gamma\cdot S\cdot dx}{2g} d\left(u_0+\Theta_0 x+\dfrac{1}{EJ}\left(M_0\cdot\dfrac{l^2}{2}+Q_0\cdot\dfrac{l^3}{3}+\int\limits_0^{l_1} q_i(x)\cdot\dfrac{x^4}{4!}dx\right)\right)}{\partial\theta(t)}\right]- \\ -\dfrac{\partial\int \dfrac{\gamma\cdot S\cdot dx}{2g} d\left(u_0+\Theta_0 x+\dfrac{1}{EJ}\left(M_0\cdot\dfrac{l^2}{2}+Q_0\cdot\dfrac{l^3}{3}+\int\limits_0^{l_1} q_i(x)\cdot\dfrac{x^4}{4!}dx\right)\right)}{\partial\theta(t)}=\sum F_i\cdot r_i \end{cases} \tag{7}$$

My solution of this systems is:

$$q_1^* = \frac{1}{C_3 e^t - Q}\int\limits_0^l \frac{\gamma\cdot S}{2g} d\left(u_0+\Theta_0 x+\frac{1}{EJ}\left(M_0\cdot\frac{l^2}{2}+Q_0\cdot\frac{l^3}{3}+\int\limits_0^{l_1} q_i(x)\cdot\frac{x^4}{4!}dx\right)\right)+C_4 \tag{8}$$

where C_i – integration constant, depends on initial conditions.

First derivative of function $\Delta Z(t)$ defines the pintle speed of drill bit movement:

$$\frac{dq_1^*}{dt}=V_Z$$

$$\frac{dq_1^*}{dt}=\frac{d\left(\dfrac{1}{C_3 e^t - Q}\displaystyle\int \frac{\gamma\cdot S}{2g} d\left(u_0+\Theta_0 x+\frac{1}{EJ}\left(M_0\cdot\frac{l^2}{2}+Q_0\cdot\frac{l^3}{3}+\int\limits_0^{l_1} q_i(x)\cdot\frac{x^4}{4!}dx\right)\right)+C_4\right)}{dt} \tag{9}$$

Thus, we get equalization, which sets connections between the face reaction force $F(t)$, structural parameters of drill bit, degree of it wearing out (by area of contact S), descriptions of drill bit and rock, conditions of drill bit installation.

So, main questions are the *u*(*t*) the PDC drill bit clearance (or it disbalance numerical value) and face reaction determination *F*(*t*).

3. PDC DRILL BIT DISBALANCE DETERMINATION

The method of final elements is used for the decision tasks.

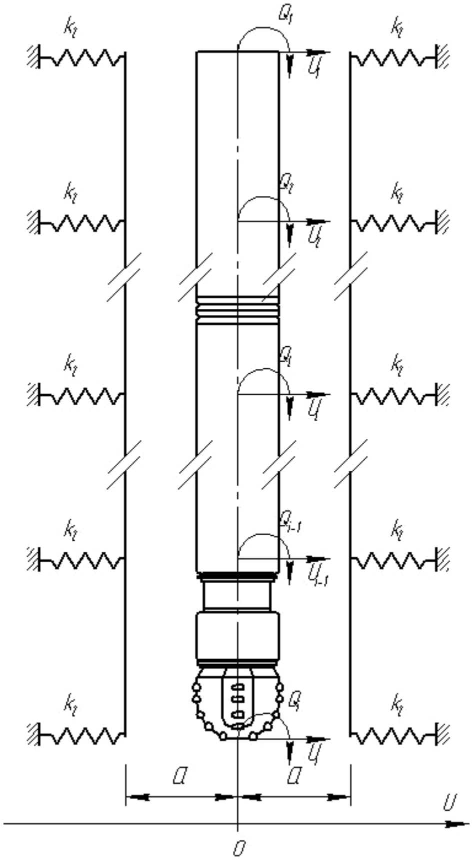

Fig. 1. Model of final elements for PDC drill bit

Every element of drill string and drill bit as the initial element of the system is characterized by the moment of inertia *I*, an area of the cross-section *A*, mass *M*, and for that model is considered that material of every structural element is resilient (Fig. 1).

Differential equalization which describes motion of the system for any the cut of model in a horizontal plane, accordant to the calculation chart is [by prof. Spanos P., 4]

$$M \cdot u(t)'' + \lambda \cdot u(t)' + k \cdot u(t) + F(u(t)) = \sum F_i(t) \quad (10)$$

where:

M – mass of the system,
λ – coefficient of vibrations,
k – coefficient of dempfing of the system;
$u(t)''$, $u(t)'$, $u(t)$ – vectors of moving, speed and acceleration of the system accordingly in a horizontal plane.

Thus, actual position of drill bit in relation to the of mining hole in any moment of time is determined from correlation

$$\breve{o}_{drill\_bit} = x_{hole} + \overline{u(t)} \tag{11}$$

where:

$\breve{o}_{drill\_bit}$, x_{hole} – co-ordinates of geometrical center of mining hole and chisel accordingly;
$g(t)$ – function of force of face reaction in a horizontal plane depending on time to proper the cut of the system;
$F(u(t))$ – function of contact of chisel with the wall of mining hole.

This function is determined the terms of contact of function of Hertz, and described by equalization:

$$F(u_i(t)) = \begin{cases} -K_2(-u(t) - a)^{3/2} & \text{if } u(t) \leq -a \\ 0 & \text{if } -a \leq u(t) \leq a,\ u_i(t) = 1, 3, 5, ...,\ n-1 \\ K_2(u(t) - a)^{3/2} & \text{if } u(t) \geq a \end{cases} \tag{12}$$

where:

K – decrement coefficient of Hertz for a concrete drilled rock;
n – general number of degrees of freedom of the system; for a instrument as system with two degrees of liberty of $n = 2$;
k – coefficient is a a – size of lateral gap between the wall of mining hole and PDC drill bit.

Using analogy to determination of coefficient of attenuation longitudinal vibrations of drill string having permanent cross-section [3], it is possible to use for determination of PDC drill bit attenuation vibrations coefficient

$$\lambda = \frac{K}{\rho A} \tag{13}$$

where:

K – decrement coefficient;
A – area cross-section;
ρ – value of conditional density of material.

For the got equalization of motion use:

– initial conditions:

$$\left.\frac{\partial u(t)}{\partial t}\right|_{t=0}=0, \qquad \left.\frac{\partial^2 u(t)}{\partial t^2}\right|_{t=0}=0,$$

– maximum terms:

$$\left.\frac{\partial u(t_{i+1})}{\partial t}\right|_{t=t_i}=u'(t_i), \quad \left.\frac{\partial^2 u(t_{i+1})}{\partial t^2}\right|_{t=t_i}=u''(t_i) \tag{14}$$

Thus, for model simplification will project the vector of moving to the co-ordinates system in a horizontal plane, so

$$\overline{u(t)}=\overline{u(t)_X}+\overline{u(t)_Y} \tag{15}$$

Passing to the polar co-ordinates and projecting the vector of moving to the wasp of co-ordinates will get the system of equalizations for determination of moving:

$$\begin{cases} M(r''-\alpha^{2'}r)+\lambda\cdot r'+kr+F(r\cdot\cos\alpha)=\sum F_{iX}(t) \\ M(\alpha''+2\alpha' r')+\lambda\cdot\alpha^{r'}+F(r\cdot\sin\alpha)=\sum F_{iY}(t) \end{cases} \tag{16}$$

Speaking about whirling, it worse to emphasize, that it rather difficult to define the beginning of this process. Besides, whirling was considered as a self-appearable process. Any PDC drill bit clearance is the cause of centrifugal force, which depends on the drill bit mass (m), clearance ($r+\Delta$) and angular velocity (w).

$$F=m(r+\Delta)^2 w \tag{17}$$

This process is like a snowball increasing. Besides, we have to include this force to calculations of generalized force Q for getting result close to real conditions. So, rconditional curved trajectory of PDC drill bit center motion is shown on a Figure 2.

Analyze of (7)–(9) assumption shows, that whirling is cause of drill bit rate of penetration changing. Besides, as more is the length to the drill bit point of rest (or to the point of any shock-absorber installing) as more is whirling is. So idea of the flexible bit [3] can be used for new designs of drill bit.

As it was shown above, the assumption [10] defines the natural oscillation and forced oscillation of drill bit. If the face reaction force tends to zero in accordance with the circumstances, the oscillations become relaxation. Natural oscillations of a drill bit depend on its inertial parameters, but (as it was shown by calculations) the mass of drill bit is not the main parameter. Value of the clearance is not periodical and could be called stochastic, but depends on the force of face reaction. Another situation is with influence of the face reaction (or generalized cutting force) amplitude to the clearance value. It well-known that amplitude of cutting force directly depends on the rock mass heterogeneity and rock mass patterns. More heterogeneous rock mass produces cutting force with more amplitude and more magnitude of the clearance as an effect of this. [5].

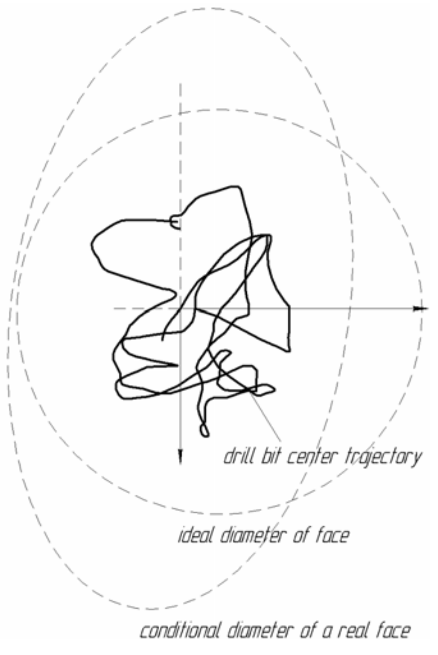

Fig. 2. PDC drill bit center motion

Speaking about the kinetic energy of the PDC bit, I like to emphasize that this characteristic is the base for modeling bit dynamics. But, in such hypothetic case, the kinetic energy is increasing during first some seconds of drill bit work, and then it tends to averaged value. Essential conclusion of this is the constancy of the general tendency.

4. CONCLUSIONS

PDC drill bit previous models were based on the assumption that PDC drill bit oscillation is self appearable or stochastic. At this work it was made the attempt to explain this process. It was shown that the cause of PDC whirling is not only the rock mass heterogeneity, but drill bit mass misbalance too and it is impossible to obtain absolutely stable system.

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