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Comparing the Results of Function Model Estimation for the Prediction of Real Estate Market Values in Additive and Multiplicative Form***

1. Introduction

The work deals with the estimation of the parameters of linear and non-linear models for the prediction of real estate market values. The problem is interesting and important. The attractiveness of the real estate as an investment of capital encourages searching the best methods to estimate its value. The law on real estates management defines several types of the values. There is among them the market value ruled by the professional standard III.1. The approaches of real estate estimation are discussed in detail in the standards III.6 and III.7. The last one deals with the estimation methods in a comparative approach. The necessity of estimating a real estate being a common question, so the standards give some simple methods of determination of real estate values. Nevertheless, it is worth, for investigation purposes, to do a detailed analysis of the market and find the best estimation models.

The subject of investigation is a database of real estates including functional premises situated in Cracow, in the administration unit – Śródmieście. The choice of such a market ensures variety of transactions and permits to select objective features to describe real estates. The analysis material was gathered in the Geodesy Department of Cracow City Office. Information on functional premises purchase-sale transactions was taken from the notarial acts as the basis of recording changes and it concerns the period of two years (from November 2003 to October 2005).

For the database of real estates being established, to describe particular premises, the following attributes were assumed: *time of transaction*, *surface*, *storey*, *number of compartments*, *situation in an administrative unit*, *communication access*, *number of public*

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communication lines, development perspectives, general sensation, situation in relation to a known landmark, standard, popularity of the place and environment. On the basis of their scaled values, a database containing 78 functional premises was set up. Then, the market analysis was done resulting in selection of features influencing significantly a real estate value.

Afterwards, the models selected for estimation of real estate market values have been presented. The analyses aimed to check if the market value should be determined by summation or by multiplication of the attributes shares. Furthermore, the effect of considering the non-linear character of the relation between an attribute and the price on modelling quality was examined as well as the answer to the question if it is possible to reduce attributes without deterioration of modelling results.

2. Statistical analysis of database

The database composed of 78 objects was submitted to the detailed analysis preparing it to the market value modelling. The purpose of the analysis was to eliminate information standing out from the other values or duplicating. The preliminary stage of analysis, consisting in examining price variation trend in time, resulted in rejecting seven real estates. The prices of the remaining 71 functional premises were corrected for the next month after the last transaction, i.e. November 2005.

Within this detailed analysis of real estate database, scatter diagrams of particular attributes and prices have been made. It aimed to reject standing out real estates, that is, such real estates, where the value of a given attribute is connected with the price corrected in an unusual for this attribute way. After eliminating such cases from the database, the information redundancy was examined on the basis of coefficients of total correlation between attributes. It was done to improve reliability of the estimated models in the subsequent stage – among the pairs of attributes, which explain the variability of the price in a very similar way, only one is left for further analysis. After that the number of real estates and attributes being reduced, correlation coefficients for all attributes with corrected prices and the shares in explanation of the price variability were calculated. In order to examine the homogeneity of the database and the diversification of the attributes, the coefficients of dispersion were examined too. As a result, the database was reduced to the 69 real estates described by 8 attributes only: *surface, storey, situation in the administrative unit, communication access, number of lines, development, sensation and landmark*. Finally, the scatter diagrams between particular attributes and prices were re-analysed, considering the choice of appropriate function representing the relation between the attribute and the corrected price. This was the preliminary stage to estimating the parameters of non-linear models. A detailed description of the presented analysis method can be found in publication [1].

3. Selection of function form for non-linear modelling

Selection of appropriate functions for the best estimation of the price variation was ruled, first of all, by the shape of scatter diagrams created within the analysis of data. Besides, the level of curvilinear correlation coefficient was taken into account – the higher is the coefficient value, the better the function represents price variability for a given attribute. The selected functions had the simplest forms to avoid unnecessary complication of modelling process and to get higher stability of estimated parameters.

Accordingly to these principles, the functions for all attributes have been selected. Their forms will be taken into consideration in the non-linear models. They are polynomials of different degrees – from the second for the attribute *situation* to the fifth for the attribute *sensation*. Presented below in pairs scatter diagrams and matched functions (Figs 1–8) reveal that the relations between the attributes and the corrected price have not linear character. It justifies the necessity of testing non-linear models for prediction of real estate values.

To facilitate the presentation, the diagrams are set in pairs. On the left, there is a scatter diagram for a given attribute with the line put on, estimated by MNK. On the right, the diagram is accompanied by a diagram of estimating function, selected using MNK with its form and with the value of curvilinear correlation coefficient.

To choose the general estimation model, the following forms of functions modelling a local market of functional real estates were analysed:

- additive function; a particular case of this function is the model multiple linear regression; the model is created by summation of the attributes shares, taking into account the non-linear relation between the price and the attribute;
- multiplicative function; a model created by the product of shares of particular attributes, expressed as exponential variation, considered in two forms:
  1) simple form, where the estimated coefficients are raised to the power corresponding to the values of particular attributes;
  2) complex form, where the estimated coefficients are raised to the power corresponding to the value of non-linear function, determined for particular attributes and estimating the relation between the price and the attribute.

Finding and determining the relations between a unit market price of a real estate and its distinctive features is the most difficult element of the analysis of real estate markets. In the case of an unstable market of real estates, the application of non-linear functions of relation between the attribute and the price can increase the correctness of real estate market value determination.

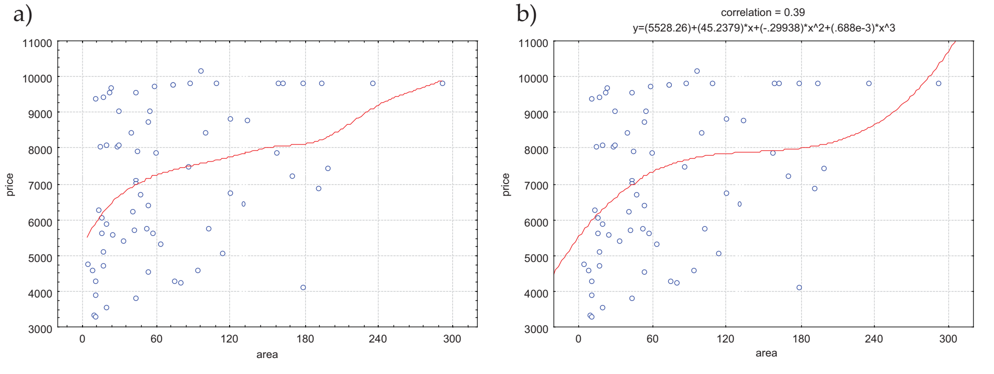

Fig. 1. Scatter plots area – price

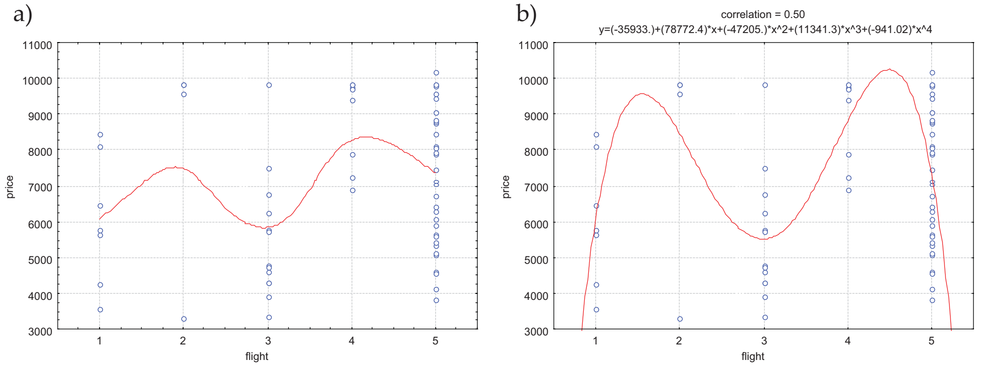

Fig. 2. Scatter plots flight – price

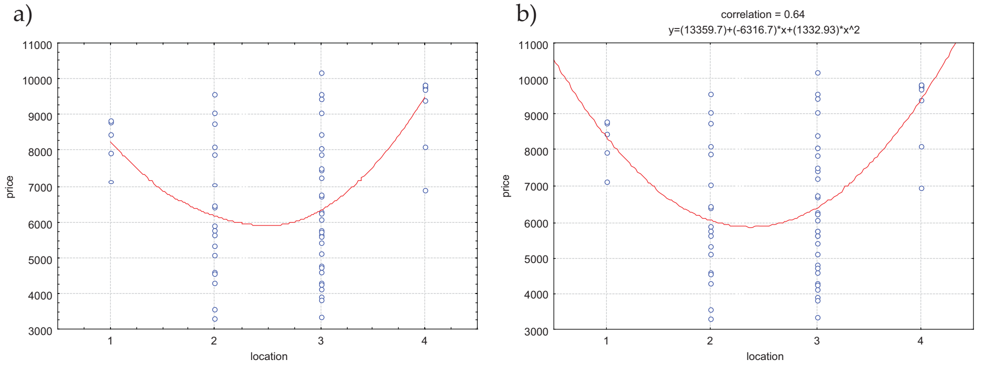

Fig. 3. Scatter plots location – price

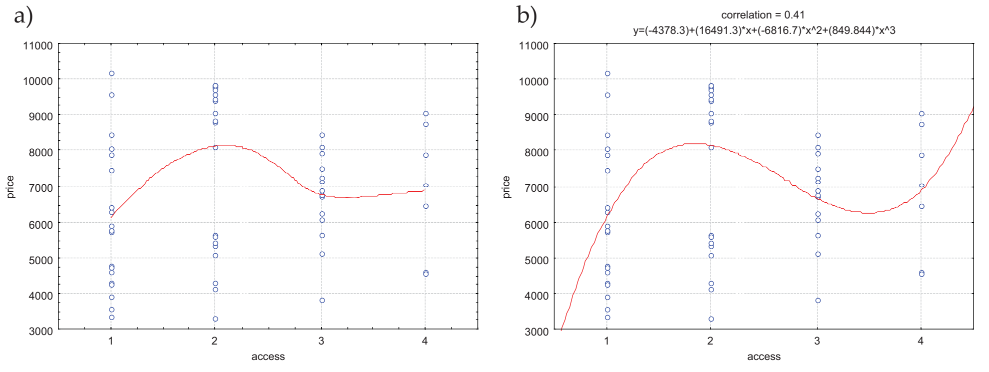

Fig. 4. Scatter plots access – price

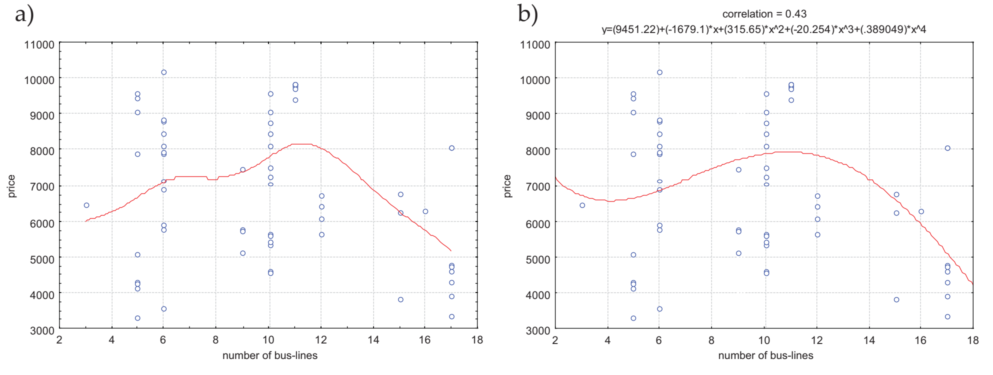

Fig. 5. Scatter plots number of bus-lines – price

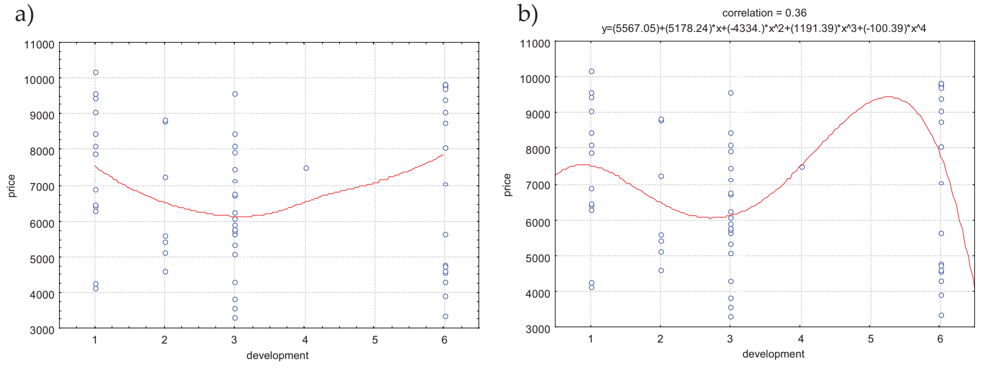

Fig. 6. Scatter plots development – price

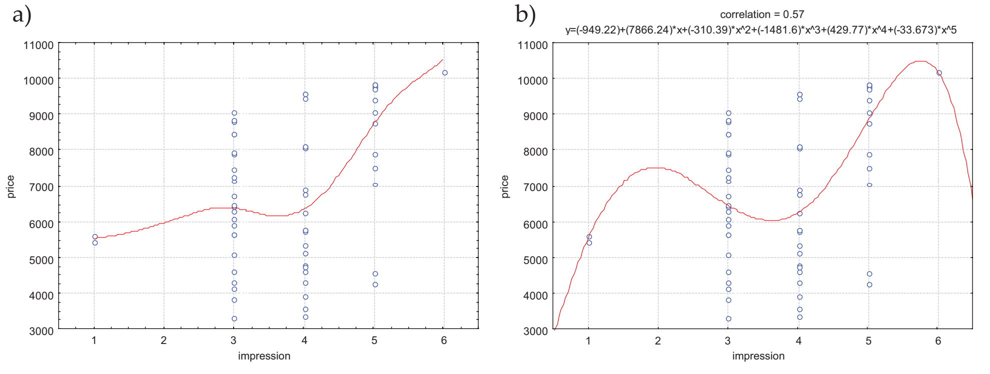

Fig. 7. Scatter plots impression – price

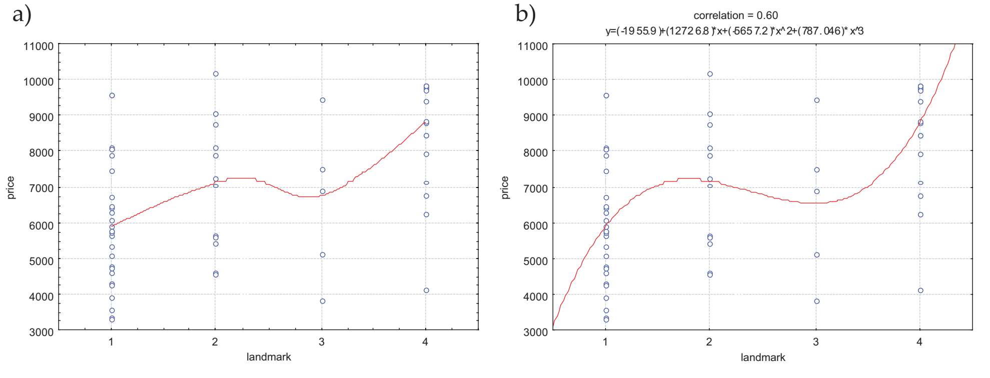

Fig. 8. Scatter plots landmark – price

3.1. Additive model

The additive model takes into account non-linear functions estimating price variation trends in relation to the particular attributes, in form of polynomials from the second to the fifth degree. General form of the model is

$$c = a_0 + \sum_{i=1}^{k} (a_{i1} \cdot x_i + a_{i2} \cdot x_i^2 + ... + a_{in_i} \cdot x_i^{n_i}) \tag{1}$$

where:

c – unit value of a real estate,
k – number of attributes considered in the model,
$x_1, x_2, ..., x_k$ – values of real estate attributes, selected for modelling the value of a real estate,
n_i – polynomial degree for i-th attribute.

3.2. Simple multiplicative model

A multiplicative model in a simple form was considered as a multiplicative exponential function

$$c = B_0 \cdot B_1^{x_1} \cdot B_2^{x_2} \cdot \ldots \cdot B_k^{x_k} \tag{2}$$

where:

c – unit value of a real estate,
$x_1, x_2, \ldots, x_k$ – values of real estate attributes, selected for modelling the value of a real estate,
B_0 – real estate unit value for zero of attribute values,
B_j – model coefficients representing bases of exponential functions for particular attributes x_j

Coefficients B_j take values near one and they operate like factors multiplying the basic value of a real estate with zero attributes. If we subtract 1 from the value of coefficient B_j the difference obtained determines the coefficient of price variation for unit of this j-th attribute.

3.3. Complex multiplicative model

The model of real estates unit values in a complex multiplicative form of exponential function was considered as well

$$c = B_0 \cdot B_1^{(a_{11} \cdot x_1 + a_{12} \cdot x_1^2 + \ldots + a_{1n_1} \cdot x_1^{n1})} \cdot \ldots \cdot B_k^{(a_{k1} \cdot x_k + a_{k2} \cdot x_k^2 + \ldots + a_{kn_k} \cdot x_k^{nk})} \tag{3}$$

where:

c – unit value of a real estate,
k – number of attributes considered in the model,
$x_1, x_2, \ldots, x_k$ – values of real estate attributes, selected for modelling the value of a real estate,
n_i – polynomial degree for i-th attribute.

4. Verification of estimated assessment models

Below, we present, in tables, the results of verification of proposed models. The models with specified numbering within an established type were created as results of a test examining the significance of estimated parameters of the model. If the statistical value of the test described in [3] falls in the critical area (at the established significance level α = 0.05), the null hypothesis $H_0 : a_i = 0$ or $H_0 : B_j = 0$ must be rejected. It means that a given coefficient effects significantly the modelling of real estate value and it should be taken into account in estimation, because it explains considerably the variability of prices. Test results were considered when the models with reduced number of attributes were created; though they were not deciding. It was so, because sometimes the test showed the necessity to reject too many attributes and this would disturb the estimation. Also, there were other reasons to leave some attributes in models with reduced number of attributes, e.g. how close was the test function value to the critical value.

Models were chosen in such a way that they could answer following questions:

- should a market value be achieved by summation of shares of particular attributes or by multiplication of them?
- should be considered, in modelling process, non-linear character of the relation between a price and an attribute?
- can we reject, in modelling process, a part of attributes, basing the estimation on these attributes only, which are the most significant for explaining the variation of the price, without lose modelling accuracy?

4.1. Estimation of a model matching quality in relation to the gathered data and investigation on the significance of its parameters

Estimation of parameters in models (1) and (2) follows the stages described in publications [1] and [3]. Within the analysis of accuracy for models in form (2) we receive covariance matrixes of parameters B_j logarithms and model values logarithms of real estate. Estimation of parameters of models in form (3) was done numerically. Among the elements of accuracy analysis we settled only the coefficient of matching model to data R^2 determining the percentage of explained variance, the value of which was verified using an appropriate statistical test (the results are presented in table 1) and the remainder variance σ^2 determining the square of estimation mean error.

The table 1, besides the basic characteristics like coefficient of variation V, coefficient of convergence φ^2 and coefficient of determination R^2, contains also results of two statistical tests verifying the models. It is the test verifying the symmetry of random element and the test verifying the significance of estimated parameter system in the model. As it can be seen, only one model (additive no 9) showed the lack of symmetry in remainders distribution. Then, in spite of a high coefficient of matching $R^2 = 0{,}83$, this model should not be applied to modelling market values of functional premises on the analysed market of real estates. Whereas, the system of parameters in every model turned out to be statistically significant.

4.2. Selection of optimal model on the grounds of invariant parameters values

Using the covariance matrix elements for coefficients of regression, we can determine the covariance matrix for predicted real estate values establishing the model (for the model of multiple linear regression as well as for the model of multiple non-linear regression, using the additive form of the function).

Table 1. Results of basic statistical analyses of estimated models

	Number of model	n	Coefficient of variation V	Coefficient of convergence φ^2	R^2	Test of deviation symmetry		Test of significance of model coefficients		
						calculated	tab.	freedom degr.	calculated	tab.
Linear	1	69	0.20	0.46	0.54	0.60	1.67	8.60	8.62	2.10
	2	67	0.17	0.36	0.64	0.12	1.67	8.58	12.71	2.11
	3	67	0.20	0.46	0.54	0.36	1.67	4.62	17.86	2.53
	4	67	0.18	0.39	0.61	0.12	1.67	5.61	19.17	2.37
	5	67	0.17	0.37	0.63	0.36	1.67	6.60	17.35	2.25
	6	67	0.18	0.39	0.61	0.61	1.67	7.59	13.00	2.17
	7	67	0.20	0.47	0.53	0.36	1.67	3.63	23.51	2.76
	8	67	0.18	0.41	0.59	0.85	1.67	4.62	22.55	2.53
	9	67	0.18	0.40	0.60	0.61	1.67	5.61	18.48	2.37
Additive	1	69	0.14	0.23	0.77	0.84	1.67	28.40	4.83	1.76
	2	67	0.11	0.14	0.86	1.10	1.67	28.38	8.04	1.77
	3	67	0.16	0.32	0.68	0.85	1.67	14.52	7.79	1.89
	6	67	0.11	0.14	0.86	1.35	1.67	27.39	8.56	1.77
	7	67	0.16	0.32	0.68	0.61	1.67	13.53	8.55	1.91
	8	67	0.19	0.43	0.57	0.36	1.67	8.58	9.61	2.11
	9	67	0.12	0.17	0.83	1.87	1.67	22.44	9.79	1.79
Simple multiplic.	1	69	0.03	0.53	0.47	1.08	1.67	8.60	6.69	2.10
	2	67	0.02	0.42	0.58	0.12	1.67	8.58	9.87	2.11
	3	67	0.02	0.50	0.50	0.85	1.67	4.62	15.73	2.53
	4	67	0.02	0.51	0.49	0.61	1.67	3.63	20.32	2.75
	5	67	0.02	0.43	0.57	0.12	1.67	6.60	13.20	2.25
	6	67	0.14	0.43	0.57	0.36	1.67	7.59	11.17	2.17
	7	67	0.16	0.50	0.50	1.61	1.67	3.63	21.00	2.76
	8	67	0.15	0.69	0.31	1.10	1.67	4.62	6.96	2.53
	9	67	0.14	0.58	0.42	0.12	1.67	5.61	8.83	2.37
Complex multiplic.	1	69	0.02	0.26	0.74	1.58	1.67	36.32	2.49	1.79
	2	67	0.01	0.19	0.81	1.10	1.67	36.30	3.49	1.84
	3	67	0.02	0.38	0.62	0.36	1.67	18.48	4.34	1.82
	6	67	0.01	0.19	0.81	1.61	1.67	35.31	3.71	1.80
	7	67	0.02	0.38	0.62	0.36	1.67	17.49	4.70	1.84

To achieve this, we have to do the following matrix multiplications

$$Cov(\hat{w}) = \underline{X} \cdot Cov(\hat{a}) \cdot \underline{X}^T \tag{4}$$

where:

$Cov(\hat{w})$ – covariance matrix of real estate model values,

X – matrix of real estate attributes values in database,

$Cov(\hat{a})$ – covariance matrix of model parameters.

Realisation of the formula (4) for a model using the multiplicative form, results in determining the covariance matrix for price logarithms.

A determined covariance matrix for predicted real estates values (prices), considered in estimation of assessment model, can be the base to formulate criteria for assessing a real estate database, used to estimate a real estate value by a selected model. For this purpose, we can use transformation invariants of symmetrical matrixes, which can be represented by the trace of a matrix

$$tr\left(Cov(\hat{w})\right) = \sum_{i=1}^{n} \sigma^2\left(\hat{w}_i\right) \tag{5}$$

and by the determinant value of this matrix

$$\det(Cov(\hat{w})) = \sum_{i}\left(m_{ij} \cdot A_{ij}\right) \tag{6}$$

where:

m_{ij} – element of matrix $Cov(\hat{w})$ in established row vector i and column j,

A_{ij} – algebraic complement of element A_{ij}.

The value of the trace of the matrix $Cov(\hat{w})$ is determined by the sum of variations of predicted market values for particular real estates forming an estimating model. On the basis of this value, behind the position [1], the following parameter of estimation model assessment has been defined

$$\sigma_{tr} = \frac{1}{w_{śr}} \cdot \sqrt{\frac{tr(Cov(\hat{w}))}{n}} \tag{7}$$

where $w_{śr}$ – mean value of predicted market values of real estate establishing the estimation model.

Considering the nature of this parameter as a measure of dispersion round the mean value of price predictions, in case of detailed estimations of real estates, its value should not exceed 0.10, i.e. 10% of mean value of predicted market values of real estates creating an estimation model.

Using the determinant of matrix $Cov(\hat{w})$, we also defined [1] mono-dimensional parameter determining the usability assessment of a database for an actual estimation model

$$\sigma_{\det} = \frac{1}{w_{śr}} \cdot \sqrt[n^2]{\det(Cov(\hat{w}))} \tag{8}$$

The form of the formula (8) indicates also the possibility to interpret the value σ_{det} as a measure of dispersion, so, in practical estimation models, it should be a small value. As a limit admissible value of this parameter, we can assume 0.15, i.e. 15% of mean value of predicted market values of real estates creating an estimation model.

For the models where it was possible (all of them except multiplicative complex models), the values of invariant parameters (7) and (8) have been calculated and presented in table 2.

The values presented in table 2 indicate that the gathered database can be applied to modelling real estate market values. In two cases only, the values of an invariant parameter exceed arbitrarily assumed limit value.

Table 2. Invariant parameters for estimated models

	Number of model	σ_{tr}	σ_{det}
Linear	1	0.08	0.00018
	2	0.07	0.00011
	3	0.06	0.00007
	4	0.06	0.00008
	5	0.06	0.00008
	6	0.07	0.00010
	7	0.05	0.00007
	8	0.05	0.00008
	9	0.06	0.00008
Additive	1	0.23	0.00013
	2	2.36	0.00013
	3		0.00832
	6	0.09	0.00014
	7	0.08	0.00009
	8	0.07	0.00716
	9	0.09	0.01066
Simple multiplicative	1	0.01	0.07129
	2	0.01	0.07044
	3	0.01	0.05367
	4	0.01	0.04955
	5	0.01	0.05589
	6	0.05	0.07275
	7	0.04	0.05523
	8	0.04	0.05914
	9	0.05	0.07369

4.3. Comparison of the models using the test comparing two random trials

The verification of reliability of estimated models could be done as well using the comparison of variance of parts explained by different models. We can apply in this purpose Fisher–Snedecor test (null hypothesis $H_0 : R_I^2 = R_{II}^2$, contra alternative hypothesis $H_1 : R_I^2 > R_{II}^2$), with the test function

$$F_{R^2} = \frac{R_I^2}{R_{II}^2} \cdot \frac{m_{II} - 1}{m_I - 1} \tag{9}$$

where:

R_I, R_{II} – coefficients of linear (or curvilinear) multiple correlation,
m_I, m_{II} – number of estimated parameters, in model I and II accordingly.

Critical testtarea for random variable F is the compartment: $[F(\alpha, m_I - 1, m_{II} - 1); +\infty)$, for which the following inferences can be formulated:

- if $F_{cal} \notin [F(\alpha, m_I - 1, m_{II} - 1); +\infty)$, then the collected sample does not deny the hypothesis under verification; in that case, the models explain price variability in an equally correct way;
- if $F_{cal} \in [F(\alpha, m_I - 1, m_{II} - 1); +\infty)$, then the hypothesis on variance equality of parts explained by models must be rejected, which means that the model *I* (with higher value R^2) significantly better models the value of the examined type of real estate on the analysed market.

An analogical test may be performed comparing remainder variances in models (null hypothesis $H_0 : \sigma_I^2 = \sigma_I^2$, contra alternative hypothesis $H_1 : \sigma_I^2 > \sigma_{II}^2$), for which the statistics has the form

$$F_{\sigma^2} = \frac{\sigma_I^2}{\sigma_{II}^2} \cdot \frac{n_{II} - m_{II}}{n_I - m_I} \tag{10}$$

where:

$\sigma_I^2, \sigma_{II}^2$ – model remainder variances,
m_I, m_{II} – number of estimated parameters in model I and II,
n_I, n_{II} – number of real estates used to estimate model I and II.

Inference is done as for the test (9):

- if $F_{cal} \notin [F(\alpha, n_I - m_I, n_{II} - m_{II}); +\infty)$, then the collected sample does not deny the hypothesis under verification; hence, estimation errors in both models are comparable;
- if $F_{cal} \in [F(\alpha, n_I - m_I, n_{II} - m_{II}); +\infty)$, the hypothesis on equality of remainder parts variances must be rejected; hence, estimation error is significantly lower in model II.

Table 3 contains data to perform these two tests, whereas selected results of comparison of models in pairs are presented in table 4.

Table 3. Coefficients of determination and remainder variances for estimated models

	Number of model	R^2	σ^2	m	n
Linear	1	0.54	2330418	9	69
	2	0.64	1788980	9	67
	3	0.54	2140061	5	67
	4	0.61	1821062	6	67
	5	0.63	1740220	7	67
	6	0.61	1904248	8	67
	7	0.53	2138858	4	67
	8	0.59	1876643	5	67
	9	0.60	1861754	6	67
Additive	1	0.77	1713920	29	69
	2	0.86	1085067	29	67
	3	0.68	1773875	15	67
	6	0.86	1057245	28	67
	7	0.68	1740406	14	67
	8	0.57	2117652	9	67
	9	0.83	1100972	23	67
Simple multiplicative	1	0.47	0.0623	9	69
	2	0.58	0.0473	9	67
	3	0.50	0.0520	5	67
	4	0.49	0.0520	4	67
	5	0.57	0.0465	7	67
	6	0.57	1.8039	8	67
	7	0.50	2.1250	4	67
	8	0.31	1.8950	5	67
	9	0.42	1.7570	6	67
Complex multiplicative	1	0.74	0.0580	37	69
	2	0.81	0.0416	37	67
	3	0.62	0.0513	19	67
	6	0.81	0.0403	36	67
	7	0.62	0.0503	18	67

Tests were performed, besides a standard level of significance $\alpha = 0.05$, also for, $\alpha = 0.10$ in order to get relatively small differences between coefficients of determination and remainder variances values.

In the table 4, following notation was assumed:

l – linear model,
a – additive model
p – simple multiplicative model
z – complex multiplicative model

e.g.: 8l – linear model number 8

In case of comparing coefficients of determination, pairs of models corresponding structurally were compared, i.e. these pairs where the model numbers are the same though they come from different input functions (1), (2) or (3). Whereas, when testing the equality of remainder variances, the pairs were formed of these determined above for R^2, with the constraint to comparing separately within additive or multiplicative models, because the difference between these two types of models [reached several orders of magnitude] was too high to compare them.

In addition, tests comparing R^2 or σ^2 were done in pairs, which seemed to give similar results of matching or of estimation exactitude, as well as in some random pairs. In the table, they are presented under a double line.

The results of the tests comparing variances of parts explained by established models reveal a regularity: in spite of many lower absolute values of R^2, generally, the models less complicated are more advantageous, i.e. within additive models – linear are better than non-linear, within multiplicative models – "simple" are better than "complex", also, additive linear are more advantageous than multiplicative "complex", and multiplicative "simple" than additive "complex" (non-linear) models. As we do not find, at the same time, statistically significant differences between "simple" models of different input form (additive and multiplicative) and "complex" at the same combination, we can conclude that a high increase of the number of estimated parameters is significantly disadvantageous to the reliability of a high coefficient of matching model to data, R^2.

Additional tests showed that one of the model pairs revealed a statistically significant difference (additive linear model no 2 and additive non-linear model no 6). However, here again, the linear model proved to be better despite of considerably lower absolute value of $R^2 = 0.64$; while, for a non-linear model it was $R^2 = 0.86$.

These observations are confirmed too, though on a smaller scale, by the results of the tests comparing remainder variances. Statistically significant differences were found between multiplicative "simple" and "complex" models. Notice that often both model types gave smaller as well as larger estimation errors, corrected to the number of degrees of freedom. Therefore, we can declare that the test comparing coefficients of matching R^2 proved to be stronger.

Table 4. Testing equality of coefficients of determination and remainder variances in models

Compared pairs of models	F_{R^2}	$F(\alpha, m_I - 1, m_{II} - 1)$		F_{σ^2}	$F(\alpha, n_I - m_I, n_{II} - m_{II})$	
		$\alpha = 0.10$	$\alpha = 0.05$		$\alpha = 0.10$	$\alpha = 0.05$
1l – 1a	2.45	**1.90**	**2.29**	1.10	1.44	1.59
1l – 1p	1.15	2.59	3.44			
1l – 1z	3.28	**1.85**	**2.21**			
1a – 1p	2.14	**1.90**	2.29			
1a – 1z	1.34	1.57	1.79			
1p – 1z	2.86	**1.85**	**2.21**	1.86	**1.48**	**1.65**
2l – 2a	2.60	**1.90**	**2.29**	1.08	1.48	1.65
2l – 2p	1.10	2.59	3.44			
2l – 2z	3.56	**1.85**	**2.21**			
2a – 2p	2.36	**1.90**	**2.29**			
2a – 2z	1.36	1.57	1.79			
2p – 2z	3.22	**1.85**	**2.21**	1.70	**1.48**	**1.65**
3l – 3a	2.78	**2.39**	3.11	1.01	1.43	1.58
3l – 3p	1.08	4.11	6.39			
3l – 3z	3.92	**2.29**	**2.93**			
3a – 3p	2.57	**2.39**	3.11			
3a – 3z	1.41	1.91	2.29			
3p – 3z	3.63	**2.29**	**2.93**	1.27	1.41	1.56
6l – 6a	2.74	**1.94**	**2.37**	1.19	1.47	1.65
6l – 6p	1.07	2.78	3.79			
6l – 6z	3.76	**1.90**	**2.28**			
6a – 6p	2.56	**1.94**	**2.37**			
6a – 6z	1.38	1.58	1.80			
6p – 6z	3.52	**1.90**	**2.28**	23.52	**1.54**	**1.72**
7l – 7a	3.38	**2.56**	3.41	1.03	1.43	1.57
7l – 7p	1.06	5.39	9.28			
7l – 7z	4.84	**2.44**	**3.20**			
7a – 7p	3.19	**2.56**	3.41			
7a – 7z	1.43	1.93	2.35			
7p – 7z	4.57	**2.44**	**3.20**	32.86	**1.43**	**1.58**

Table 4 cd.

Compared pairs of models	F_{R^2}	$F(\alpha, m_I - 1, m_{II} - 1)$		F_{σ^2}	$F(\alpha, n_I - m_I, n_{II} - m_{II})$	
		$\alpha = 0.10$	$\alpha = 0.05$		$\alpha = 0.10$	$\alpha = 0.05$
4l – 4p	1.34	3.62	5.41			
5l – 5p	1.10	3.05	4.28			
8l – 8a	2.07	2.81	3.84	1.21	1.40	1.53
8l – 8p	1.90	4.11	6.39			
8a – 8p	1.09	2.81	3.84			
9l – 9a	3.18	**2.13**	**2.66**	1.22	1.47	1.62
9l – 9p	1.43	3.45	5.05			
9a – 9p	2.23	**2.13**	2.66			
2l – 1a				1.39	1.45	1.60
2l – 3a				1.11	1.41	1.56
2l – 7a	1.53	2.20	2.77	1.06	1.41	1.56
1l – 8a	1.06	2.59	3.44	1.06	1.40	1.53
3l – 8a				1.06	1.40	1.53
7l – 8a				1.07	1.40	1.53
2l – 6a	2.50	**1.91**	**2.31**	1.14	1.48	1.65
8a – 5p	1.33	2.67	3.58			
8l – 2p	2.03	2.81	3.84			
1a – 7z	1.33	1.72	2.01			
9a – 6z	1.63	1.69	1.96			
3p – 7z				1.22	1.41	1.56
4p – 3z				1.29	1.41	1.56
4p – 7z				1.24	1.41	1.56

5. Conclusions and closing remarks

Performed analyses allowed to formulate the following conclusions:

- Modelling by summation of attributes shares and by multiplication of these shares may give statistically equally good results.
- Considering in modelling process the non-linear relation between a given attribute and the price improves significantly the quality of modelling. How-

ever, when it requires an important growth of estimated parameters number, the reliability of a high coefficient of determination decreases quickly.

– According to the results of the test examining significance of particular model coefficients, we can reduce the number of attributes, thereby simplifying the model form without impair estimation results.

On the basis of the performed researches, we can affirm that the application of non-linear models in estimation process may contribute to ameliorate modelling quality. It requires however detailed analyses and does not permit to use automatically formulas determined by standards. It obliges also the expert to a more substantial preparation, making of the estimation process an interesting scientific problem.

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