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Study on GPS Phase Ambiguity Resolution Effectiveness with use of LAMBDA De-Correlation Method**

1. Introduction

Actually used methods of GPS signals handling allow calculation of geodetic coordinates with high accuracy and reliability. Moreover, a tendency of satellite methods improvement via application of algorithms used for short observations session, is observed. Advanced procedures of phase cycle initial ambiguities calculation are used in algorithms of this type. Calculation of ambiguities in form of integers is a key for high accuracy calculation of point coordinates [4, 7]. In case of short measurement sessions, calculation of ambiguities is difficult, because of the ill-condition of observation equations [1]. Moreover, increase of distance between points results in more and more inconvenient influence of differential ionospheric and tropospheric refraction. In consequence considerable errors in calculation of unknowns in "float"-type solutions are generated, what considerably complicates process of integer-valued ambiguities calculation [5]. Application of the LAMBDA method improves the mentioned process effectiveness via ambiguity search space reduction [10]. In the next part of the present study, in order to use shorter description, the term ambiguity understood as phase cycle initial ambiguity, will be used.

2. Theoretical Base

System of observation equations for a case of GPS phase measurements in matrix description is given in the following form [11]:

$$\mathbf{L} = \mathbf{AX} + \mathbf{Bn} + \mathbf{v} \tag{1}$$

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In equation (1) **A** and **B** are observation plan matrices for both resolution point coordinates and ambiguity. Observation vector is marked as **L**, whereas vector of random deviations is marked as **v**. Vectors of calculated point coordinates and ambiguities are marked as **x** and **n** respectively.

Vector of calculated unknowns $\hat{\mathbf{x}}$ and $\hat{\mathbf{n}}$, together with variance-covariance matrix $\hat{\mathbf{Q}}$, is obtained in result of solution of the system of equations (1) using least squares method [10]

$$\begin{pmatrix} \hat{\mathbf{x}} \\ \hat{\mathbf{n}} \end{pmatrix}, \begin{pmatrix} \mathbf{Q}_{\hat{x}} & \mathbf{Q}_{\hat{x}\hat{n}} \\ \mathbf{Q}_{\hat{n}\hat{x}} & \mathbf{Q}_{\hat{n}} \end{pmatrix} \tag{2}$$

It is so called "float"-type solution, in which the ambiguities are expressed in form of real numbers. In the next algorithm step, the "float" ambiguities should be rounded off into integers. The commonly used method used for calculation of integer-valued ambiguities is called as "search method". In this method, ambiguities obtained from "float" solution, together with suitable variance-covariance matrix, are used as input data. On this base a set of potential integer-valued ambiguities is developed (so called search space). Then the optimal ambiguity is searched, testing all possible combinations of integers belonging to this space [4]. Identification criterion for optimal set of phase cycles integer-valued ambiguities is formed on the basis of following relation [10]

$$R_i = (\hat{\mathbf{n}} - \mathbf{n}_\mathrm{i})^T \mathbf{Q}_{\hat{n}}^{-1} (\hat{\mathbf{n}} - \mathbf{n}_\mathrm{i}) \tag{3}$$

In equation (3) vector $\mathbf{n}_\mathrm{i}$ contains "*i*" set of integer-valued ambiguities, being a subset of the search space. Whereas R_i is a value of equation (3) calculated for vector $\mathbf{n}_\mathrm{i}$. It was assumed that vector $\mathbf{n}_\mathrm{i}$, which as substituted into equation (3), gives the smallest value of the parameter R_i, is considered as potentially optimal phase cycle integer-valued ambiguities set. This set is considered as optimal if the following relation is satisfied

$$\frac{R_2}{R_1} > \alpha \tag{4}$$

In the inequality (4), the lowest value was marked as R_1, whereas the successive value in the assemblage R_i was marked as R_2. In the literature, quotient *R2*/*R1* is known as Integer Search Ratio (ISR), whereas α is an empirically determined constant [11]. Value α = 1.8 was assumed in numerical examples presented in the present study. Number of potential sets of integer-valued ambiguities is generated with use of standard deviation σ_i values obtained for "float" type ambiguities.

Thus this number depends on values of variance-covariance matrix diagonal elements. In case of short measuring sessions, the diagonal elements of variance-covariance matrix have usually high values. In consequence, number of ambiguity sets, which should be tested, considerably increases. This leads to considerable acceleration of the calculation process. The LAMBDA method is an effective tool allowing reduction of the search space. This method is based on ambiguity de-correlation via application of so called *Z*-transformation [10], what is used for transformation of original ambiguities $\hat{\mathbf{n}}$, together with corresponding variance-covariance matrix $\mathbf{Q}_{\hat{\mathbf{n}}}$. Ambiguity vector $\hat{\mathbf{z}}$ and variance-covariance matrix $\mathbf{Q}_{\hat{\mathbf{z}}}$, in which values of diagonal elements are considerably lower than original ones, is obtained in result of this transformation [3]:

$$\hat{\mathbf{z}} = \mathbf{Z} \cdot \hat{\mathbf{n}} \tag{5}$$

$$\mathbf{Q}_{\hat{\mathbf{z}}} = \mathbf{Z} \cdot \mathbf{Q}_{\hat{\mathbf{n}}} \cdot \mathbf{Z}^T \tag{6}$$

In consecutive calculations original procedure of ambiguity resolution based on relation (3) is replaced by search among ambiguity set $\mathbf{z}$ in the following equation

$$R_i = (\hat{\mathbf{z}} - \mathbf{z}_\mathbf{i})^T \mathbf{Q}_{\hat{\mathbf{z}}}^{-1} (\hat{\mathbf{z}} - \mathbf{z}_\mathbf{i}) \tag{7}$$

When the search process is completed, calculated values of the integer-valued ambiguities should be transformed using reverse transformation

$$\mathbf{n} = \mathbf{Z}^{-1} \cdot \mathbf{z} \tag{8}$$

In order to obtain integer-valued ambiguities $\mathbf{n}$ in result of the transformation, individual elements of matrix $\mathbf{Z}^{-1}$ should be expressed as integers. Moreover, it is assumed that the determinant of matrix $\mathbf{Z}$ and $\mathbf{Z}^{-1}$ equals to ±1 [3]. Detailed description of the algorithm used for integer-valued ambiguities calculation, including corresponding example, is cited in [3].

3. Description of Calculation Experiments

An experiment comprising calculation of integer-valued ambiguities of phase cycles expressed in double differences using standard algorithm described in equation (1)–(4), as well as with use of the LAMBDA method, has been conducted in order to test the LAMBDA method effectiveness for short GPS measurement sessions. GPS measurements registered in 20.06.2007 in points PGOR and STAB located in Trzebinia region, were handled. Vector PGOR-STAB has length d = 3310 m, and geodetic height difference between the points Δh = 60 m. Observations of GPS signals were conducted in the time period 6:45 – 10:15 TU. The measurements were made with use of Ashtech μZ-12 receivers with ASH701008.01B antennas. GPS signals from satellites located over 10° over the testing stand horizon have been registered. Measurement time interval amounted for 15 s. Nine five-minutes sessions started in 25 minutes intervals have been separated from the observation set. Chosen sessions were suitably handled in order to calculate initial phase cycle ambiguities within the band L_1, in two calculation variants. Standard procedure was used in variant 1, whereas LAMBDA de-correlation method was used in variant 2. In both calculation variants, phase GPS signals from satellites located over 15° over testing stand horizon, have been used. Tropospheric refraction H. Hopfield, with use of mapping function Chao [6], has been used in the study. It was assumed in the calculations that atmospheric conditions were identical in both measurement points. Coordinates of GPS satellites were calculated with use of broadcast ephemeris. Search space of potential integer-valued ambiguities were generated with use of values $4\sigma_i$, where σ_i is a standard deviation obtained from "float"-type solution of ambiguity i [11]. Control calculations were conducted in order to verify the calculated ambiguities, assuming that coordinates of both GPS vector ends are known, and the only unknowns comprise the ambiguities [9]. Geodetic coordinates of pints PGOR and STAB calculated with high precision on the basis of cyclic measurements conducted in Faculty of Mining Surveying and Environmental Engineering, AGH University of Science and Technology [2] were used in control calculations. It was assumed in the next steps that received in this manner ambiguities comprise true values. Integer-valued ambiguities obtained in result of nine 5-minutes long calculation sessions were numbered, prescribing them consecutive integers. Comparison of deviations dN_i of ambiguities obtained from "float"-type $\nabla\Delta\hat{N}_i$, from their integer-valued values, where i means consecutive number of the ambiguity, is shown in figure 1.

$$dN_i = \nabla\Delta\hat{N}_i - \nabla\Delta N_i \tag{9}$$

It should be noted that for the second calculation variant, values $\nabla\Delta\hat{N}_i$ correspond to suitable ambiguities obtained from the result of the Z-transformation.

In the second variant of the ambiguity calculation, the ambiguity values after transformation were essentially closer to integer-valued values, as compared with "float"-type ambiguities from the first variant. Arithmetic mean calculated for a set of absolute values $|dN_i|$, expressed in cycles L_1, in the first calculation variant amounted for 0.72, whereas in the second variant 0.14, respectively.

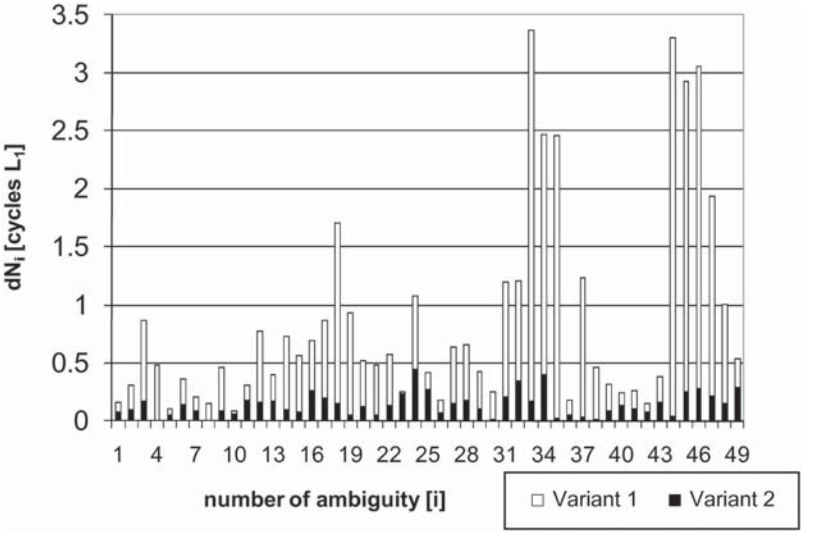

Fig. 1. Deviations dNi of ambiguities obtained in two calculation variants.

Size of search space was also compared. In the first calculation variant the size of the space ranged from 3360 to 299 520 of potential ambiguity sets. In the second calculation variant the search space was radically reduced in all calculation sessions in result of application of the LAMBDA method (Tab. 1).

Table 1. Size of the ambiguity search space

Session number	1	2	3	4	5	6	7	8	9
Variant 1	132 000	3360	153 600	299 520	38 720	29 400	32 130	110 880	156 000
Variant 2	12	18	64	600	96	161	162	72	144

Values of the ISR factor were also compared. In sessions 4–7 ambiguity sets having minimal value R_i (equation (3)) did not satisfy criterion that ISR > 1.8. Moreover, in 6 calculation sessions, ISR values in both variants were identical, whereas in three cases value of ISR factor was reduced in result of application of the LAMBDA method (Tab. 2).

Table 2. Values of the ISR factor

Session number	1	2	3	4	5	6	7	8	9
Variant 1	5.6	5.1	6.7	1.6	1.6	1.6	1.6	6.1	2.2
Variant 2	5.5	5.1	5.7	1.6	1.6	1.6	1.6	3.1	2.2

However it should be noted that in the second calculation variant, ambiguity sets *N* minimizing values of R_i, were in all calculation sessions equal to true values N_0, calculated with the assumption that coordinates of GPS vector ends are not changed. Thus the LAMDA method was successful, irrespectively to negative verification by the ISR factor. In case of standard procedure (variant 1), ambiguities *N* different than true values N_0 (Tab. 3), have been obtained in session no. 6.

Table 3. Comparison of integer ambiguities *N* obtained in session no. 6 with its true values N_0

Prn	*N*	N_0
4	20 597 588	20 597 588
13	19 027 357	19 027 357
21	17 334 233	17 334 233
25	18 377 562	18 377 563
31	3 505 363	3 505 364

Obtained results indicate that this stage of examinations should be re--analyzed in context of selection of suitable identification criterion for optimal phase cycles ambiguity sets, obtained with use of the LAMBDA method.

4. Final Conclusions

The executed analysis proved usefulness of the LAMBDA method in the process of short GPS sessions handling. Application of the method results in reduction of search space, what in turn considerably accelerates calculation of phase cycles integer-valued ambiguities. It was proved on the basis of executed calculation tests that application of standard ISR criterion can lead to rejection of correct ambiguity sets, determined with use of the LAMBDA method. The present study is a continuation of the author studies on phase GPS measurements registered on the area bounded with network of permanent stations. The former stages of the studies were focused on the problem of estimation of local differential refraction models [8], as well as on calculation of phase cycles ambiguities, with the assumption that coordinates of the permanent stations network points are not changed [9]. The calculations were made with use of RBS program developed in the Department of Geomatics of the AGH University of Science and Technology in Krakow. Source codes from Internet NGS (National Geodetic Survey) service were also used [3].

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