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Repetitive Measurements of the Strain State in the Rock Mass Persistently Disturbed by the Mining Exploitation – Focus on the Triangular Rosette****

1. Introduction

Underground mining exploitation causes many negative effects on the surface of the mining area. Among the biggest nuisances for the buildings, one should count the arising of squeezing and stretching stress, and the description of the arising deformations is an important element of the safety assessment of the objects on the surface. The measurements and calculations, carried out on measurement rosettes, allow defining strains occurring in the ground in any direction. The article presents the results of the measurements with a triangular measurement rosette situated over the exploited deposit during the whole process of the wall exploitation. The theoretical description to define the surface tensor of strains was also given.

2. The Description of the Study Area

As the study area we chose the area located over wall 108 in seam 501/1, exploited by the mine "Centrum" in 2003–2004. This region is in the Odrzańska street in Bytom. This was the exploitation of hard coal and its parameters were:

- mean depth of the deposit: 615 m;
- mean thickness of the exploited deposit: 2.70 m;
- parameters of S. Knothe's theory: $a = 0.80$, $\mathrm{tg}\beta = 2.1$, $c = 5.0$ year^{-1}.

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Additionally it should be mentioned that the rock mass in this area was frequently disturbed in the past by mining exploitations (about 25 earlier exploitations caused the subsidence of the area exceeding 23 m).

A triangular measurement rosette (type "delta"), presented in figure 1, was stabilized in a central part of the wall over the south-east edge of the exploitation.

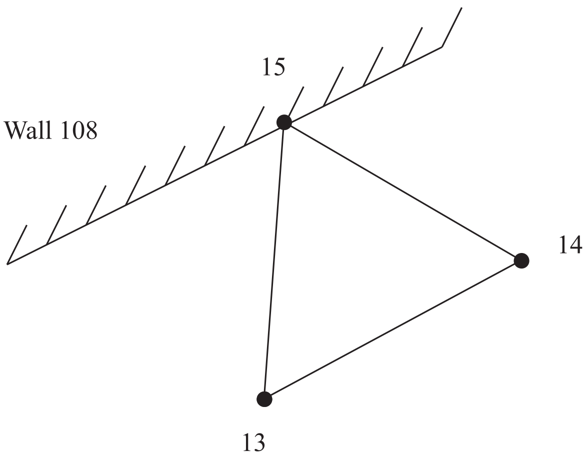


Fig. 1. A triangular measurement rosette with the numbers of the points making it

The lengths of the sides of the rosette were ca. 20 m while the inner angles were about 60°. The rosette was marked by stabilizing three earth measurement points, number 13, 14, 15 (stabilization to the depth of ca. 1.5 m from the surface). Side 13–14 of the rosette was approximately parallel to the axis of the exploited wall.

Before starting the exploitation of seam 108 the first series of measurements was made, during the exploitation there were subsequent 19 measurements series, and just after ending the exploitation additional 2 series were made. The measurements were made in time intervals adjusted to changeable intensity of the influences (during the time of passing the exploitation front under the rosette the measurements were made in a week intervals). The last 23rd series was made over 5 years after finishing the exploitation in the seam. The scheme of situating the edge of the exploitation front in the moments of measurements is presented in figure 2.

Every measurement series included 8 times measurement of all the lengths of sides in the rosette (side 13–14, side 14–15, side 15–13) and the measurement of every inner angle in two series. Every time the measurements were made with the high class measurement equipment ensuring accuracy of defining the length of side $m_l \leq \pm 1$ mm and accuracy of defining the horizontal direction $m_k \leq \pm 2^{cc}$. Based on equalized angle-linear observation the co-ordinates of each point marking the rosette were defined and the length of sides and inner angles of the rosette were calculated. These values were put together and made base for further calculations.

As a result of equalization the accuracy of determining the angle in the rosette was $m_\alpha \leq \pm 10^{cc}$ and the accuracy of calculating the lengths of the side of the rosette was $m_l \leq \pm 0.6$ mm.

Fig. 2. The scheme of situating the edge of the exploitation front in the moments of measurements

3. The Results of the Measurements

Based on the length of sides determined in every measurement series temporary horizontal strains (between series) and total strains (compared to the initial measurement series) in the discussed rosette were determined.

The obtained for side 13–14 results are presented in figure 3. In the presented graphs the last (twenty third) measurement series was not shown, due to the limitation of the legibility of graphs caused by a large time difference between 22nd and 23rd measurement series (about 5 years).

Marked based on the last (23rd measurement series) value total horizontal strain equalled:

- for side 13–14 $\varepsilon_{I-XXIII} = 0.15$ mm/m;
- for side 14–15 $\varepsilon_{I-XXIII} = 0.35$ mm/m;
- for side 15–13 $\varepsilon_{I-XXIII} = 0.09$ mm/m.

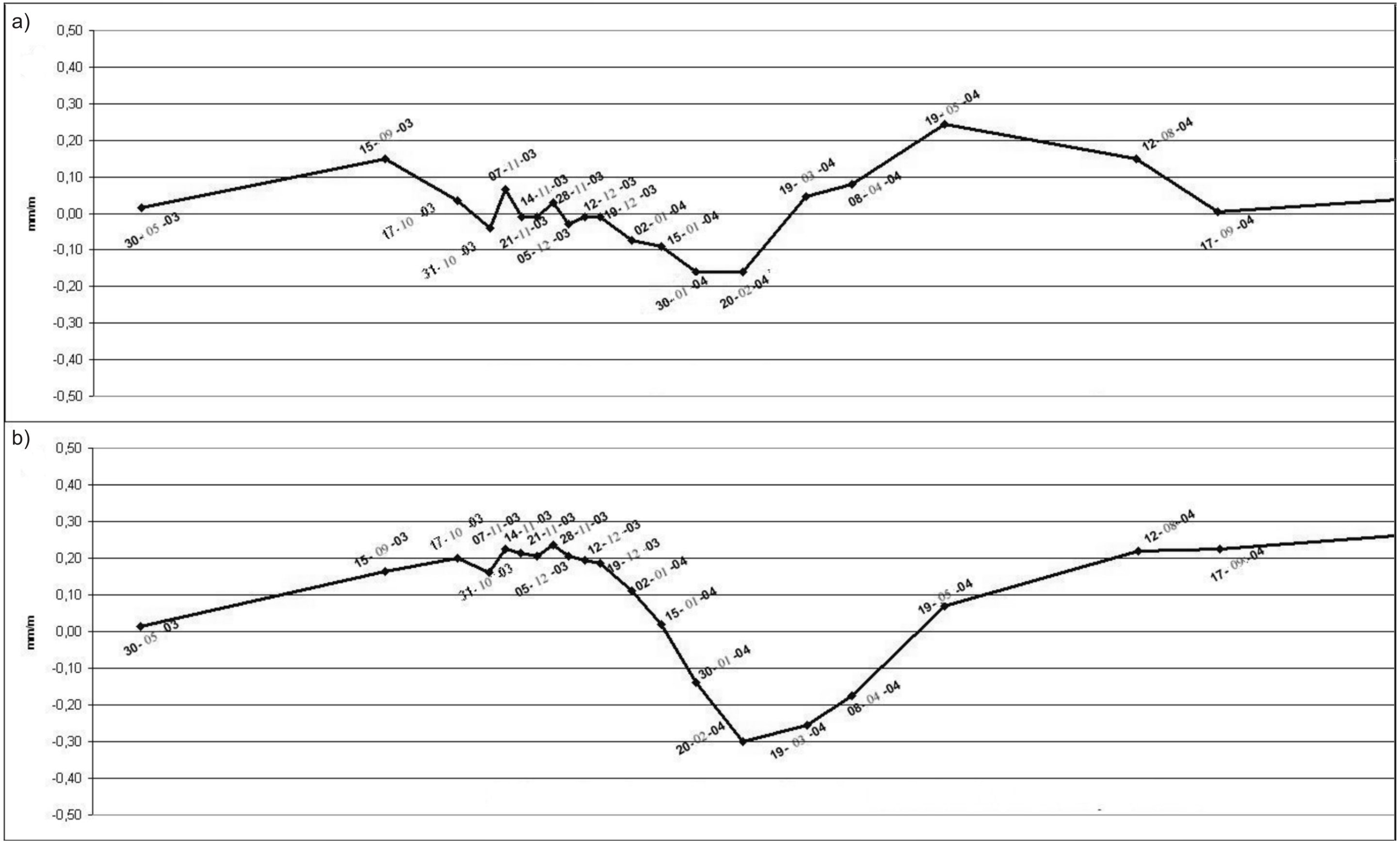


Fig. 3. The graph value of linear horizontal strains of side 13–14 of the "delta" rosette in the function of time: a) temporary horizontal strains; b) total horizontal strains

Additionally also the values of temporary horizontal strain (between 22nd and 23rd measurement series) equalled:
- for side 13–14 $\varepsilon_{XXII–XXIII}$ = –0.12 mm/m;
- for side 14–15 $\varepsilon_{XXII–XXIII}$ = 0.10 mm/m;
- for side 15–13 $\varepsilon_{XXII-XXIII}$ = 0.25 mm/m.

The above values indicate the occurrence of additional influence of the completed exploitation after making second to last (twenty second) measurement series.

Changes of temporary and total strains for side 13–14 of the studied rosette were presented in figure 3. This side was selected, because it was perpendicular to the direction of the exploitation front of wall 108, and lied in the half of the height of the wall, so it should show strains occurring towards the axis following the progress of exploitation. Of course these strains will not be maximal, because this side is not situated in the axis wall, but on the side of the wall.

4. Theoretical Relationships for Surface Tensor of Strains

The measurements to mark the value of horizontal strains arising under the influence of underground mining exploitation from the beginning have been carried out by geodetic methods. Up till now they are based on the measurement of relative change of the length of the section, between two permanently stabilized points in the field, and the determined value is attributed to the centre of the section. Despite the fact that geodetic methods do not give practically the possibility of determining the value of horizontal strains in the point, due to the lack of other methods, they are widely applied while determining horizontal strains in mining areas.

The state of strain is defined in the point, if for any direction coming out from this point, a proper measure of strain is defined and expressed in the strain in a numeral way.

The measure of linear strain in the sense of Cauchy's e^c is, for a finite section its relative change of length:

$$e^c = \varepsilon = \lambda - 1 = \frac{l - l_0}{l_0} \tag{1}$$

where l_0 is the length of section in a non-deformed, and l length of this section after deformation. the value known as λ called extension (stretch) is defined as:

$$\lambda = \frac{l}{l_0} \tag{2}$$

To define the measure of the linear strain any function of variable *l* can be applied, provided that the following conditions are fulfilled:
- it take the value of 0 for $\lambda = 1$,
- it can be reduced to the Cauchy's strain, when λ is close to 1,
- it is a dimensionless quantity.

By determining the value of a linear strain in different directions, it is possible to describe the state of the strain in the point, by attributing to this point a strain tensor. The measurement of strains in different directions is carried out on the sides of so-called geodetic rosettes.

The shapes of the rosettes used in the geodetic measurements are taken from the tensometric rosettes of a small size. They have been applied for many years, usually put on the walls of the buildings to examine the state of their strains. A feature distinguishing the examined rosette from other measurement rosettes is the way of the situation of their arms (Fig. 4.).

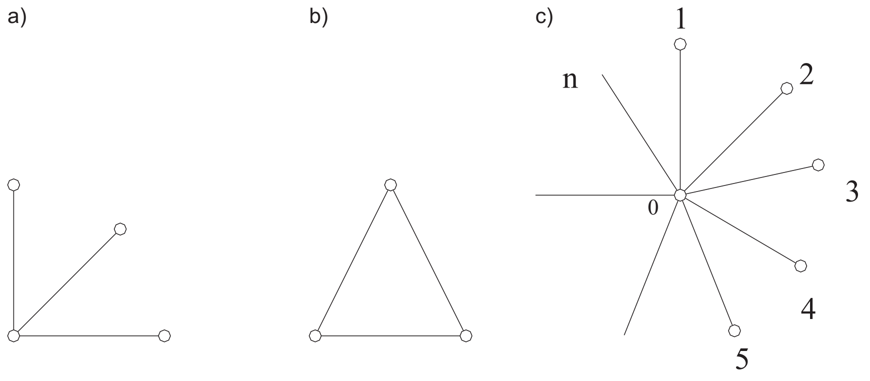


Fig. 4. The shapes of the measurement rosettes:
a) rectangular rosette; b) triangular rosette "delta"; c) central rosette "star"

The determination of the co-ordinates of the tensor of strains for the 2D space, using geodetic measurements is based on the assumption that the determined values of the components of the tensor are the mean values referring to the area of the figure such as a geodetic rosette and they are attributed to its geometric centre. Such an assumption should be made due to the large (about 20 m) lengths of the sides in the rosette, while, according to the physical definition of the value of the strain, they should be marked on the infinitely small sections. In the geodetic measurements, it is impossible to take infinitely short bases, thus in practice such bases are defined that their lengths correspond to such, for which in a given region (Upper Silesia, Legnica-Głogów Copper Area) the parameters of the theory of influences (Knothe–Budryk theory) were determined and are obtained by the measurements of the values of the deformation indexes. The determined this way value of the strain, also defined as the relative change of the length, is attributed as the mean value of strain on the full length of a given section (for the point making its centre).

The tensor for the 2D case can be described as a matrix as [1]:

$$T_{\varepsilon} = \begin{Vmatrix} \varepsilon_{11} & \varepsilon_{12} \\ \varepsilon_{21} & \varepsilon_{22} \end{Vmatrix} \tag{3}$$

where co-ordinates of the tensor for the 2D state of deformation are defined by the following relationships:

$$\varepsilon_{11} = \frac{\partial u_x}{\partial x}$$

$$\varepsilon_{22} = \frac{\partial u_y}{\partial y} \tag{4}$$

$$\varepsilon_{12} = \varepsilon_{21} = \frac{1}{2}\left(\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y}\right)$$

Based on the measured lengths the horizontal strains ε_i are marked for each side of the rosette. To determine the co-ordinates of the tensor it is necessary to additionally know the azimuths of the sides of the rosette φ_i in the accepted system of co-ordinates. For each side of the rosette the equation is formed:

$$\varepsilon_i = \varepsilon_{11} \cos^2 \varphi_i + 2\varepsilon_{12} \cos\varphi_i \sin\varphi_i + \varepsilon_{22} \sin^2 \varphi_i \tag{5}$$

this is to (subsequently) determine, by solving the system of n equations of three variables of the co-ordinates of the tensor: $\varepsilon_{11}, \varepsilon_{22}, \varepsilon_{12}$.

For the rectangular rosette and delta rosette (Fig. 4) – determination of the co- -ordinates of the tensor of strains is based on three equations, whale for the central rosette from the number of arms (*n*) this rosette has, and in this case it is possible to solve such a system of equations with the equalization of the determined variables. It should be, however, made sure that the arms of the central rosette are not lying on the complanary directions.

In the point there is a pair of mutually perpendicular directions called main directions, which during the deformation are not affected by form deformation, that means that the angle between them remains unchanged. In main directions linear strain reaches extreme values ε_{max} and ε_{min} called main strains.

The value of the strains in main directions is determined from the relation:

$$\varepsilon_{max} = \frac{\varepsilon_{11} + \varepsilon_{22}}{2} + \frac{1}{2}\sqrt{\left(\varepsilon_{11} - \varepsilon_{22}\right)^2 + 4\varepsilon_{12}^2} \tag{6}$$

$$\varepsilon_{min} = \frac{\varepsilon_{11} + \varepsilon_{22}}{2} - \frac{1}{2}\sqrt{\left(\varepsilon_{11} - \varepsilon_{22}\right)^2 + 4\varepsilon_{12}^2} \tag{7}$$

where as ε_{max} the main strain the strain of the greater relative value (except of the sign) is accepted.

The extreme form strains are found for the orthogonal sections, which are bisections of the main directions and are calculated according to the formula:

$$\gamma_{ekstr} = \varepsilon_{max} - \varepsilon_{min} = \pm[(\varepsilon_{11} - \varepsilon_{22})^2 + 4\varepsilon_{12}^2]^{\frac{1}{2}} \tag{8}$$

To orient the occurring deformations in the accepted system of co-ordinates the angle β is defined between the main direction of the strain ε_{max}, and the axis *x* of the system from the formula:

$$\chi = \frac{1}{2}\operatorname{arctg}\frac{2\varepsilon_{12}}{\varepsilon_{11} - \varepsilon_{22}}.$$

5. The Analysis of Strains Calculated for the Surface Tensor

Presented here values of the equalized lengths of the sides in the rosette (coming from 23 measurement series) were subdued to the analysis that included:
- marking the co-ordinates of the tensor of the strain in each observation series, putting them into a table and presenting in a form of graphs;
- the interpretation of the changes of the state of the strain for the rosette in subsequent observation series, regarding the progress of the exploitation front and the situation of the rosette towards the exploit field.

Using the calculation program DEF, the components of the tensor of the strain of all the observation series were found. The calculated for the discussed rosette, in a given measurement series, co-ordinates of the tensor of strains with the errors, were used in the further determination of the value of extreme strains ε_{max}, ε_{min} and the value of the form strain γ and the direction of the occurrence of maximal strains β (azimuth in the respective system of reference) together with the errors of these values. The results of the calculations were put in the form of table (Tabs 1 and 2) presenting total values (towards the first measurement series) and temporary values (between subsequent series).

Table 1. Temporary values of the strain tensor components, the value of the form strain and extreme strains together with the direction of the occurrence for the triangular rosette stabilized at the Odrzańska street in Bytom

Data	ε_{11}	$\varepsilon_{12} = \varepsilon_{21}$	ε_{22}	ε_{max}	ε_{min}	γ	β
30.05.2003	0.01	0.01	0.00	0.02	–0.01	0.02	38.90
15.09.2003	–0.36	0.18	0.28	0.33	–0.41	0.74	83.56
17.10.2003	–0.22	0.03	0.23	0.23	–0.22	0.45	96.49
31.10.2003	–0.11	–0.05	0.12	0.13	–0.12	0.25	113.18
7.11.2003	0.05	0.07	–0.04	0.08	–0.08	0.16	30.36
14.11.2003	–0.10	–0.01	0.09	0.09	–0.10	0.19	102.81
21.11.2003	–0.10	–0.03	0.14	0.14	–0.11	0.25	107.98
28.11.2003	–0.05	0.04	0.03	0.05	–0.06	0.11	76.67
5.12.2003	0.00	–0.03	0.00	0.03	–0.03	0.06	146.25
12.12.2003	–0.01	–0.01	0.00	0.00	–0.01	0.01	139.00
19.12.2003	–0.07	0.00	0.04	0.04	–0.07	0.10	98.04
02.01.2004	–0.07	–0.06	0.03	0.05	–0.10	0.15	126.61
15.01.2004	–0.05	–0.06	–0.02	0.02	–0.09	0.12	142.86
30.01.2004	–0.01	–0.11	–0.09	0.07	–0.17	0.23	160.79
20.02.2004	0.01	–0.12	–0.09	0.09	–0.17	0.26	163.48
19.03.2004	0.21	0.02	–0.15	0.21	–0.15	0.36	3.79
08.04.2004	0.16	0.07	–0.12	0.17	–0.14	0.31	14.29
19.05.2004	0.33	0.14	–0.10	0.37	–0.14	0.51	18.07
12.08.2004	0.20	0.07	–0.02	0.22	–0.04	0.26	16.70
17.09.2004	–0.07	0.02	0.04	0.05	–0.07	0.12	90.44
14.01.2005	0.11	0.01	–0.02	0.11	–0.02	0.13	5.16
10.09.2009	0.11	–0.19	0.02	0.26	–0.13	0.39	156.94

Table 2. Total values of the strain tensor components, value the value of the form strain and extreme strains together with the direction of the occurrence for the triangular rosette stabilized at the Odrzańska street in Bytom

Data	ε_{11}	$\varepsilon_{12} = \varepsilon_{21}$	ε_{22}	ε_{max}	ε_{min}	γ	β
30.05.2003	0.01	0.01	0.00	0.02	–0.01	0.02	38.90
15.09.2003	–0.36	0.19	0.28	0.33	–0.41	0.74	82.51
17.10.2003	–0.58	0.22	0.51	0.55	–0.62	1.17	87.79
31.10.2003	–0.69	0.17	0.63	0.65	–0.71	1.36	92.04
7.11.2003	–0.64	0.23	0.58	0.63	–0.68	1.31	88.42
14.11.2003	–0.74	0.22	0.68	0.71	–0.78	1.49	90.23
21.11.2003	–0.84	0.19	0.81	0.84	–0.87	1.70	92.67
28.11.2003	–0.89	0.23	0.85	0.88	–0.92	1.79	91.77
5.12.2003	–0.89	0.20	0.85	0.87	–0.91	1.79	92.84
12.12.2003	–0.90	0.19	0.85	0.87	–0.92	1.79	93.06
19.12.2003	–0.96	0.20	0.88	0.90	–0.99	1.89	93.33
02.01.2004	–1.04	0.14	0.91	0.92	–1.05	1.97	95.41
15.01.2004	–1.08	0.09	0.89	0.89	–1.09	1.98	97.26
30.01.2004	–1.09	–0.02	0.80	0.80	–1.09	1.90	100.82
20.02.2004	–1.08	–0.14	0.71	0.72	–1.09	1.81	105.07
19.03.2004	–0.87	–0.12	0.56	0.57	–0.88	1.45	105.39
08.04.2004	–0.71	–0.05	0.44	0.44	–0.72	1.15	103.02
19.05.2004	–0.38	0.08	0.34	0.35	–0.39	0.74	92.98
12.08.2004	–0.18	0.15	0.31	0.35	–0.22	0.58	83.10
17.09.2004	–0.25	0.16	0.36	0.40	–0.30	0.69	84.34
14.01.2005	–0.14	0.17	0.34	0.40	–0.20	0.60	80.19
10.09.2009	–0.04	–0.02	0.36	0.37	–0.04	0.40	102.43

For each side of the rosette graph of total strains referring to the first measurement series in a given region was made and its course in time was presented in figure 5. In the same graph the values extreme strains determined from the components of the tensor based on formulas (6) and (7), for respective periods of time, were also presented.

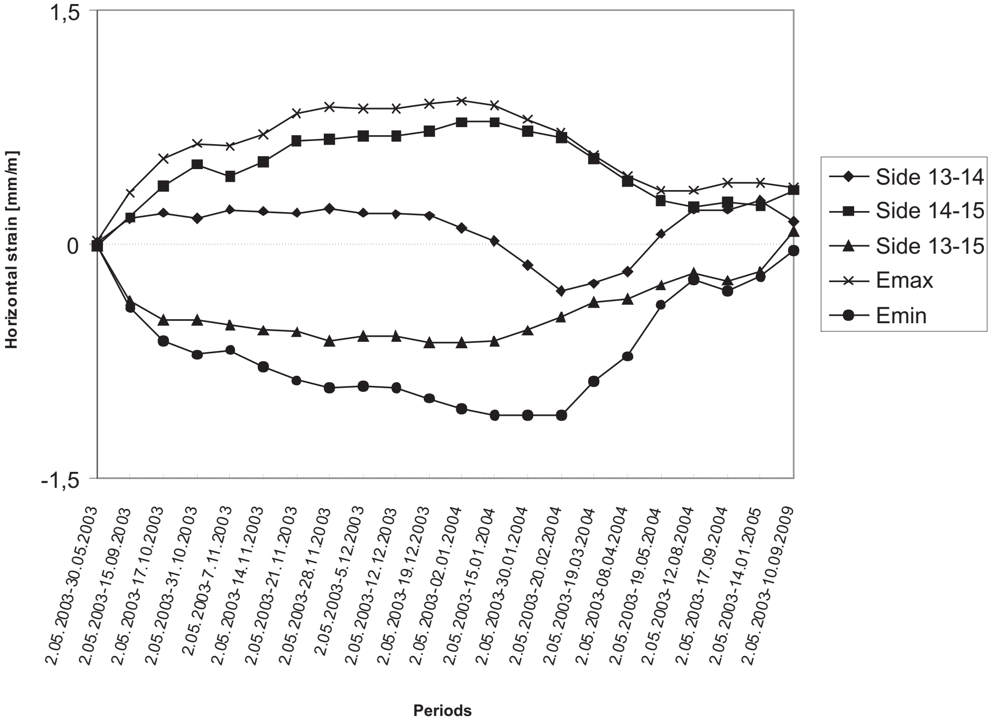
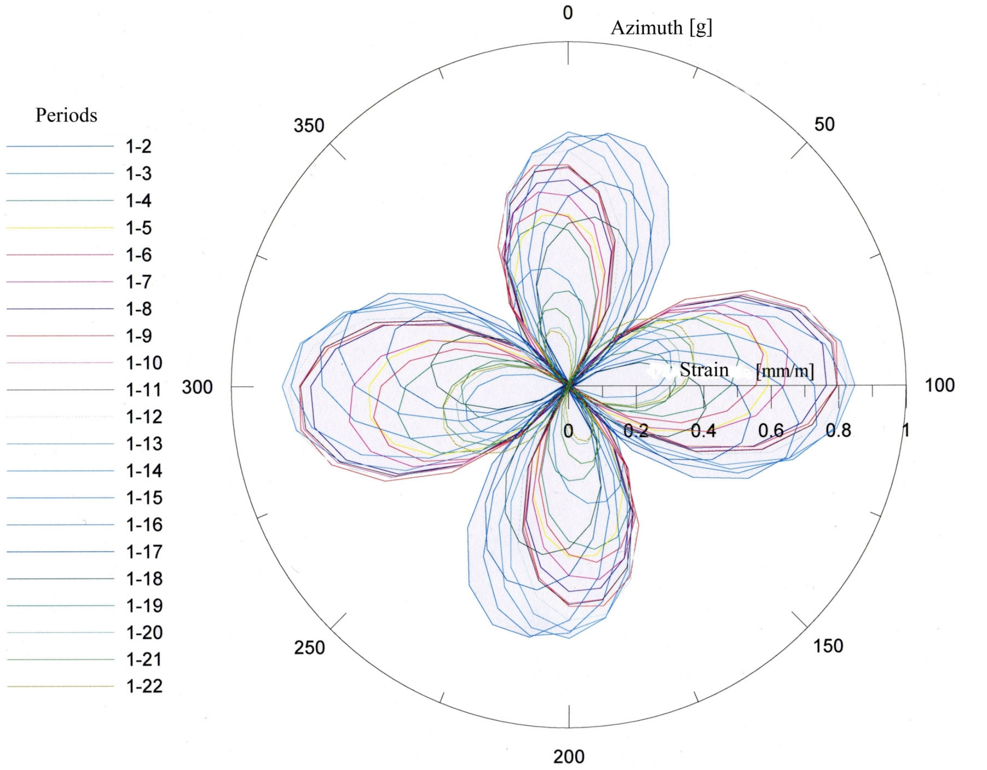


Fig. 5. Presentation of the changes of strains during extreme values determined by the measurement and calculated for each side of the rosette

Based on the calculated components of the tensor circular graphs of direction distribution of the strain in each measurement series were constructed (Fig. 6). The distance from the beginning of the axis of the strain to the selected point of the graph reflects the value of the strain, and the direction in the accepted system complies with the direction of the occurrence of this strain in the area. Such a presentation of the state of strains allows visualisation of the values of the occurring strains, directions of the occurrence of extreme strains in a given rosette and their possible change in time.



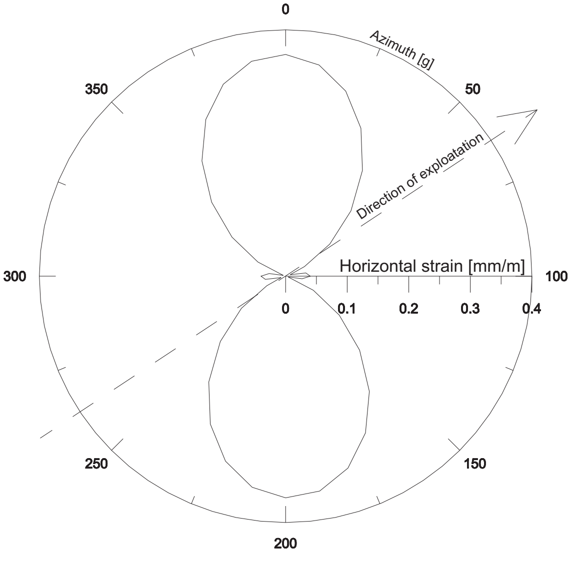


Fig. 6. The distribution of strains in the triangular rosette situated at the permanent edge of the exploitation field in Bytom

6. The Interpretation of the Results

In the case of the examined rosette, situated outside the side (permanent) edge of the exploited wall 108 in seam 501/1, according to the presented in the previous chapter figure 6, one can observe two-direction state of strains, meaning the simultaneous occurrence of stretching strains and compressing strains in the mutually perpendicular directions. Compressing strains occur in the directions close to the direction of the progress of the exploitation front and grow when the exploitation front is approaching, achieving its maximal value about 6 months after the front goes near the rosette, and then systematically diminish. Similar changes are in stretching strains, detected in the direction perpendicular to the stress, which, for the outer profile of the formed subsidence trough (situated outside the edge of the exploitation edge) shows a full compliance with the theory.

The presented in figure 3 graph values of total horizontal strains in the function of time (for side 13–14) is compliant to the theoretical model [3]. Qualitatively it also refers to the other sides of the rosette.

The states of the strains is determined by measurements in geodetic rosettes, after their graphical processing, give a very clear image of the occurring deformations and should be applied to assess the degree of threat for the objects particularly sensitive to deformations causes, among others by the underground mining exploitation. The results of the measurements of geodetic rosettes give more complete analysis of the course of deformation, because they allow the determination of the direction of the occurrence of extreme strains, with the form strain, which cannot be defined by the results of the measuring the observation line.

References

[1] Pielok J.: *Badania deformacji powierzchni terenu i górotworu wywołanych eksploatacją górniczą.* Uczelniane Wydawnictwa Naukowo-Dydaktyczne AGH, Kraków 2002.

[2] Pielok J.: *Wyznaczanie powierzchniowego tensora odkształceń na terenach górniczych w oparciu o pomiary geodezyjne*. Uczelniane Wydawnictwa Naukowo-Dydaktyczne AGH, Kraków 2005.

[3] Popiołek E.: *Ochrona terenów górniczych.* Wydawnictwa AGH, Kraków 2009.

[4] Szafarczyk A.: *Wyznaczanie odkształceń powierzchni terenu górniczego przy zastosowaniu rozet geodezyjnych*. Kraków 2008 (Ph.D. Thesis, unpubl.).