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Refined Astrogravimetric Geoid in Poland – Part I**

1. Introduction

Astronomical levelling as a method of geoid determination was developed by Helmert and applied first time for modelling geoid in Harz Mountains, Germany. In the middle of 20th century the method has already been used in numerous countries for the determination of local geoid that was mainly required for reductions of geodetic observations to the reference ellipsoid.

Works on geoid determination on a continental scale using astronomical levelling were conducted in North America [5] and Europe [10]. The accuracy of those regional geoid models was at the level of a few metres. Much more accurate were local geoid models developed in mountainous countries such as Switzerland and Austria with the use of large number of uniformly distributed deflections of the vertical together with high resolution digital terrain model [4, 12].

The first astrogravimetric geoid model for Poland was developed at the Institute of Geodesy and Cartography in Warsaw in 1961 [3] from about a hundred of astrogeodetic deflections of the vertical and gravity anomalies obtained from gravity maps. In the framework of modernisation of geodetic control network in Poland that model was refined in 1970 and 1978 with the use of more data and more detail gravity maps.

The next astrogravimetric geoid model for Poland was calculated in 2005 in the framework of the project on precise geoid modelling in Poland, conducted in 2002–2005. That model was developed using the archival deflections of the verti-

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cal, both astrogeodetic and astrogravimetric, after transforming them to uniform standards and systems [14, 18]. The geoid model was computed using classical astronomical levelling approach. Accuracy of that astrogravimetric geoid model was estimated by comparing it with the existing geoid models and standard deviation of computed geoid heights was estimated on ±30 cm [18].

These developed models suffer from the lack of several effects, that were not fully considered, and problems that were not completely solved. They concern quality of archival astrogravimetric data, problem of weighting, the effects of plumb line curvature, elimination of outlying observations. In addition, all those geoid models were determined with the use of classical astronomical levelling approach.

The aim of this study is to improve the astrogravimetric geoid model in Poland by application least squares collocation and by using improved data. First the realistic accuracy of the deflections of the vertical was estimated and the proper weights of astrogeodetic and astrogravimetric deflections of the vertical were determined. Then the astrogeodetic and astrogravimetric geoid models were computed from improved archival deflections of the vertical with the use of Helmert astronomical levelling. Finally new astrogeodetic and astrogravimetric geoid models were determined by least squares collocation from improved archival astrogravimetric deflections of the vertical and gravity anomalies. All version of computed geoid models were mutually compared. They were also were compared with the GPS/levelling geoid spanned on the sites of the POLREF network and the differences obtained were analysed and discussed.

2. Astronomical Levelling by Helmert's Method and by Least Squares Collocation

When at point *P* of geodetic coordinates (φ, λ) its astronomical coordinates (Φ, Λ) are measured then the deflection of the vertical θ can be determined. The components ξ, η of the deflection of the vertical θ in the meridional and prime vertical plane, respectively, are given as follows:

$$\begin{aligned} \xi &= \Phi - \varphi \\ \eta &= (\Lambda - \lambda)\cos\varphi \end{aligned} \qquad (1)$$

The deflection of the vertical ε in the plane of the geodetic azimuth α expressed in terms of ξ and η is

$$\varepsilon = \xi\cos\alpha + \eta\sin\alpha \qquad (2)$$

and reflects the tilt of the profile of the equipotential surface with regard to the respective spheropotential surface

$$dN = -\varepsilon \cdot ds \tag{3}$$

The corresponding deflection of the vertical ε_0 on the geoid in the plane of the geodetic azimuth α represents the tilt of the geoid with respect to the ellipsoid [19]

$$dN_0 = -\varepsilon_0 \cdot ds \tag{4}$$

Astronomical levelling provides geoid height differences along the profiles on physical surface of the Earth or on the geoid by integrating deflections of the vertical. Geoid height difference between two points P_i and P_j on the geoid is given as follows

$$\Delta N_{P_iP_j} = N_{P_j} - N_{P_i} = -\int_{P_i}^{P_j} \varepsilon_0 \cdot ds \tag{5}$$

In the formula (5) it has been assumed that deflections of the vertical are provided continuously along the profile P_iP_j of the astronomical levelling. Practically, due to high cost and laboriousness, deflections of the vertical are measured at points distant by at least few tens of kilometres. They might be densified either with the use of numerical interpolation or with the use of prediction with taking into consideration physical features of the gravity field [1].

When using numerical interpolation of deflections of the vertical between points P_i and P_j one assumes its linear change from ε_{P_i} to ε_{P_j}. Then

$$\Delta N_{P_iP_j} = -\frac{\varepsilon_{P_i} + \varepsilon_{P_j}}{2} s_{P_iP_j} \tag{6}$$

where is the distance between points P_i and P_j.

Theory and algorithms concerning the interpolation of astrogravimetric deflections of the vertical in Poland has been given e.g. in [2]. Components ξ_A, η_A of deflections of the vertical interpolated at point A are expressed as follows:

$$\begin{aligned} \xi_A &= \xi_A^{gr} + \Delta\xi_A \\ \eta_A &= \eta_A^{gr} + \Delta\eta_A \end{aligned} \tag{7}$$

where ξ_A^{gr}, η_A^{gr} are components of deflections of the vertical calculated at point A from gravity data within the radius r around A. Terms $\Delta\xi_A$, $\Delta\eta_A$ are corrections transforming components of the gravimetric deflection of the vertical at A into the astrogeodetic one.

Astrogravimetric levelling by least squares collocation. If we consider the area σ_1 inside of which either components ξ, η of the deflection of the vertical or gravity anomalies Δg or both observables are available the geoid undulation N can be determined in the points of this area with the use of data available in σ_1 applying least squares collocation.

The vector **l** of measurements can be partitioned accordingly

$$\mathbf{l} = \begin{bmatrix} \mathbf{l}_1 \\ \mathbf{l}_2 \end{bmatrix} \tag{8}$$

where $\mathbf{l}_1$ contains the ξ, η measurements and $\mathbf{l}_2$ contains gravity anomalies Δg available in σ_1. The measured functionals of the disturbing potential T, i.e. ξ, η, Δg are affected by random measuring errors n (noise). Hence the vector **l** is the sum of the vector **t** of signals and the vector **n** of noise. The signals correspond to the true values (unknown) of the functionals ξ, η, Δg. The covariance matrix $\mathbf{C}_{nn}$ is determined with the use of measuring errors **n**. The covariance matrix $\mathbf{C}_{tt}$ can be split up as

$$\mathbf{C}_{tt} = \begin{bmatrix} \mathbf{C}_{t_1 t_1} & \mathbf{C}_{t_1 t_2} \\ \mathbf{C}_{t_2 t_1} & \mathbf{C}_{t_2 t_2} \end{bmatrix} \tag{9}$$

where $\mathbf{C}_{t_1 t_1}$ and $\mathbf{C}_{t_2 t_2}$ are the autocovariance matrices of the components of the deflections of the vertical and gravity anomalies, respectively, and $\mathbf{C}_{t_1 t_2}$ is the crosscovariance matrix between components of the deflection of the vertical and gravity anomalies, with $\mathbf{C}_{t_1 t_2} = \mathbf{C}_{t_2 t_1}$. Once the matrix $\mathbf{C}_{tt}$ is determined, the matrix $\mathbf{C}_{ll}$ can be evaluated from

$$\mathbf{C}_{ll} = \mathbf{C}_{tt} + \mathbf{C}_{nn} \tag{10}$$

The signals **s** that are to be predicted are geoid undulations at points P_i. They can be computed from [13]

$$\mathbf{s} = \mathbf{C}_{sl} \mathbf{C}_{ll}^{-1} \mathbf{l} \tag{11}$$

where $\mathbf{C}_{sl}$ is the vector of covariance between the signals **s** to be predicted and the measurements **l** and can be written in a partitioned form as

$$\mathbf{C}_{sl} = \begin{bmatrix} C_{sl_1} \\ C_{sl_2} \end{bmatrix} \tag{12}$$

where $\mathbf{C}_{sl_1}$ and $\mathbf{C}_{sl_2}$ are the vectors of covariance between the signals **s** and the components $\mathbf{l}_1$ of deflection of the vertical, and the gravity anomalies $\mathbf{l}_2$, respectively. It should be noted that all covariances between the different functionals of the disturbing potential T do not depend on the measurements **l** but only on their relative positions on the terrestrial sphere.

The choice of covariance function appropriate to compute necessary covariances is described e.g. in [9].

3. Data Used for Geoid Determination

Archival astrogeodetic data from Poland set up in the catalogues in 1967–1981, unfortunately do not contain astronomic measurements from before 1939. Moreover, source materials such as observational sheets and lists of calculations of astronomical coordinates have not been preserved.

Remained astronomical data contain averaged astronomical coordinates and the epoch of observation (date). They also contain information on the number of stars (or pairs of stars) observed, observation method applied, observation instrument and star catalogue used for processing observations. Majority of astronomic observations were processed with the use of FK3 catalogue. FK4 catalogue was used for processing ten observations and Chelberger catalogue – for processing three observations.

Remained information contains also terms of reductions to the conventional mean pole and corrections to UT1 in accordance with IAU conventions effective in the epoch of the discussed data processing.

Different instruments and different methods were used for the determination of astronomical coordinates. The astronomic latitude was determined mainly by Pevtsov and Talcott method (in few cases by Sterneck method) while the astronomic longitude was determined by Zinger or Mayer method.

Astronomical coordinates were determined at 171 points (Fig. 1). Geodetic coordinates of those points were referred to Krassowski's ellipsoid and "Pulkowo42" datum. The standard deviations of the components of astrogeodetic deflections of the vertical were estimated in [3] as $\sigma_\xi = 0{,}2''$, $\sigma_\eta = 0{,}3''$ while in [6] as $\sigma_\xi = \sigma_\eta = 0.45''$.

Archival astrogravimetric deflections of the vertical were determined in 370 points that were densifying the astrogeodetic network. Standard deviations of the components of astrogeodetic deflections of the vertical varied dependently on the roughness of terrain within the range from 0.54'' to 0.69'' [2].

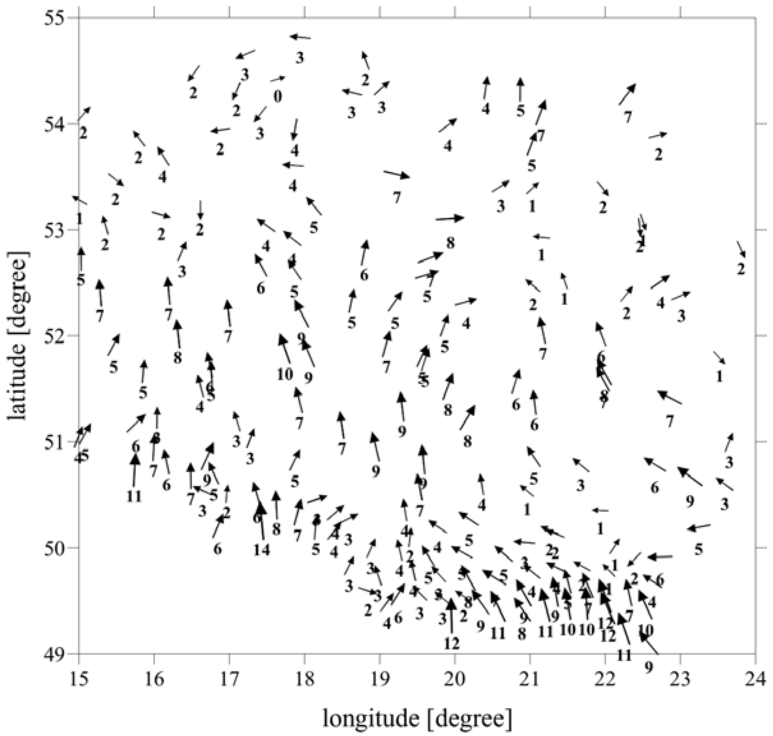

Fig. 1. Astrogeodetic deflections of the vertical (in arcseconds) referred to Krassowski ellipsoid

Archival relative deflections of the vertical [7] are given in JSAG system referred to Krassowski's ellipsoid with initial (fundamental) point in Pulkovo.

The original astrogeodetic and astrogravimetric deflections of the vertical, which were measured nearly forty years ago, required a number of corrections [14]. In 2003 they were unified and referred to the presently effective reference system, i.e. they were corrected due to star catalogue, polar motion, time system and geodetic datum. The corrected deflections of the vertical are referred to ETRF89 (GRS80 ellipsoid), IERS pole and TDT and UT1 [16].

The works on unification of archival deflections of the vertical were preceded with their independent quality estimate. The standard deviation of components of astrogeodetic deflections of the vertical was estimated as 0.5" while for astrogravimetric ones as 0.7" [15].

In 2003–2004, complementary astronomic and GPS observations were conducted in 29 points of geodetic control in Poland with the use of the circumzenithal VUGT 50/500 [17]. According to BIH recommendations the observational program consisted of stars from FK5 catalogue for which the ephemeris of their transit through the almucantar were calculated. The astronomic observations were processed by Jan Hefty of Slovak University of Technology, Bratislava, while GPS observations – at Warsaw University of Technology. The standard deviation of components of astrogeodetic deflections of the vertical was estimated as 0.3"–0.5" [17].

4. Accuracy Estimation of the Astrogeodetic Deflections of the Vertical

The estimates of accuracy of the components of the deflections of the vertical substantially diverge due to applying different methods and statistical tools. It indicates the need of redoing the accuracy estimate that could be considered reliable. The new estimate of accuracy of the components of the deflections of the vertical is based on comparison of the existing astrogeodetic deflections of the vertical with the respective ones computed from gravity data.

Having gravity data of sufficient coverage and eventually digital terrain model (DTM) of high resolution in the area of interest enable to calculate so called gravimetric deflections of the vertical as follows:

$$\begin{aligned} \xi^{grav} &= \xi_{GM} + \xi_{\Delta g_{res}} + \xi_{DTM} \\ \eta^{grav} &= \eta_{GM} + \eta_{\Delta g_{res}} + \eta_{DTM} \end{aligned} \tag{13}$$

where ξ_{GM}, η_{GM} are low frequency components that are determined from the global geopotential model (GM), $\xi_{\Delta g_{res}}$, $\eta_{\Delta g_{res}}$ are medium frequency components that are calculated from the residual gravity anomalies with the use of Vening–Meinesz integral, and ξ_{DTM}, η_{DTM} are high frequency components that are computed using DTM.

Gravimetric deflections of the vertical ξ^{grav}, η^{grav} computed with (13) can be compared with the respective astrogeodetic deflections of the vertical ξ^{astr}, η^{astr}. The differences obtained:

$$\begin{aligned} \Delta\xi &= \xi^{astr} - \xi^{grav} \\ \Delta\eta &= \eta^{astr} - \eta^{grav} \end{aligned} \tag{14}$$

can further be used for quality estimation of the existing astrogeodetic deflections of the vertical.

Gravimetric deflections of the vertical have been calculated at 171 archival astrogeodetic points, 370 archival astrogravimetric points and 23 newly surveyed astrogeodetic points with the use of most recent set of gravity data (set no. 7 in [8] consisting of mean 1' × 1' Faye gravity anomalies) and the global geopotential model EGM96. The high frequency components due to topography in (13) were neglected since residual Faye anomalies used to generate medium frequency components contained already the high frequency terms through terrain corrections. Statistics of differences between computed gravimetric deflections of the vertical and astrogeodetic or astrogravimetric deflections of the vertical are given in tables 1, 2 and 3.

Table 1. Statistics of differences between 171 unified astrogeodetic deflections of the vertical and the respective gravimetric deflections of the vertical [arcsec]

Statistics	$\Delta\xi$	$\Delta\eta$	$\Delta\theta$
Mean	–0.14	0.11	–0.04
Std dev.	0.53	0.57	0.54
Min	–1.61	–2.32	–2.17
Max	1.45	2.20	2.47

Table 2. Statistics of differences between 370 unified astrogravimetric deflections of the vertical and the respective gravimetric deflections of the vertical [arcsec]

Statistics	$\Delta\xi$	$\Delta\eta$	$\Delta\theta$
Mean	–0.23	0.04	–0.08
Std dev.	0.49	0.59	0.56
Min	–3.50	–5.35	–3.81
Max	2.17	1.51	4.15

Table 3. Statistics of differences between 23 newly surveyed astrogeodetic deflections of the vertical and the respective gravimetric deflections of the vertical [arcsec]

Statistics	$\Delta\xi$	$\Delta\eta$	$\Delta\theta$
Mean	–0.79	0.19	–0.36
Std dev.	1.02	0.47	0.77
Min	–5.09	–0.40	–3.22
Max	0.31	1.89	0.77

Figure 2 shows the mean and the standard deviation of differences of the deflections of the vertical for three data sets investigated. In case of unified archival data (Tabs 1 and 2) both the mean and the standard deviation do not practically differ. It means that the accuracy of unified archival 171 astrogeodetic deflections of the vertical can be considered equal to the accuracy of unified archival 370 astrogravimetric deflections of the vertical. For 23 newly surveyed astrogeodetic deflections of the vertical the mean is much larger than for the archival data but the standard deviation – slightly larger. That divergence results probably from small number of data analysed (23 only), and thus the statistics obtained cannot be considered representative.

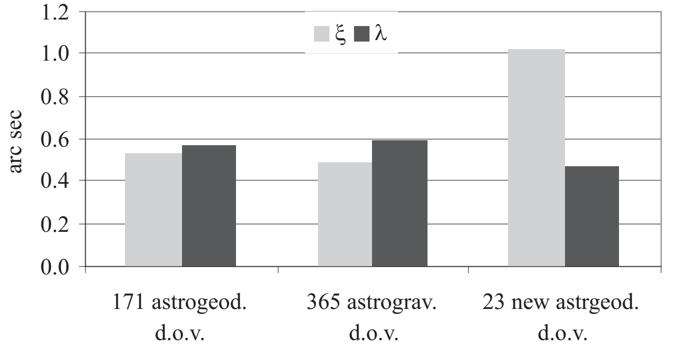

Fig. 2. The standard deviation of the differences (eq. (14) for three sets of deflections of the vertical (d.o.v.)

It can thus be concluded that the accuracy of the components of deflections of the vertical investigated is uniform and represented by standard deviation equal to 0.5".

Quality analysis performed and estimation of accuracy of available sets of deflections of the vertical enables the development of new, more reliable models of the astrogeodetic and astrogravimetric geoid.

5. Conclusion

The estimates of accuracy of the components of the archival deflections of the vertical substantially differed from 0.2" to 0.7" due to applying different estimation methods. The new estimate of accuracy of the components of the deflections of the vertical based on comparison of the existing astrogeodetic and astrogeodetic deflections of the vertical with the respective ones computed from gravity data shows that both sets of data have the same accuracy express by standard deviation 0.5''. Estimated accuracy of the 23 newly surveyed astrogeodetic deflections of the vertical is slightly different, probably due to small number of data analysed,and therefore calculated statistics are not representative. Therefore in this situation reasonable is assume, for further calculation, that its accuracy is 0.5''.

References

[1] Arnold K.: *Astrogravimetric levelling*. [in:] *International Dictionary of Geophysics*. Pergamon Press, Oxford 1967, pp. 84–87.

[2] Bokun J.: *Analysis and conclusions resulting from the utilization of gravimetric data in geodetic measurements in Poland*. Proceedings of the Institute of Geodesy and Cartography, t. VIII, z. 1 (17), 1961 (in Polish).

[3] Bokun J.: *Geoid determination in Poland on the base of astrogravimetric and gravity data*. Proceedings of the Institute of Geodesy and Cartography, t. VIII, z. 1 (17), 1961 (in Polish).

[4] Ecker E.: A *refined solution of the Austrian geoid using modified astronomical and gravimetric levelling*. Paper presented at the XIX General Assembly of IAG, Vancouver (Canada) 1987.

[5] Fisher I., Slutsky M., Campbell J.: *New pieces in the Picture puzzle of an astrogeodetic geoid map of the Word*. Bull. Géod, No. 88, 1968.

[6] Kamela Cz.: *Report of experts on the improvement of horizontal control network in Poland*. Główny Urząd Geodezji i Kartografii, Warszawa 1975 (in Polish).

[7] *Catalogue of deflections of the vertical in Poland*. Institute of Geodesy and Cartography, Warsaw 1981 (in Polish).

[8] Kryński J.: *Precise quasigeoid modelling in Poland – results and accuracy estimation*. Institute of Geodesy and Cartography, Monographic Series No. 13, 2007 (in Polish).

[9] Lachapelle G.: *Determination of the geoid using heterogeneous data*. Mitteilungen der geodätischen Institute der Technischen Univesität Graz, Folge 19, 1975.

[10] Levallois J., Monge H.: *Le géoide astrogéodésique Européen: version 1978*. Proceedings of the International Symposium on the geoid in Europe and Mediterranean area, Ancona – Numana 25–29.09.1978.

[11] Łyszkowicz A.: *Gravimetric vertical deflections for the area of Poland*. Artificial Satellites, Journal of Planetary Geodesy, Vol. 38, No. 4, 2003, pp. 107–118.

[12] Marti U.: *Geoid der Schweiz*. Geod. geophysik. Arbeiten in der Schweiz 56, Schweiz. Geod. Komm., Zürich 1997.

[13] Moritz H.: *Advanced Physical Geodesy*. Herbert Wichmann Verlag, Karlsruhe 1980.

[14] Rogowski J.B., Kłęk M.: *The unification of astrogeodetic data and inserting them into the database*. Warsaw University of Technology (report for the Institute of Geodesy and Cartography), Warsaw 2003 (in Polish).

[15] Rogowski J.B., Barlik M., Kujawa L., Kurka W., Kłęk M.: *Qualitative and quantitative analysis of the existing deflections of the vertical*. Warsaw University of Technology (report for the Institute of Geodesy and Cartography), Warsaw 2003 (in Polish).

[16] Rogowski J.B., Barlik M., Kujawa L., Kurka W., Kłęk M.: *The study of the method of unification of existing astrogeodetic data (points position, height, reference ellipsoid, reductions, systematic error of star position, time scales, polar motion)*. Warsaw University of Technology (report for the Institute of Geodesy and Cartography), Warsaw 2003 (in Polish).

[17] Rogowski J.B., Hefty J., Bogusz J., Moskwiński M., Kłęk M.: *The realization of supplementary astronomical measurements, their study and introduction to the base of the data*. Warsaw University of Technology (report for the Institute of Geodesy and Cartography), Warsaw 2004 (in Polish).

[18] Rogowski J.B., Bogusz J., Kujawa L., Kłęk M., Barlik M.: *New astro-geodetic geoid in Poland*. II Workshop "Summary of the project on a cm geoid in Poland", Warsaw 16–17.11.2005 (CD).

[19] Torge W.: *Geodesy*. 3rd edition, de Gruyter, Berlin – New York 2001.