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Principles of Calculating the Cadastral Value

1. Mass Appraisal Method

Pursuant to the ordinance of the Council of Ministers of 29 June, 2005, on mass appraisal of real estate, the cadastral value of the real estate should be calculated based on a comparative approach, using the market statistical analysis method.

Comparative approach is a process in which the market value of a given real estate is calculated based on the analysis of the similar real estate market and based on the comparison of the similar real estate with the appraised real estate. The application of this method requires the knowledge of transaction prices paid for real estate properties similar to the appraised real estate, and the knowledge attributes of these real estate properties. Similar real estates should be selected from a representative market, and the scope of scale of their attributes should include the attributes of the appraised real estate.

The criteria for the choice of attributes should be as objective as possible, and the selection of attributes should reflect the trends observed in the real estate market.

Each appraised real estate should be identified from the formal, legal and technical points of view. The purpose and the date of valuation should be determined, and the appraised real estate and the real estates in the base should be described in details according to the accepted attributes. Transaction prices of real estates selected for comparison should be calculated per area unit, and then corrected based on the date of valuation. If the correction is impossible, the time that lapsed from the date of sale to the date of valuation should be interpreted as an attribute.

Calculating the real estate value using the comparative approach is always preceded by the analysis of market information, which consists in selecting from among many attributes those which have material influence on the price and determining their role in explaining the real estate value.

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Unfortunately, neither the aforementioned ordinance, nor the Best Practices for Real Estate Professionals regulate the manner of calculating the value of representative real estates. The only issue that has been addressed is that this method should be applied by the statistical methods used in statistical analyses. Therefore, it is often suggested to use the modified method of correcting the average price which would fit to the widely understood statistical analysis according to the new Best Practices.

2. Real Estate Base

The analysis has been conducted on the example of big real estate market of Cracow. 205 undeveloped real estates have been selected for the analysis. The studied transactions have taken place over the course of two years.

The real estates in the base are located in three districts of the city of Cracow: Krowodrza, Podgórze and Nowa Huta. Real estates located in the city centre (Śródmieście district) are not included in the base. No purchase/sale transactions of undeveloped real estate have been recorded within the studied period.

The real estates in the base have been described using eight attributes which may have material influence on the price changeability. The attributes, their description and the accepted scale are presented in table 1.

Table 1. The attributes, their description and the accepted scale

Attribute	Attribute description, scale
Time	number of months that have passed from the date of transaction to the date of valuation
Utilities	existing utilities or utilities that can be brought to the land, number of systems - scale from 0 to 6 (power, water supply, gas mains, sewage system, heat supply system, telecommunications)
Location	real estate location – distance to the city centre, places considered attractive and public institutions (scale from –1 to 2)
Transportation	availability of public transportation services, access to public road (scale from 0 to 3)
Trends	location of real estate in Cracow according to current trends (scale from 0 to 5)
Surroundings	influence of the existing infrastructure and surrounding land development on the real estate (scale from –1 to 2)
Purpose	taking into account the land development plans, planning permissions issued and the purpose attributed to the real estate in the land register (scale from 0 to 2).
Surface area	surface area of the real estate in square metres

Figure 1 presents the location of all real estates from the base on the area map of Cracow.

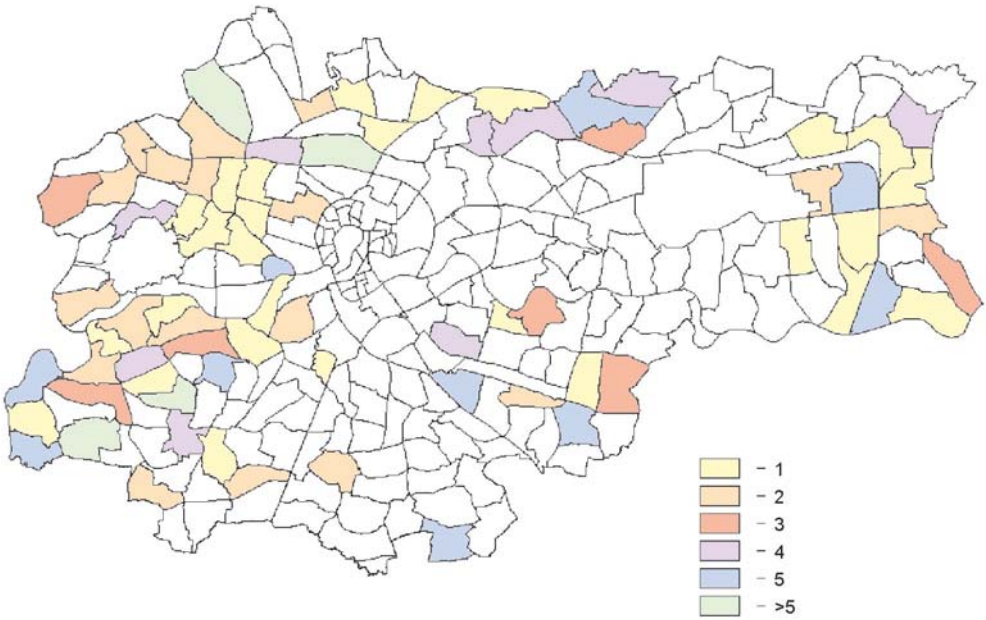

Fig. 1. Area map of Cracow with the number of real estates from the base in given bounds

3. Real Estate Market Analysis

All information concerning the real estate market is random. This means that random factors shall decide both which real estates are to be appraised, and which real estates shall be selected for the comparison, and which attributes shall be used to describe them. Analogically, the selection of the valuation method and the real estate professional, who is to prepare his opinion on the real estate value, seem to be a random event. Therefore, the conclusion is that the result of the valuation is also of random character, which, in turn, makes the real estate professionals apply rules for random quantities. Therefore, the market analysis is conducted using the estimation method.

A multidimensional random variable is employed in methods for statistical market analysis. The variable representing the real estate price is a dependent variable, and other variables representing the attributes of the real estate are explanatory, independent variables.

In the process of analyzing data employing many variables, calculating the correlation matrix and seeking significant dependencies are commonly applied methods of analysis.

The elements of correlation matrix are Pearson product-moment correlation coefficients between given variables.

While analyzing the correlation matrix, one can not only study the relations between the transaction prices of real estates and attributes that describe them, but also the dependencies that can be observed between specific attributes. Pairs of attributes that indicate high correlation coefficient have similar influence on real estate prices, and therefore, they explain the respective price changeability. In case of such pairs, in order to improve the credibility of the model and to guarantee the stability of their coefficients estimation, one should give up one attribute that is less correlated with the price.

Based on the correlation coefficients, the multiple correlation coefficient (R) and its square, the coefficient of determination (R^2), are defined: The multiple correlation coefficient indicates to what extent all attributes are linearly correlated with prices, and the coefficient of determination indicates to what extent all attributes can explain price changeability of real estates in the base.

In the first stage of the analysis of the obtained market information, calculation has been made on all real estates in the base. The Pearson product-moment coefficients of correlation between attributes and the transaction price have been calculated. Based on these coefficients, the standardized weight percentage of attributes in explaining prices has been calculated. The results of these calculations are presented in table 2.

Table 2. Coefficients of correlation between attributes and the price, their squares and weight percentage of attributes in explaining prices for the whole city of Cracow

	Date	Utilities	Location	Transportation	Trends	Surroundings	Purpose	Surface area
Product-moment correlation coefficient	0.30	0.37	0.30	0.03	0.05	0.27	0.37	–0.19
The square of the product-moment correlation coefficient	0.09	0.13	0.09	0.00	0.00	0.07	0.13	0.04
Weight percentage [%]	16	24	16	0	0	13	24	7

The abovementioned values of squares of product-moment correlation coefficients lead to a conclusion that in the analyzed set of market information on real estates, unit prices are influenced by the following attributes (percentage, in de-

creasing order): UTILITIES (13%), PURPOSE (13%), LOCATION (9%), DATE OF VALUATION (8%), SURROUNDINGS (7%) and the negative correlation with SURFACE AREA (4%).

TRANSPORTATION and TRENDS have no influence on the real estate price. One may attempt to explain this phenomenon in the following way. Real estates located far from noisy roads are characterized by higher transaction prices. Buyers tend to look for peaceful and quiet places. When it comes to trends, in the researched period it was of no importance, as currently there are so many investments in Cracow that real estates in trendy districts are no longer marketed, as they have been sold in the first instance. Nowadays, *all* available real estates are purchased and sold. Investors do not pay attention to the trends concerning the location of the real estate. A developer who buys a land property knows that houses and apartments built on this land will not stay empty for too long, regardless of the location of the property.

Having calculated these values into 100% (after dividing the squares of product-moment correlation with their total of 0.55), weight percentages (standardized percentages) of influence of analyzed attributes on changes in prices of real estates have been calculated: UTILITIES (24%), PURPOSE (24%), LOCATION (16%), DATE OF VALUATION (16%), SURROUNDINGS (13%) and SURFACE AREA (7%).

Standardized weight percentage of particular attributes in explaining price changeability is shown in figure 2.

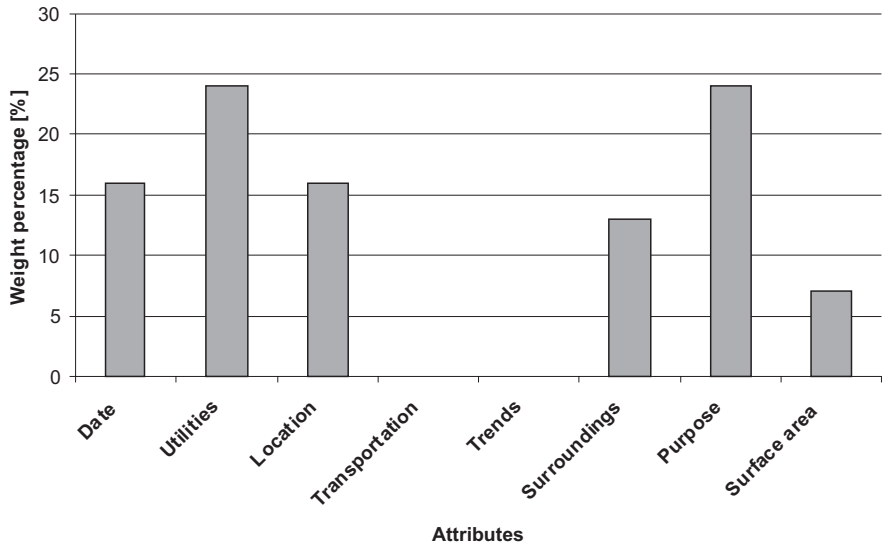

Fig. 2. Weight percentage of attributes in explaining price changeability for the whole city of Cracow

Moreover, the characteristic parameters of this market have the following values:

- average unit price: $\hat{c}$ = 109.80 zł/m^2,
- standard dispersion of unit prices around the average value: σ_n = 105.10 zł/m^2,
- dispersion coefficient: $\lambda = \sigma_n / \hat{c}$ = 0.96.

The obtained dispersion coefficient indicates that the base of analyzed real estates is inconsistent – it is commonly accepted that the base is consistent when the dispersion coefficient $\lambda \leq 0.25$. However, it was predictable, as the base contains real estates from all over the city, and the real estate market in Cracow is strongly diversified and should not be analyzed as a uniform market. Based on the correlation coefficients, the multiple correlation coefficient (R) and its square, coefficient of determination (R^2) have been defined: R = 0.56, R^2 = 0.34.

The calculated values show that all attributes are linearly correlated with real estate prices with strength of 0.56 and that in the linear multiple regression model, the analyzed attributes may explain only 34% of the changeability of prices of real estates included in the base.

Due to the low value coefficient of determination it has been decided to check what would happen if:

- real estates which stand out were removed from the base, so that the coefficient of determination could reach the maximum value;
- the city was divided into zones;
- the number of attributes describing the real estates in the base was reduced.

4. Division into Zones

Depending on the location, the city has been divided into 9 zones so that the bases of real estates in each of them to be compared have the coefficient of determination higher than 60%. Each new base included at least a dozen real estates for comparison.

Table 3 shows the description of zones. Figure 3 presents the division into zones. Table 4 shows figures concerning each zone and information used in the comparison process.

One representative real estate per each zone has been selected for appraisal. In order for them not to influence the result of the appraisal, they have been removed from the base for comparison, contradictory to the provisions of the ordinance on mass appraisal of real estate.

Table 3. Description of zones

Zone	District	Number of real estates	Number of bounds
Zone 1 (Nowa Huta 1)	Mistrzejowice, Bieńczyce	18	6
Zone 2 (Nowa Huta 2)	Branice	29	13
Zone 3 (Podgórze 1)	Bieżanów, Prokocim	20	8
Zone 4 (Podgórze 2)	Kostrze, Bodzów, Skotniki	21	10
Zone 5 (Podgórze 3)	Łagiewniki	12	5
Zone 6 (Podgórze 4)	Tyniec	20	7
Zone 7 (Krowodrza 1)	Prądnik Biały, Prądnik Czerwony	10	6
Zone 8 (Krowodrza 2)	Krowodrza	14	9
Zone 9 (Krowodrza 3)	Bronowice	25	7

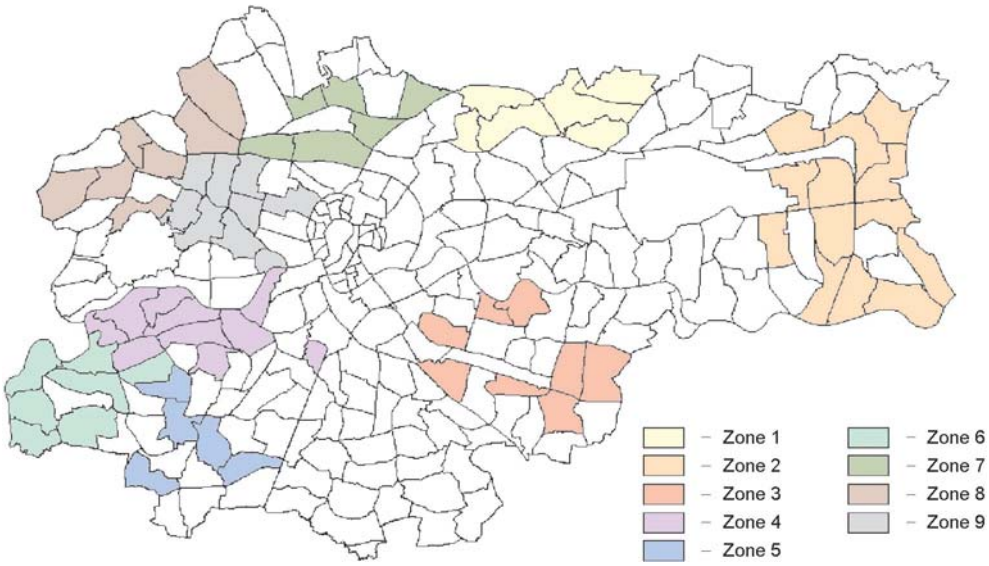

Fig. 3. Division into zones

Table 4. Comparable information on real estates per zone

	Zone 1	Zone 2	Zone 3	Zone 4	Zone 5	Zone 6	Zone 7	Zone 8	Zone 9
Minimum price	2	1	12	5	14	2	17	44	89
Maximum price	208	48	218	364	214	120	365	599	351
Average value	118.0	12.3	68.5	101.1	125.5	54.6	205.8	230.3	210.7
Standard deviation	46.52	14.24	52.23	95.42	67.47	42.48	106.94	199.39	68.19

Table 5 presents standardized weight percentage for all bases created for the purpose of comparison.

Table 5. Weight percentage of attributes in explaining prices concerning all bases created for the purpose of comparison [%]

	Zone 1	Zone 2	Zone 3	Zone 4	Zone 5	Zone 6	Zone 7	Zone 8	Zone 9	Cracow	Cracow corrected
Date	3	14	3	33	0	2	0	27	2	8	15
Utilities	19	36	4	49	21	14	15	1	5	48	24
Location	39	2	33	10	16	25	0	11	0	15	16
Transportation	1	7	0	1	17	10	8	0	7	1	0
Trends	0	6	3	3	10	7	0	30	14	0	0
Surroundings	25	4	8	1	6	15	19	10	0	17	15
Purpose	0	24	39	1	6	18	2	21	38	9	24
Surface area	13	6	11	1	24	9	56	0	34	1	7

CRACOW base includes 196 real estates and it is the whole base, diminished by the appraised real estates.

CRACOW CORRECTED is the CRACOW base without real estates that stand out, so that the coefficient of determination value is as high as possible.

5. Smaller Number of Attributes

As it was assumed that real estates in all bases have to be described with the same attributes, the attributes to be rejected have been selected based on the analysis of weight coefficients in particular zones.

TRENDS and SURROUNDINGS attributes were removed from the base to be valuated. This was caused by the fact that in most zones TRANSPORTATION had a significant influence on prices, whereas TRENDS and SURROUNDINGS explained the prices to the extent similar to the influence of the PURPOSE attribute and were strongly correlated with them.

Weight coefficients obtained after the reduction of the number of attributes have been presented in table 6.

Table 6. Weight percentage of attributes in explaining prices concerning all created bases after the reduction in the number of attributes [%]

	Zone 1	Zone 2	Zone 3	Zone 4	Zone 5	Zone 6	Zone 7	Zone 8	Zone 9	Cracow	Cracow corrected
Date	3	16	3	3	0	3	0	46	2	10	17
Utilities	26	40	4	26	26	18	18	1	6	58	28
Location	52	2	37	52	19	32	0	18	0	18	19
Transportation	2	8	0	2	20	12	10	0	8	1	0
Purpose	0	27	44	0	8	23	3	36	44	11	28
Surface area	17	7	13	17	28	11	69	0	40	1	9

6. Valuation

While performing valuation using the modified method of correcting the average price, the base containing at least eleven real estates is employed. These real estates are the basis for determining the average transaction price and its standard deviation, as well as average values for particular attributes. The real property value is determined by means of correction of the average transaction price from the base, using appropriate weight coefficients of attributes and the difference in value of attributes of the appraised real estate and their average values in the base.

The following formula is employed while calculating the unit market value of a real estate

$$w = \hat{c} + 3 \cdot \sigma^c \cdot \sum_{j=1}^{u} k_j \frac{(\bar{a}_j - \hat{a}_j)}{(a_{j/\max} - a_{j/\min})},$$

where:

$\hat{c}$ – average transaction price from the base, used for comparison,
σ_c – standard dispersion of unit prices from the base, used for comparison,
k_j – weight percentage,
$\bar{a}_j$ – value of attributes of the appraised real property,
$\hat{a}_j$ – average value of j-th attribute from the base, used for comparison,
$a_{j/\max}$ – maximum value of j-th attribute from the base, used for comparison,
$a_{j/\min}$ – minimum value of j-th attribute from the base, used for comparison.

In order to evaluate the consistency of the appraisal model, one should calculate the cadastral value of each real estate from the base for comparison. σ_n standard deviation from differences of unit transaction prices from the base and estimated market values of each real estate from the subbase is the average measure of inconsistency that can be observed between the appraisal model and the base of real estates for appraisal.

The coefficient of inconsistency between the appraisal model and the base to be compared shall be expressed by means of the dispersion coefficient, which can be presented using the following formula

$$\lambda = \frac{\sigma_n}{\hat{c}}.$$

For the purpose of evaluation of credibility coefficient of the mass appraisal model, agreed scale is used – appraisal is unacceptable if $\lambda > 0.25$.

In case one obtains such result, the real estate base should be verified, and the calculations should be carried out one more time.

Selected representative real estates have been valuated using the method described earlier. Each real estate has been appraised 6 times, based on the real estate base from the whole city of Cracow, and using all collected data, having rejected real estates which stood out, within the whole city and in zones, each time using 8 attributes first, and then 6 attributes for the purpose of analysis.

7. Results and Conclusions

Having taken into account the division of the market into zones, different appraisal options have been analyzed. Data collected during the appraisal have been presented in table 7, and in order to illustrate the results, figure 4 has been created (value of representative real estates based on different bases).

The obtained results show significant discrepancies between the transaction price and the calculated cadastral value, depending on the appraised base. By far the smallest discrepancies appeared in the appraisal conducted in zones.

Using the base from the whole city shall not be of use for the mass appraisal. Appraisal values do not correspond with their transaction counterparts. Analogical situation can be observed in the corrected base, which consists of a smaller number of real estates {142}. In case of this base, cadastral values are understated in most cases. The cause for this situation is probably the statistical method with all its imperfections.

Table 7. Comparison of transaction prices and cadastral values in PLN/m²

Column No.	1	2	3	4	5	6	7	8	9	10	11	12	13
	Transaction price	Value in a zone – 8 attributes	Value in a zone – 6 attributes	% change in col. 2 presented in col. 3	% change in col. 3 presented in col. 1	Value in the 196 N base – 8 attributes	Value in the 196 N base – 6 attributes	% change in col. 6 presented in col. 7	% change in col. 7 presented in col. 1	Value in the 142 N base – 8 attributes	Value in the 142 N base – 6 attributes	% change in col. 10 presented in col. 11	% change in col. 11 presented in col. 1
Zone 1	110	127	130	2%	18%	128	129	1%	17%	97	101	4%	–8%
Zone 2	37	16	16	0%	–57%	70	79	11%	114%	47	53	11%	43%
Zone 3	199	125	138	9%	–31%	144	183	21%	–8%	91	118	23%	–41%
Zone 4	130	82	78	–5%	–40%	207	204	–1%	57%	104	98	–6%	–25%
Zone 5	113	134	128	–5%	13%	81	74	–9%	–35%	56	51	–10%	–55%
Zone 6	67	44	42	–5%	–37%	80	72	–11%	7%	61	57	–7%	–15%
Zone 7	206	219	250	12%	21%	83	76	–9%	–63%	53	48	–10%	–77%
Zone 8	317	264	324	19%	2%	148	153	3%	–52%	81	82	1%	–74%
Zone 9	194	256	262	2%	35%	157	163	4%	–16%	84	85	1%	–56%

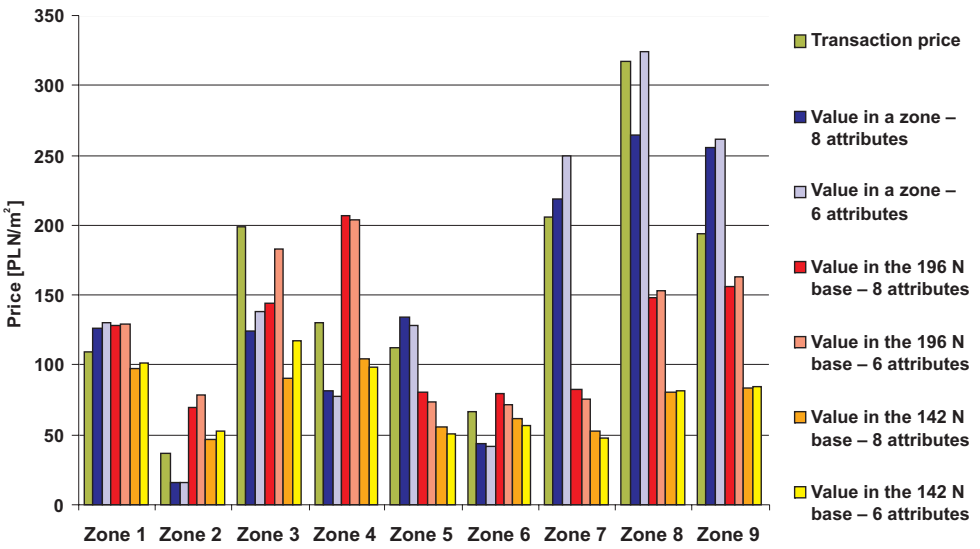

Fig. 4. Comparison of transaction prices and cadastral values

In order to increase the value of the coefficient of determination, the outliers have been eliminated. In this case, the outliers were real estates characterized by high transaction prices. After the analysis, the model became flat in price.

In all cases there was a significant dispersion of bases, which is reflected by the value of dispersion coefficients presented in table 8.

Table 8. Dispersion coefficients

Real estate	Dispersion coefficients – 8 attributes			Dispersion coefficients – 6 attributes		
	Cracow	Cracow corrected	Zone	Cracow	Cracow corrected	Zone
Zone 1	0.46	0.69	0.28	0.47	0.77	0.27
Zone 2	0.55	0.54	0.69	0.51	0.54	0.74
Zone 3	0.53	0.63	0.91	0.80	0.97	1.09
Zone 4	1.00	0.77	0.65	0.97	0.97	0.69
Zone 5	0.49	0.49	0.35	0.54	0.73	0.35
Zone 6	0.50	0.47	0.48	0.55	0.55	0.52
Zone 7	0.48	0.50	0.40	0.52	0.52	0.46
Zone 8	0.56	0.54	0.57	0.59	0.58	0.70
Zone 9	0.61	0.56	0.51	0.66	0.60	0.56

To sum up, in order to conduct mass appraisal of real estate, it is essential to create small zones, whose features correspond to the conditions of local micromarket, encompassing districts or their parts in urban areas, and communes or their parts in rural areas. The key to success in this method is the proper manner of dividing and selecting these zones. The best method to create zones is to prepare and study the data gathered for the whole area of a given region. Market analysis conducted using statistical methods shall make it possible to set apart zones, based on the whole set of information in the base. These zones need to be small enough to reflect the trends on the local market, and, at the same time, they need to be big enough to minimize appraisal costs. The preliminary analysis shall also eliminate the information noise and misleading information, as well as hidden factors which can distort the model. Valuation also requires the reduction of the number of attributes, so that it is as small as possible, but still provides equal amount of information. These conditions allow to effectively calculate the cadastral value. However, one should put a lot of effort into developing an accurate method algorithm which would fully reflect the actual value of a given real estate.

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