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Determining Density of Regular Grid for Creating DTM Using Bicubic Spline Interpolation

Abstract: The article present the accuracy of DTM reproduction created by authors program of bicubic spline interpolation on the example of different categories of relief complexity. Correlations are established between the size of regular grid, category of relief complexity and a mean-square error of height finding. The graph is drawn showing the dependence of accuracy of point height finding from DTM, the size of regular grid and category of relief complexity.

Keywords: interpolation, bicubic spline, DTM, regular grid

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1. Problem Statement

As investigated in [1], a surface constructed with the use of bicubic spline interpolation (BSI) reproduces a relief with high accuracy; this allows us to recommend such an interpolation for the more accurate reproduction of real surfaces. The initial data for calculating a bicubic polynomial that defines the surface are the values of the heights of regular grid nodes; therefore, an important question when using BSI for building a digital terrain model (DTM) is the choice of a regular grid density. Clearly, the accuracy of a terrain reproduction will increase with a greater-density grid. The calculation of BSI polynomial coefficients is a laborious process that takes a lot of time – even for modern computers. In addition, the numbers of nodes and points accordingly increase when the density of the grid increases (the heights of which must be determined). This will result in excessive expenses regarding computer time.

Therefore, there is a problem in investigating the optimal density of a regular grid that is used in automatically constructing a DTM.

2. Analysis of Latest Research and Publications

In [2], the author points to the problem of selecting a regular grid density according to the type of terrain; therefore, for constructing a DTM, it is suggested to use the data obtained from a combined method in a process of digitizing the structural lines of the basic and vertex types, and horizontals. In turn, I.G. Chervanov [3] offers a map at a scale of 1:50,000 for a plain terrain to build a relatively correct DTM using a regular grid, recording it into the computer memory as 400 dots per 1 km²; that is, a grid with 50 m sides. Many scholars have repeatedly argued that an optimal decision is creating a DTM while taking into account the characteristic elements of the relief [4–6]. Nevertheless, M.P. Kumler [7] showed that a DTM created on a square grid may be more accurate than a triangulation irregular grid (TIN).

Various authors have worked in the area of shape preservation [8, 9]. In these papers, the shape preserving interpolation were studied for positive, monotonic, and convex data using a rational cubic spline.

In [10], a family of recursive interpolation schemes based on B-spline representation and its inverse gradient weighting version was employed to enhance the accuracy of DIC analysis. Theories and simulation results were also introduced to illustrate the effectiveness of the method as compared to common bicubic interpolation.

An in-depth review of the differential geometry of curves, a wide range of exercises with selected solutions, and complete computer programs for several forms of splines and smoothing splines are presented in [11].

In [12], a technique is presented for creating a digital elevation model (DEM) from grid-based contour data. The method computes new intermediate contours in between the existing isolines. These are found by finding the shortest line segment that connects the points of two neighboring contours with differing elevations.

In [13], Podobnikar mentioned that interpolation methods are very important during the pre-processing, respecting the nature of the data source and DTM resolution. The final resolution chosen for the DTM was 20 m. This DTM was made with an average vertical precision of 3.5 m. For a DTM with a resolution of 25 m, bilinear interpolation was better [14], and for contour lines, a Linear Inverse Distance Interpolation from Contours (LIDIC) [15] method with many improvements regarding the basic method was applied.

Therefore, one of the main questions that appears in the process of DTM construction is the choice of the grid step for surface creation with required accuracy.

3. Problem Definition

Determining the sufficient size of a regular grid to achieve the required accuracy of terrain reproduction and calculation of the correlation dependencies between regular grid density and terrain accuracy in automatically constructing a DTM.

4. Exposition of Basic Material

The authors of this article used an example of a typical relief taken from [16]; namely, the standards for determining relief complexity. It should be noted that, in the above-mentioned work, a relief of the first complexity category was investigated. In this work are the results of accuracy research of terrain mapping of a higher complexity category. Three samples of terrain were investigated; namely, the second, fourth, and fifth categories of complexity. For each of these categories, a DTM was created (specified by a bicubic polynomial).

The method for determining the accuracy of the reproduction of the surface relief assigned by such a polynomial has been described in a previous work [10]. The accuracy analysis consisted of finding the height deviation values obtained using a mathematical model that was based on a linear interpolation, the original cartographic material, and the values found using the developed program.

Figure 1 shows the original samples of relief with a 100-m-length-side regular grid mapped on it.

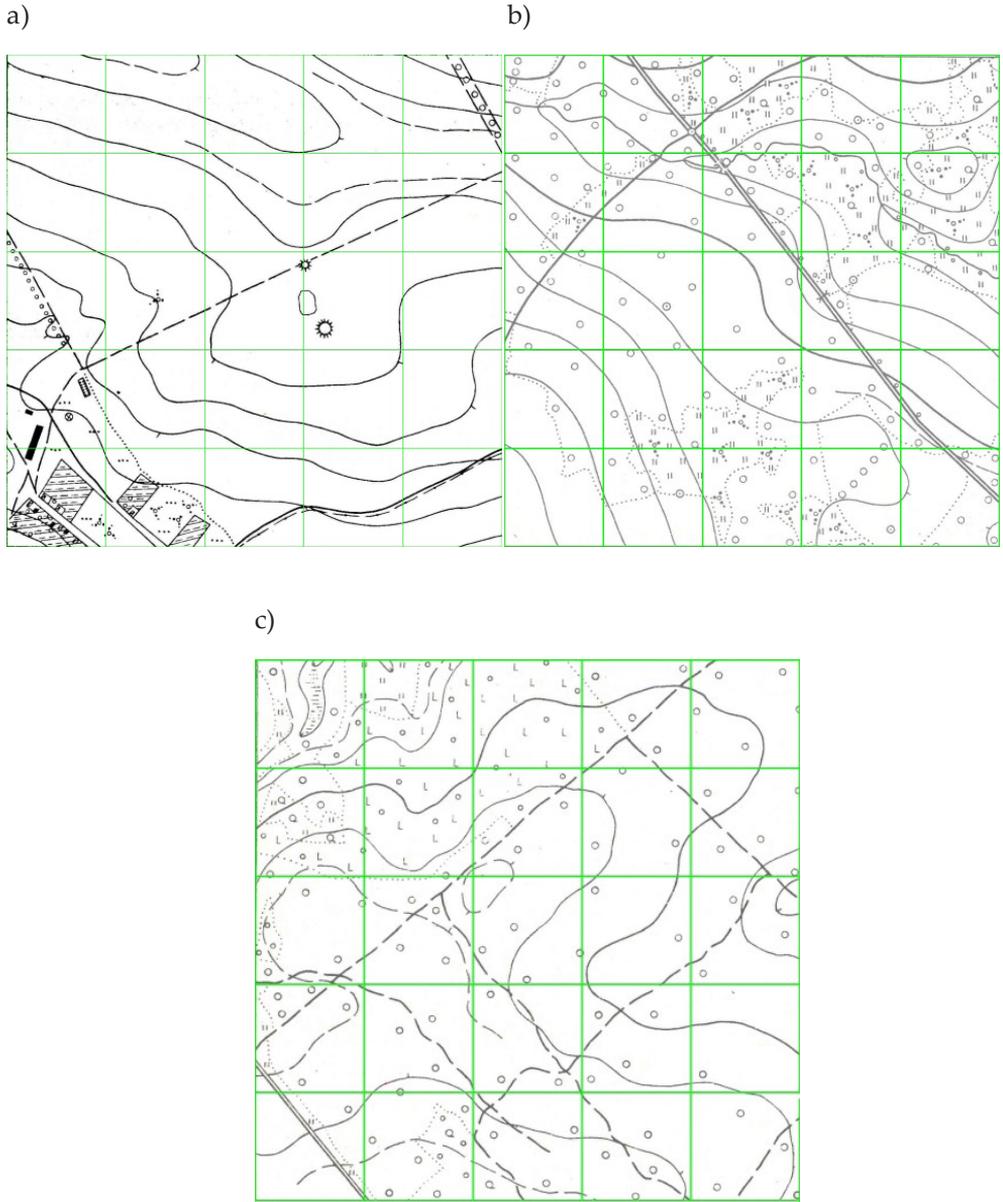


Fig. 1. Original cartographic material of second (a), fourth (b), and fifth (c) categories with 100-meter grid

Having formed the surfaces assigned by the polynomial and using the method of accuracy calculation developed in [1], the accuracy of DTM reproduction created with BSI was investigated. These results are shown in Table 1.

Taking the values obtained from a mathematical model as the ideal, a calculation of mean square errors (MSEs) using Gauss' formula was made:

$$m = \sqrt{\frac{\sum_{i=1}^n \Delta_i^2}{n}} \tag{1}$$

Table 1. Calculation of mean square error of finding heights on DTM surface assigned by bicubic polynomial at regular grid with density 100 m

No.	Second complexity category			Fourth complexity category			Fifth complexity category			
	H [m] linear	bicubic spline interpolation		H [m] linear	bicubic spline interpolation		H [m] linear	bicubic spline interpolation		
		H [m]	Δ [m]		H [m]	Δ [m]		H [m]	Δ [m]	
1	251.987	252.041	0.054	70.168	70.244	0.076	75.614	75.902	0.288	
2	253.651	254.414	0.763	67.897	67.800	-0.097	75.627	76.644	1.017	
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	
124	244.506	243.870	-0.636	81.463	80.519	-0.944	79.377	77.209	-2.168	
125	245.931	245.607	-0.324	77.109	77.162	0.053	81.791	79.799	-1.992	
MSE = 0.885 m				MSE = 1.339 m				MSE = 0.891 m		

Having analyzed the results, we can conclude that the polynomial for the DTM that was calculated using the developed BSI program with a regular grid density of 100 m created with MSEs amounted to 0.885 m for reliefs of the second category of complexity, 1.339 m for the fourth category, and 0.891 m for the fifth category.

To find the dependence between the density of a regular grid and the accuracy of the DTM construction, the thickening of a regular grid was performed, and a change in the accuracy of the relief reconstitution was calculated. When using a regular grid with 100-meter sides, 36 nodal points and 25 squares are obtained. When thickening a grid threefold to obtain a grid with sides of 33 m, we obtain 256 nodal points and 225 squares. Due to the huge amount of required work in the process of thickening a grid on the whole surface, two 2500-meter squares were used that had the greatest and lowest MSE relief reproduction for assessing the accuracy (Fig. 2).

The thickening of a regular grid with sides of 100 m that was built on an example of a relief of the second complexity category with a grid featuring 33-m sides, we calculated the MSE relief reproduction (the results are shown in Table 2). The MSEs for the second-complexity category were 0.885 m (as mentioned earlier); for our investigation, we selected 100-meter squares with a maximum value of MSE (1.683 m) and a minimum value of MSEs (0.140 m). After thickening these squares with a 33-m-side grid, the MSE_{min} equaled 0.137 m, and the MSE_{max} – 0.597 m.

MSEs, the accuracy improved from 0.140 m to 0.098 m, and for the square with the maximum MSEs – from 1.683 m to 0.752 m (when using a regular grid thickening threefold).

Table 3. Comparative table of mean square errors

Second complexity category (MSE = 0.885 m)							
MSEmin = 0.140 m							
No.	H [m]	BSI_33 [m]	Δ [m]	Δ^2 [m ²]	BSI_100 [m]	Δ [m]	Δ^2 [m ²]
106	256.210	256.0726	-0.137	0.0189	256.4269	0.217	0.0470
107	258.842	258.9937	0.152	0.0230	258.8537	0.012	0.0001
108	252.691	252.6567	-0.034	0.0012	252.7708	0.080	0.0064
109	255.583	255.5404	-0.043	0.0018	255.3845	-0.199	0.0394
110	255.945	256.0012	0.056	0.0032	256.0147	0.070	0.0049
MSE = 0.098 m at 33 m					MSE = 0.140 m at 100 m		
MSEmax = 1.683 m							
No.	H [m]	BSI_33 [m]	Δ [m]	Δ^2 [m ²]	BSI_100 [m]	Δ [m]	Δ^2 [m ²]
216	239.667	239.4990	-0.168	0.0282	239.9947	0.328	0.1074
217	242.541	242.3898	-0.151	0.0228	243.2292	0.688	0.4736
218	236.477	237.4886	1.012	1.0241	239.4128	2.936	8.6192
219	234.379	233.3012	-1.078	1.1620	235.8850	1.506	2.2681
220	238.878	239.6486	0.771	0.5944	240.5170	1.639	2.6865
MSE = 0.752 m at 33 m					MSE = 1.683 m at 100 m		

To assess the accuracy of the obtained data, the same investigations were conducted for cartographic materials from the fourth (Tab. 4) and fifth (Tab. 5) complexity categories.

Table 4. Calculation of mean square errors for fourth complexity category

Fourth complexity category (MSE = 1.3395 m at 100 m)							
MSEmin = 0.1249 m at 100 m				MSEmax = 3.5095 m at 100 m			
No.	H [m] linear	bicubic spline interpolation		No.	H [m] linear	bicubic spline interpolation	
		H [m]	Δ [m]			H [m]	Δ [m]
55	93.2605	93.3867	0.1262	241	69.9596	69.6918	-0.2678
56	92.8988	93.0440	0.1452	242	70.8093	71.1884	0.3791
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
98	93.9944	93.9746	-0.0198	284	59.0941	58.6174	-0.4767
99	94.5903	94.5917	0.0014	285	61.0275	60.6557	-0.3718
MSE = 0.067 m at 33 m				MSE = 0.367 m at 33 m			

Table 5. Calculation of mean square errors for fifth complexity category

Fifth complexity category (MSE = 0.891 m at 100 m)							
MSEmin = 0.208 m at 100 m				MSEmax = 1.866 m at 100 m			
No.	H [m] linear	bicubic spline interpolation		No.	H [m] linear	bicubic spline interpolation	
		H [m]	Δ [m]			H [m]	Δ [m]
53	81.745	81.717	-0.028	1	67.500	68.946	1.446
54	80.384	80.422	0.038	2	70.662	70.714	0.052
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
96	81.900	81.891	-0.009	44	81.767	81.744	-0.023
97	81.227	81.127	-0.099	45	81.758	81.651	-0.107
MSE = 0.144 m at 33 m				MSE = 0.268 m at 33 m			

By using a regular grid with a threefold lower side, the MSEs were lowered to the level of the BSI terrain reproduction of the first complexity category. Therefore, we can assume that, by increasing the density of regular grid - the squares of this modified grid reflect areas of simpler reliefs, which as this investigation shows are reproduced with a higher accuracy. Ostensive results of thickening are shown in Figures 3–5, where an evident decrease of MSEs with decreasing sizes of regular grids can clearly be traced.

Thus, the MSEs of relief reconstruction of the second complexity category with BSI using a 100-meter grid are equal to 0.885 m, with a 33-meter grid – 0.433 m, and with an 11-meter grid – 0.096 m (Fig. 3). Obviously, we can conclude that there is a dependence between the size of the grid and the accuracy of the surface reproduction. To find this dependence, Pearson’s correlation coefficient and the level of its significance were calculated. It is known that the level of significance is a measure of the statistical authenticity of the calculation results. Pearson’s correlation coefficient between the density of grid and the MSEs is 0.685, which indicates a significant direct dependence of these variables and a bilateral significance equal to 0, which allows us to consider that the correlation is statistically true.

Similar investigations were conducted for the fourth complexity category (Fig. 4). For this complexity category, a terrain accuracy reproduction with BSI using a regular grid with a density of 100 m was 1.399 m, using a 33-m regular grid, MSEs were 0.255 m, and for a grid with an 11-m grid, MSEs dropped to 0.088 m. For the cartographic sample of the fourth complexity category, the Pearson correlation value and significance level were also calculated. The Pearson coefficient is 0.665 with a 0-significance level, which also shows a high direct correlation between MSEs and the density of the grid.

Similarly, for the fifth complexity category, the MSEs at 100 meters were 0.891 m, at 33 m, MSEs = 0.214 m, and at 11 m – 0.038 m. The values of the maximum and minimum MSE reproduction surfaces are presented in Figure 5. The Pearson correlation coefficient was also calculated, which equaled 0.759 and a bi-directional 0 significance.

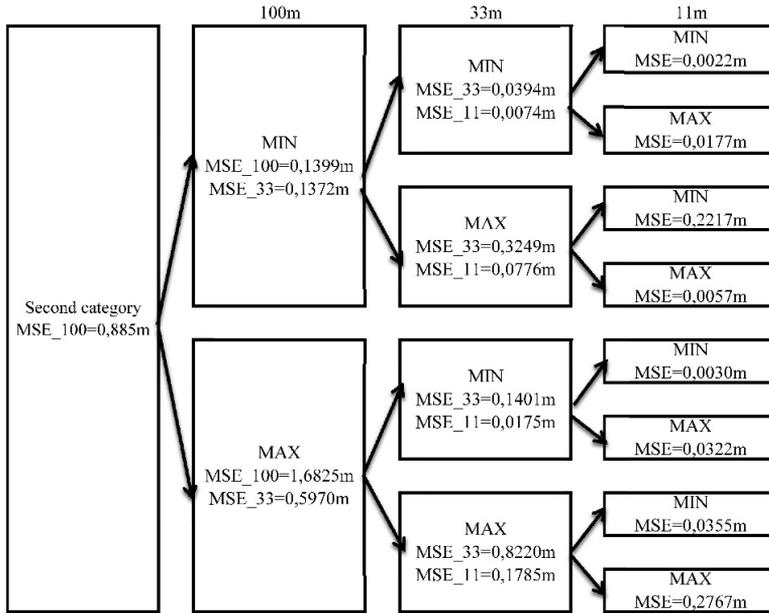


Fig. 3. Results of investigation of regular grid thickening (second complexity category)

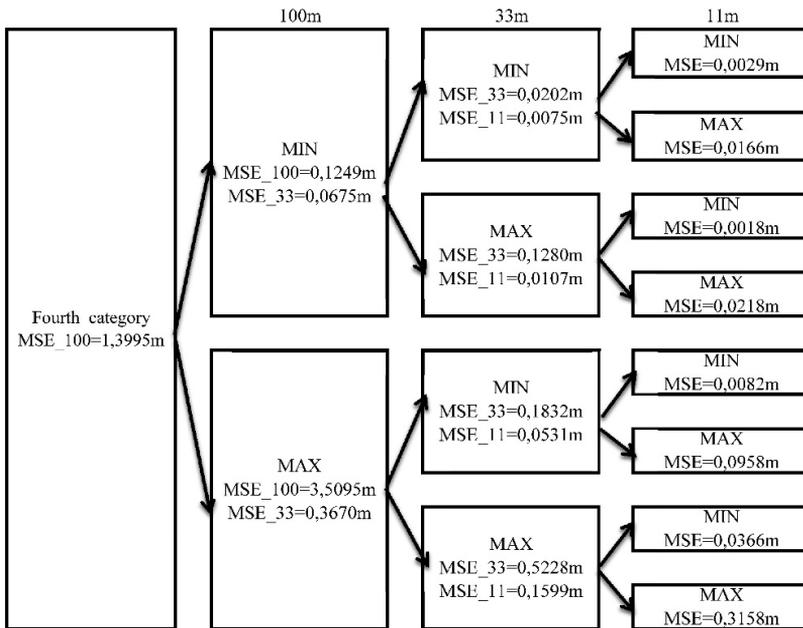


Fig. 4. Results of accuracy estimation of DTM reconstruction at thickening a regular grid (fourth category of complexity)

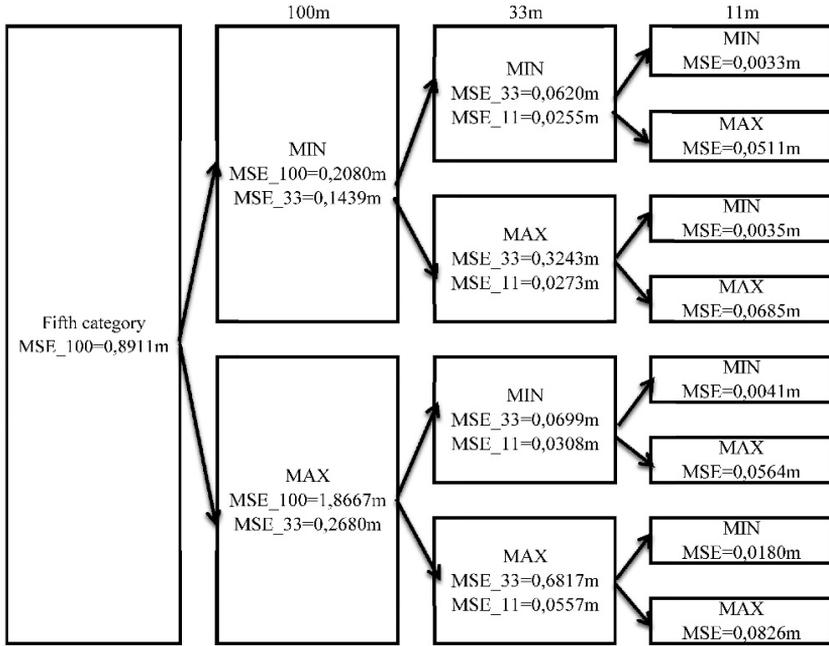


Fig. 5. Results of accuracy estimation of DTM reconstruction at thickening a regular grid (fifth category of complexity)

By means of known formulas based on higher conducted investigations, a general correlation can be established between the MSEs of height finding, terrain category, and grid density (Tab. 6):

$$r_{L,n} = \frac{\sum_{i=1}^m (L_i - L)(n_i - \bar{n})}{\sqrt{\sum_{i=1}^m (L_i - L)^2 \sum_{i=1}^m (n_i - \bar{n})^2}} \tag{2}$$

where:

- L – regular grid step,
- n – terrain category,
- m – the number of pairs of the investigation results.

After analyzing 237 pairs of values (whose results are presented in Table 6), we received a bilateral significance at a level of 0.00, which allowed us to conclude that there is a high direct dependence between MSEs and the density of a grid.

Based on established dependencies and calculated mean square errors, we received the dependency of accuracy of point heights obtained from DTM, which was constructed using BSI, on density of this regular grid and on complexity of relief (Fig. 6).

Table 6. Correlation connections of investigated criteria

	Relief category	Grid density	MSE
Relief category	1	0	-0.022
Grid density	0	1	0.685
MSE	-0.022	0.685	1

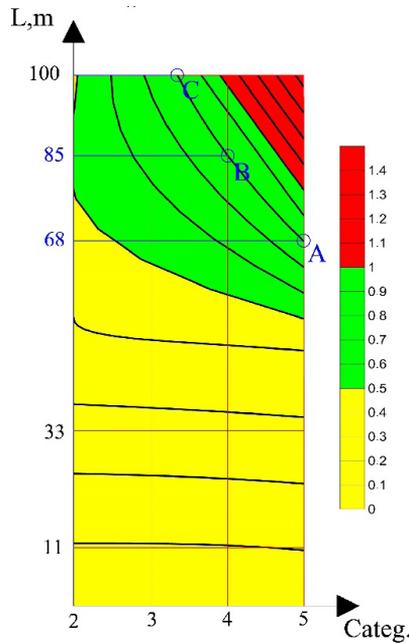


Fig. 6. Dependency of accuracy of point heights obtained from DTM on density of regular grid and on complexity of relief

Guided by the provisions [17] that are obligatory for all enterprises, organizations, and institutions that perform surveying and cartographic works regardless of their ownership and departmental dependence, the authors have established the necessary size of a square grid for achieving a regimented accuracy (Tab. 7). Using Figure 6, let us establish the required L for each relief complexity category by a graphical method. Based on Table 7, it is known that the MSEs should not exceed 0.1 m for flat plain areas of work for maps at a scale of 1:10,000; therefore, using BSI for DTM construction for the second, fourth, and fifth relief complexity categories, it is then needed to use a regular grid with an 11 m side. To ensure the accuracy of maps at a scale of 1:50,000 for flat, crossed, and hilly areas with superior terrain slopes of up to 6° for the fifth complexity category, it is sufficient to use a grid with a 68-m side (Fig. 6 – p. A), for the fourth category – 85 m (Fig. 6 – p. B), and to reproduce the DTM of the second category terrain, a 100-m grid can be used (Fig. 6 – p. C).

Table 7. Regulatory requirements of finding height accuracy

Areas of work	The map scale			
	1:10,000	1:25,000	1:50,000	1:100,000
	Mean square error [m]			
Flat plain areas with slopes of up to 2°	0.1	0.25	0.8	1.5
The same in forested areas	0.2	0.5	0.8	1.5
Plain, crossed, and hilly areas with slopes of superior terrain up to 6°	0.25	0.5	0.8	1.5
The same in open areas with slopes of up to 4°	0.25	0.25	0.8	1.5
Lowland and midland areas	0.5	0.5	1.2	2.5
Highlands	–	1.0	2.6	5.0

5. Conclusions

The results of our investigations are an *a posteriori* assessment of the impact of square regular grid size and the complexity category of relief on the mean square error of height finding of a digital elevation model created by bicubic spline interpolation as well as the established correlations between these parameters. The graph of dependence of point height finding accuracy on a DTM from a regular grid step and complexity category of relief is created, which allows us to choose the optimal size of a regular grid.

The possibility of using BSI for constructing DTMs for the second, fourth, and fifth complexity category of relief with normative accuracy has been proven. The sizes of a regular grid that satisfy the normative requirements for the accuracy of height finding in these areas of work have been calculated.

References

- [1] Burak K., Kovtun V., Levyts'kyi R., Nychvyd M.: *Doslidzhennya osoblyvostey ta tochnosti pobudovy TSMR za dopomohoyu bikubichnoyi splayn-interpolyatsiyi*. Suchasni dosyahnennya heodezychnoyi nauky ta vyrobnytstva, no. 2 (28), 2014, pp. 32–36 [Бурак К., Ковтун В., Левицький Р., Ничвид М.: *Дослідження особливостей та точності побудови ЦМР за допомогою бікубічної сплайн-інтерполяції*. Сучасні досягнення геодезичної науки та виробництва, № 2 (28), 2014, с. 32–36].
- [2] Bilous L.F.: *Tsyfrova model' rel'yefu v heohrafichnomu y heoinformatsiynomu prostori*. Uchenyye zapiski Krymskogo federal'nogo universiteta imeni V.I. Vernadskogo. Geografiya, t. 21, no. 2 (60), 2008, pp. 115–125 [Білоус Л.Ф.: *Цифрова модель рельєфу в географічному й геoinформаційному просторі*. Ученые записки таврического национального университета имени В.И. Вернадского. Серия: География, Т. 21 (60), № 2, 2008, с. 115–125].

- [3] Chervanev I.G.: *Struktura rel'yefa i yeye vliyaniye na landshaft*. Fizicheskaya geografiya i geomorfologiya, no. 30, 1983, pp. 104–107 [Черванев И.Г.: *Структура рельефа и ее влияние на ландшафт*. Физическая география и геоморфология, № 30, 1983, с. 104–107].
- [4] Weibel R., Heller M.: *Digital terrain modelling*. [in:] Maguire D.J., Goodchild M.F., Rhind D.W. (Eds.), *Geographical Information Systems: Principles and Applications*. Vol. 1, Longman Scientific & Technical, 1991, pp. 269–297
- [5] Kulazhskiy A.V.: *Primeneniye neyronnykh setey na osnovoveradial'nykh bazisnykh funktsiy v modelirovani rel'yefa mestnosti*. Vestnik Rostovskogo gosudarstvennogo universiteta putey soobshcheniya, no. 3 (35), 2009, pp. 118–124 [Кулажский А.В.: *Применение нейронных сетей на основе радиальных базисных функций в моделировании рельефа местности*. Вестник Ростовского государственного университета путей сообщения, № 3 (35), 2009, с. 118–124].
- [6] Kulazhskiy A.V., Portnov A.V.: *Modelirovaniye rel'yefa pri interaktivnom trasirovani lineynykh sooruzheniy*. Nauchno-tekhnicheskii i proizvodstvennyy zhurnal «Transportnoye stroitel'stvo», no. 5, 2010, pp. 20–21 [Кулажский А.В., Портнов А.В.: *Моделирование рельефа при интерактивном трассировании линейных сооружений*. Научно-технический и производственный журнал «Транспортное строительство», № 5, 2010, с. 20–21].
- [7] Kumlér M.: *An intensive comparison of triangulated irregular networks (TINs) and digital elevation models (DEMs)*. Cartographica: The International Journal for Geographic Information and Geovisualization, vol. 31, iss. 2, 1994, pp. 1–99.
- [8] Sarfraz M., Al-Mulhem M., Ashraf F.: *Preserving monotonic shape of the data using piecewise rational cubic functions*. Computers & Graphics, vol. 21, iss. 1, 1997, pp. 5–14.
- [9] Sarfraz M., Butt S., Hussain M.Z.: *Visualization of shaped data by a rational cubic spline interpolation*. Computers & Graphics, vol. 25, iss. 5, 2001, pp. 833–845.
- [10] Luu L., Wang Z., Vo M., Hoang T., Ma J.: *Accuracy enhancement of digital image correlation with B-spline interpolation*. Optics Letters, vol. 36, no. 16, 2011, pp. 3070–3072.
- [11] Knott G.D.: *Interpolating Cubic Splines*. Birkhäuser, Boston 1999.
- [12] Gousie M.B., Franklin W. R.: *Constructing a DEM from grid-based data by computing intermediate contours*. [in:] *Proceedings of the eleventh ACM International Symposium on Advances in Geographic Information Systems*, Association for Computing Machinery, 2003, pp. 71–77.
- [13] Podobnikar T.: *Various data sources of different quality for DTM production*. [in:] Ruiz M., Gould M., Ramon J. (Eds.), *Proceedings of the 5th AGILE Conference on Geographic Information Science, Palma de Mallorca, Spain, April 25–27*, Universitat de les Illes Balears, Palma, pp. 517–521.

- [14] Jaakkola O., Oksanen J.: *Creating DEMs from contour lines: Interpolation techniques which save terrain morphology*. GIM International, vol. 14, iss. 9, 2000, pp. 46–49.
- [15] Longley P.A., Goodchild M.F., Maguire D.J., Rhind D.W.: *Geographic Information Systems and Science*. John Wiley & Sons, 2005.
- [16] *Yedinyye normy vyrabotki (vremeni) na geodezicheskiye i topografograficheskiye raboty. Chast' II. Kameral'nyye raboty*. Utv. Goskomtrud, VTSSPS Post. 29/3-1, Moskva 2003 [Единые нормы выработки (времени) на геодезические и топографографические работы. Часть II. Камеральные работы. УТВ. Госкомтруда, ВЦСПС Пост. 29/3-1, Москва 2003].
- [17] Basic provisions for creating and updating topographic maps of scale 1:10,000, 1:25,000, 1:50,000, 1:100,000, 1:200,000, 1:500,000, 1:1,000,000 Approved by the order of the Chief Directorate of Geodesy, Cartography and Cadastre of Ukraine No. 156 dated December 31, 1999, and coordinated with the Military Planning Directorate of the General Staff of the Armed Forces of Ukraine.

Wyznaczanie gęstości regularnej siatki do tworzenia NMT przy użyciu interpolacji splajnami dwusześciennymi

Streszczenie: Za pomocą stworzonego przez autorów programu do interpolacji splajnami dwusześciennymi (BSI, *bicubic spline interpolation*) zbadano dokładność odtworzenia terenu w numerycznym modelu terenu (NMT). Analizowano kategorie terenu o różnym stopniu skomplikowania morfologii. Ustalono korelacje między rozmiarem regularnej sieci, kategorią skomplikowania rzeźby powierzchni terenu i średniokwadratowym błędem ustalania wysokości punktu. Wykonano wykres zależności dokładności wyznaczenia wysokości punktów z wykorzystaniem NMT od kroku regularnej siatki i stopnia skomplikowania morfologii terenu.

Słowa

kluczowe: interpolacja, dwusześcienny splajn, NMT, regularna siatka