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The Potential Application of the GNSS Leveling Method in Local Areas by Means of Sector Analysis

Abstract: The purpose of this work is to perform the comparison of heights of global geoid models EGM08, EIGEN-6C4, GECO, and XGM2019e based on sector analysis that are obtained relative to the ellipsoid WGS84 and GRS80 in order to implement the method of GNSS leveling in local areas. The heights of the global geoid models determined from the ellipsoid WGS84 should be reduced by −41 cm ("zero-degree term") in order to scale them to the calculated geoid by GNSS leveling. Heights determined from the ellipsoid GRS80 should be increased by +52 cm. Spatial analysis of the heights of geoid models in the relative system for the northern territory shows that the standard deviation of the heights of geoid models is 13.6 cm, and for the southern territory it is 36.5 cm. The elevation errors of the geoid models in the relative system were estimated to be standard deviations of 2.9 cm within the northern area and 2.3 cm within the southern one. The root mean square values of initial errors of the models EGM08, EIGEN-6C4, GECO, and XGM2019e are 8.6 cm, 4.6 cm, 4.4 cm, and 3.8 cm, respectively, and standard deviation values are 2.0 cm, 2.2 cm, 3.2 cm, and 2.4 cm. The paper also performs a sector analysis of the geoid model errors in order to correct them for the application of the GNSS leveling method within the research area. The standard deviations of the residual error of the corrected model heights are 1.8 cm, 1.9 cm, 2.5 cm, and 2.0 cm for EGM08, EIGEN-6C4, GECO, and XGM2019e. The root mean square values of these residual errors for the geoid models are 1.9 cm, 2.0 cm, 2.5 cm, and 2.0 cm, respectively.

Keywords: global geoid model, GNSS leveling, errors of geoid model, correction

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1. Introduction

To date, a large number of the different geoid models have been developed. A mathematical function that approximates the real gravitational potential of the Earth in space is called a global model of the gravitational field or a global geopotential model (GGM) [1]. In other words, global geoid models are those extending to the entire surface of the Earth.

The peculiarity of global models is that they are publicly available to users and can be used for any region of the planet. Such access is provided by gravity field services, one of which is the International Center for Global Earth Models (ICGEM) [2]. ICGEM offers the world's largest collection of gravitational field models, including models from the 1960s to the 1990s, as well as the latest ones developed using data from special satellite gravity missions such as CHAMP, GRACE, GOCE, satellite altimetry and ground gravity information [3, 4]. Of particular note are global models designed for up to 2190 degree and over, such as the EGM08 [5], EIGEN-6C4 [6] gravitational models and the combined models GECO [7], XGM2019e [8]. The independent testing of global models is performed by GNSS leveling at the national height systems to study their accuracy.

A detailed review and analysis of publications on testing the accuracy of EGM08, EIGEN-6C4, GECO, and XGM2019e in different countries suggests that the accuracy of the same model can range from a few to tens of centimeters in a single region area [9–15]. Therefore, the actual accuracy of global models can only be determined at the local area.

This article deals with research on the analysis of the height errors of the EGM08, EIGEN-6C4, GECO, and XGM2019e global geoid models identified at the points of high-precision geometric leveling according to GNSS leveling. The paper also uses the method of sector analysis to model the errors of the considered models of global geoid in the research area, after which these errors can be used as corrections. This is important for the application of the GNSS leveling method, because the geoid heights obtained from the models need to be corrected to match the heights of the calculated geoid derived from GNSS leveling at the local level.

2. Material and Methods

On the ICGEM website, the root mean square value of the EGM08 model is 12.5 cm, about the mean value of GNSS leveling for Europe, including 1,047 values; for the EIGEN-6C4 model it is 12.1 cm, for the GECO model it is 12.3 cm, and for the XGM2019e model it is 12.7 cm [2].

In this work, the analysis is performed at 22 points of high-precision geometric leveling for which the values of ellipsoidal heights were obtained according to the results of GNSS observations. For research on geoid models, a good prospect is including the data from permanent GNSS-stations in the calculations. In practice, this is very difficult to implement, as the network of such stations is not tied to national height systems. However, in this study, the normal height of the SULP permanent station of the National University "Lviv Polytechnic" is used, as it was determined by the method of trigonometric leveling with reference to the point of geometric leveling. The research area extends for about 38 km to the north relative to the SULP station and for about 62 km to the south. The location of both points and station are shown in Figure 1.

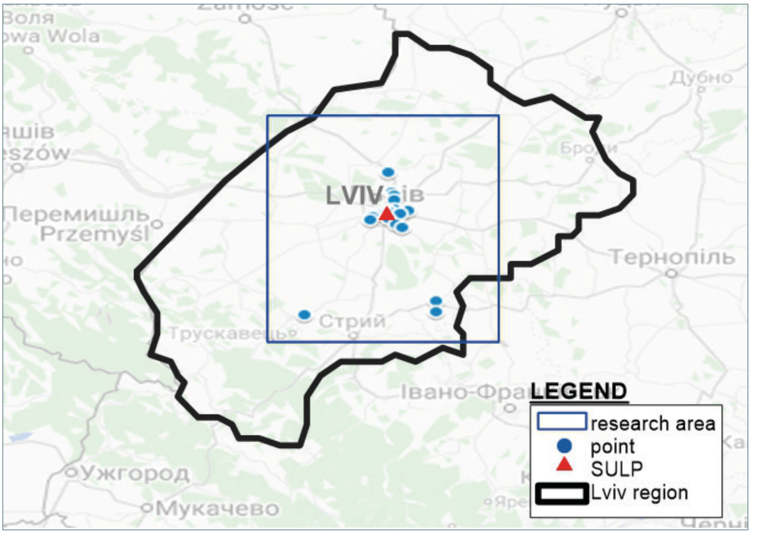

Fig. 1. Map of the location of points and a permanent station on the research area

The modern approach to determining the height of the Earth's physical surface based on the Global Navigation Satellite Systems (GNSS) [16, 17] is called GNSS leveling. In general, the essence of the method is that the normal or orthometric height is obtained from the following communication equations:

$$H^{\gamma} = H - \zeta_{\text{mod}} \tag{1}$$

$$H^{O} = H - N_{\text{mod}} \tag{2}$$

where:

H – the ellipsoidal height determined from GNSS observations,

ζ_{mod} – the height of the regional or local quasi-geoid obtained from a mathematical model,

N_{mod} – the height of the global, regional or local geoid taken from models.

The disadvantage of this method is that the heights of the quasi-geoid and geoid are the results of a mathematical function that perfectly describes the physical meaning of each of the surfaces, but is not the actual value of these quantities.

According to formula (1), if we determine normal heights by means of GNSS leveling, we have to use a quasi-geoid model, but in some cases the quasi-geoid model can be replaced by a global geoid model ($\zeta_{\mathrm{mod}} = N_{\mathrm{mod}} = N_{\mathrm{GGM}}$) [17]. This equality will be correct for those territories where the "real" quasi-geoid and the geoid differ by a few centimeters. Such a replacement will not lead to significant losses of accuracy in determining the normal heights of points on the Earth's surface by means of GNSS leveling. The main problem of the method comes down to the very fact of using mathematically modeled quantities of a quasi-geoid or geoid, and not their directly defined values.

For this research, the normal height of leveling points ($H^{\gamma}_{(k)}$) is taken from the catalog of heights of the 1st and 2nd classes. Therefore, having substituted these values in formula (1) and taking ($\zeta_{\mathrm{mod}} = N_{\mathrm{GGM}}$) we can calculate the height of the geoid with GNSS leveling:

$$N_{(o)} = H - H^{\gamma}_{(k)} \tag{3}$$

The heights calculated as follows can be used as independent data to compare the accuracy of global geoid models.

Model values of geoid heights can be obtained using special software packages for interpolation. Geoid heights for EGM08, EIGEN-6C4, GECO, and XGM2019e were determined for the ellipsoid WGS84 and GRS80 using the online service "User-defined points" provided by the ICGEM center [2]. To calculate these values, the "tide free" system is adopted, which eliminates the affect of the tides which are caused by the interaction of the Earth with the Moon and the Sun.

The differences of the calculated geoid from GNSS leveling and global geoid models were obtained by the formula:

$$\Delta N^{s}_{\mathrm{GGM}} = N_{(o)} - N^{s}_{\mathrm{GGM}} \tag{4}$$

where N^{s}_{GGM} is the height of the geoid model based on a specific reference ellipsoid (eg. WGS84 or GRS80), top index (s) indicates the system of the reference ellipsoid relative to which the height of the geoid model is determined (eg. s = W = WGS84 or s = G = GRS80), and lower index (GGM) is the name of a specific global geoid model.

In this paper, the differences obtained in this way are revised as the height errors of the geoid model. Such an analysis of errors prepares the ground for concluding that the accuracy of global models of the Earth and the prospects of using them can be applied to determining normal heights by GNSS leveling.

However, the heights of the models determined through the ICGEM site utility relative to the surface of the ellipsoid WGS84 ($N^{W}_{\_GGM}$) are greater than the heights of the calculated geoid, and the heights determined relative to the surface GRS80 $N^{G}_{\_GGM}$ are smaller. The wave difference of geoid models between WGS84 and GRS80 is 0.93 m and is a constant value for all models (the same difference was obtained by the authors [18]). From this it is concluded that the heights of the geoid obtained from the website do not take into account the value of the undulation of the geoid of zero order – "zero-degree term" – which is equal to −41 cm for the system WGS84 [5, 19]. Based on this information, the heights $N^{W}_{\_GGM}$ should be reduced by 41 cm, and the heights $N^{G}_{\_GGM}$ increased by 52 cm (93 cm – 41 cm = 52 cm). In this way, all the heights obtained with the help of the site utility are reduced to one scale. After doing so, it was found that the difference between the eponymous models defined relative to the surface of the ellipsoids WGS84 and GRS80 is very small and equals 1–5 mm. This means that analysis can only be performed for one data set. For geodetic works, it is advisable to use the GRS80 system, as it consists of a global ellipsoid and a model of the gravitational field, so in the future the analysis of all quantities will be presented for this system.

After a detailed analysis of the errors obtained by formula (4), the heights of the global geoid models can be corrected on the basis of the following formula:

$$N^{\Delta}_{\mathrm{GGM}} = N^{s}_{\mathrm{GGM}} + \Delta N^{\sec}_{\mathrm{GGM}}(B, L) \tag{5}$$

where:

$N^{\Delta}_{\mathrm{GGM}}$ – the corrected height of the geoid model,

N^{s}_{GGM} – the height of the global geoid model relative to the surface of the ellipsoid GRS80 (or WGS84) which is brought to the scale of the calculated geoid,

$\Delta N^{\sec}_{\mathrm{GGM}}(B, L)$ – the average error of the relevant section which is formed on the basis of sector analysis, and which includes the desired point.

3. Results

The main data in this study are the errors of the EGM08, EIGEN-6C4, GECO, and XGM2019e geoid models determined by the results of GNSS leveling performed at leveling points of 1 and 2 accuracy class. During the first stage, the model values of the geoid heights obtained relative to the reference surfaces of the WGS84 and GRS80 systems were considered and this data will be regarded as initial. For analysis, all points were sorted by latitude from north to south and compared with the heights of the calculated geoid from GNSS leveling. A graphical comparison is shown in Figure 2.

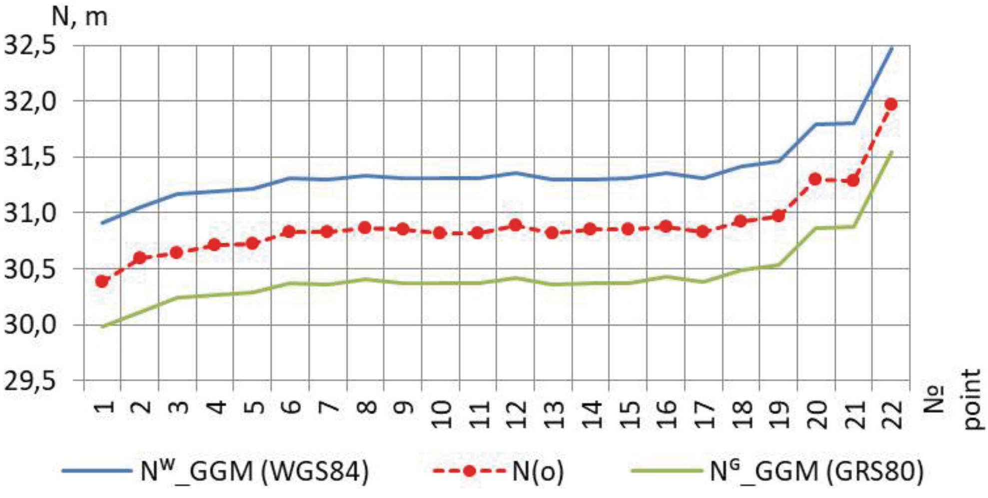

Fig. 2. Comparison heights of the calculated geoid and heights of the geoid model from the surface of the ellipsoid WGS84 and GRS80

As it can be seen from Figure 2, the values of the heights of global geoid models do not take into account the "zero-degree term". For further use they need to be translated to the scale of the calculated geoid with GNSS leveling by adding the value of "zero-degree term".

The heights of the geoid models and the values of the obtained errors were analyzed for the spatial change of these values in the northern and southern directions. A relative system with a starting point as a permanent SULP station was adopted for analysis. From the station to the north there are 10 points with a length of about 38 km and to the south there are 12 points, covering an area of approximately 62 km. Such a relative reference system makes it possible to estimate all the heights of the models and errors as a single data set and then, on the basis of statistical characteristics, to analyze the trend of spatial change of these values. When comparing the heights of geoid models with the heights of the calculated geoid in the relative reference system, five points were found, where the heights of all four geoid models differ from the heights of the calculated geoid in sign and size, as it is shown in Table 1. This effect may indicate gross inaccuracies in the modeling of the geoid waves of the models themselves, so for further analysis, three points located in the northern part and two points in the southern part were removed from the data set.

Thus, the statistical analysis based on mean (Δ^*), maximum (Δ_{max}), minimum (Δ_{min}), standard deviation (σ), and data range (R) will cover seven points in the north, ten points in the south of relatively permanent station. Table 2 shows the results of the analysis of the heights of the geoid models, and Table 3 shows the results of the analysis of the errors of the geoid models in the relative system.

Table 1. Differences in the heights of the calculated geoid with the model values in the relative system

Differences name	Northern points [m]			Southern points [m]	
	NOVOZ	700R	OPER2	HORO3	AVIA
$N^{SULP}_{(o)}$	**−0.02**	**−0.01**	**−0.04**	**−0.03**	**−0.03**
N^{SULP}_{egm}	0.01	0.02	0.01	0.01	0.01
N^{SULP}_{eigen}	0.01	0.02	0.01	0.01	0.01
N^{SULP}_{geco}	0.01	0.02	0.01	0.01	0.01
N^{SULP}_{xgm}	0.01	0.02	0.01	0.01	0.01

Table 2. Height statistics of geoid models for the northern and southern research areas in the relative system

Differences name	Δ^* [m]	Δ_{max} [m]	Δ_{min} [m]	σ [m]	R [m]
N^{SULP}_{north}	0.14	0.40	−0.04	0.136	0.43
N^{SULP}_{south}	−0.25	0.02	−1.21	0.365	1.23

The data in Table 2 shows that the statistical characteristics of heights in the northern direction are much smaller than in the south. Standard deviation for the northern heights of the geoid model is 13.6 cm, while for the southern data it is 36.5 cm. The range of values to the north is 0.43 m, and to the south 1.23 m, which is almost three times more than Northern ones. The northern average values are 0.14 m, and the southern ones are −0.25 m. The minimum – maximum range varies from −0.04 m to 0.40 m for the northern part and from −1.21 m to 0.02 m for the southern part. Such results may indicate increased undulation of the heights of the geoid to the south. The northern area is only 24 km smaller than the southern one, which may mean that in the general sense of the research the area is at the junction of significant differences in the heights of the "real" geoid.

The values in Table 3 show that the errors of the models are characterized by more systematic nature of the change in two directions. The average values are equal to zero, the minimum and maximum values differ by 1 cm, and the range of values is the same. The standard deviation of the northern values is 2.9 cm, and for the southern ones it is 2.3 cm. This means that a significant spatial change in errors in two directions is not observed.

Table 3. Error statistics of geoid models in the northern and southern directions (relative system)

Errors name	Δ^* [m]	Δ_{max} [m]	Δ_{min} [m]	σ [m]	R [m]
ΔN_{north}^{SULP}	0.00	0.06	−0.04	0.029	0.10
ΔN_{south}^{SULP}	0.00	0.07	−0.03	0.023	0.10

From the analysis of errors in the relative system we can see that this made it possible to identify gross errors of geoid models and compare the spatial nature of changes in geoid waves and errors around the permanent SULP station in the northern and southern parts of the area.

The next stage of research was the comparison of the heights of geoid models with the heights of the calculated geoid. To visualize the results, contour graphs of errors were constructed, as shown in Figure 3 (the distance between isolines is 1 cm). A darker gradation means that the error values increase with a minus sign and the lightest gradation indicates only positive values.

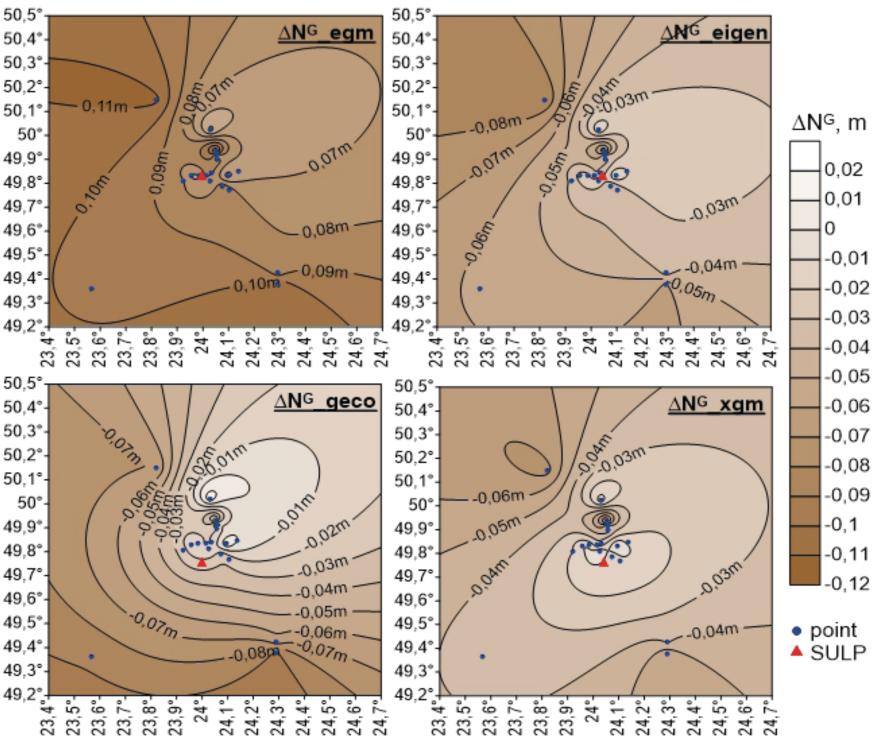

Fig. 3. Visualization of height errors of global geoid models based on the results of GNSS leveling

As it can be seen from Figure 3, the errors of the EGM08 model increase from northeast to southwest. The errors of the EIGEN-6C4 model show the same tendency, but are smaller in size. The range of error values of the GECO and XGM2019e models is almost the same, but has a slightly different spatial nature of change. The errors of the GECO model change from the northeast to the southwest with intense density, and the errors of the XGM2019e have a gentler change. The errors of the EGM08 model are exclusively negative values, and for the other three models they have both positive and negative values. All errors were also analyzed for minimum, maximum, mean, standard deviation and range of values. The results of the statistical characteristics are presented in Table 4.

Table 4. Statistical characteristics of geoid model errors of the research area

Statistical characteristic	ΔN^{G}_{egm} [cm]	ΔN^{G}_{eigen} [cm]	ΔN^{G}_{geco} [cm]	ΔN^{G}_{xgm} [cm]
Average (Δ^*)	−8.2	−3.9	−2.9	−2.8
Minimum (Δ_{min})	−4.7	−1.0	1.7	0.2
Maximum (Δ_{max})	−12.1	−8.9	−9.2	−8.5
Range (R)	7.4	7.9	10.9	8.7
Standard deviation (σ)	2.0	2.2	3.2	2.4
Root mean square (RMS)	8.6	4.6	4.4	3.8

Table 4 shows that the root mean square errors of the EGM08, EIGEN-6C4, GECO, and XGM2019e models are 8.6 cm, 4.6 cm, 4.4 cm, and 3.8 cm and standart deviation is 2.0 cm, 2.2 cm, 3.2 cm, and 2.4 cm, respectively.

In general, summarizing the statistical characteristics of each model, we can conclude that the errors derived from the XGM2019e model are the smallest. However, it should also be noted that the errors of geoid models vary within a few centimeters with positive and negative values, even between the nearest points of the height of the leveling network at which GNSS leveling is performed. Theoretically, such a variation in error differences should not occur in a relatively small area.

Analyzing the obtained errors of geoid models at the points of leveling heights, we can conclude that they have quite diverse properties, which makes it impossible to use a geoid model directly for practical purposes, such as for more accurate geodetic work. That is, when obtaining a normal height by GNSS leveling at any point of the research area, it will be determined with a certain total error, which in turn will largely depend on the error of a particular model of the geoid at a concrete point on the Earth's surface. The error of the geoid model for an unknown point can be taken into account by averaging the values of all errors obtained at points of heights of high-precision geometric leveling, where GNSS-observations are performed. However, preliminary data analysis indicates that this method should only be used for a local area of about 10–15 km [20]. The research area in this work is several times larger, so to determine the average errors of geoid models, the method of sector analysis was used.

The essence of such an analysis is to clarify in more detail the average error of the geoid model around a certain "starting" point in a given direction. In this case, the SULP permanent station was taken as such a point, as it is located in the central part of the research area. Thus, the territory around the station was divided into eight sectors: North, North_East, East, South_East, South, South_West, West, and North_West. For each sector, the average values of the errors of geoid models were calculated on the basis of leveling points that were located in a corresponding sector. Figure 4 shows the principle of averaging and distributing errors based on the results of sector analysis.

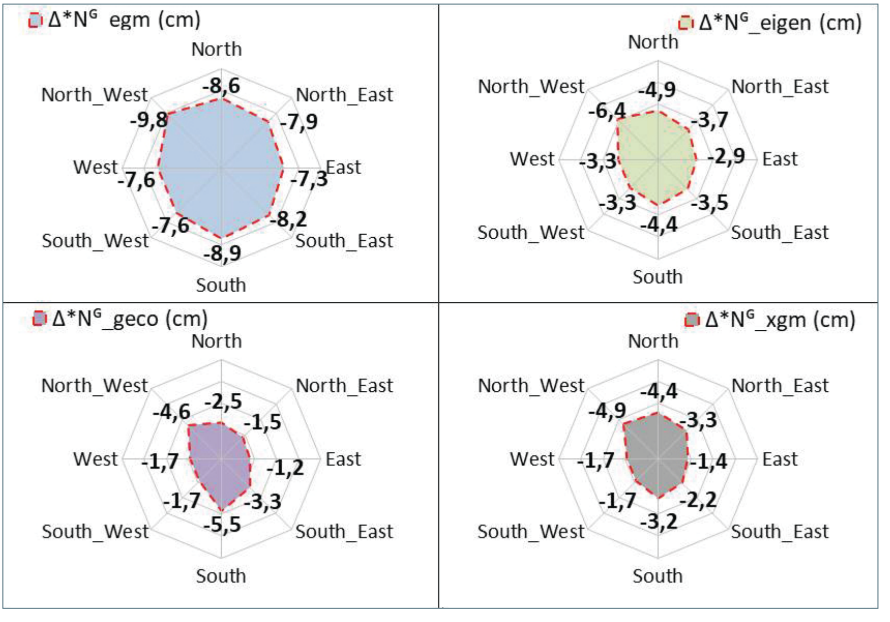

Fig. 4. Distribution of averaged errors of geoid models based on the results of sector analysis with respect to the permanent station SULP

From Figure 4, we can see that the average errors in the sectors are calculated so that they maximally preserve the spatial nature of the change in the errors of geoid models that are determined at the points of the leveling network by GNSS leveling.

The range of mean values is from −7.3 cm to −9.8 cm, from −2.9 cm to −6.4 cm, from −1.2 cm to −5.5 cm and from −1.4 cm to −4.9 cm for geoid models EGM08, EIGEN-6C4, GECO, and XGM2019e, respectively. The standard deviation of the average errors according to the sector analysis was 0.8 cm for the model of the geoid EGM08, 1.2 cm for EIGEN-6C4, 1.6 cm for GECO, and 1.3 cm for XGM2019e. Such results are 1.5–2 times less than the standard deviations of the errors of the geoid models identified at the points of the leveling network (see Tab. 4).

Based on the results of sector analysis of the geoid model errors on the research territory, contour maps of the spatial distribution of averaged errors were constructed as shown in Figure 5.

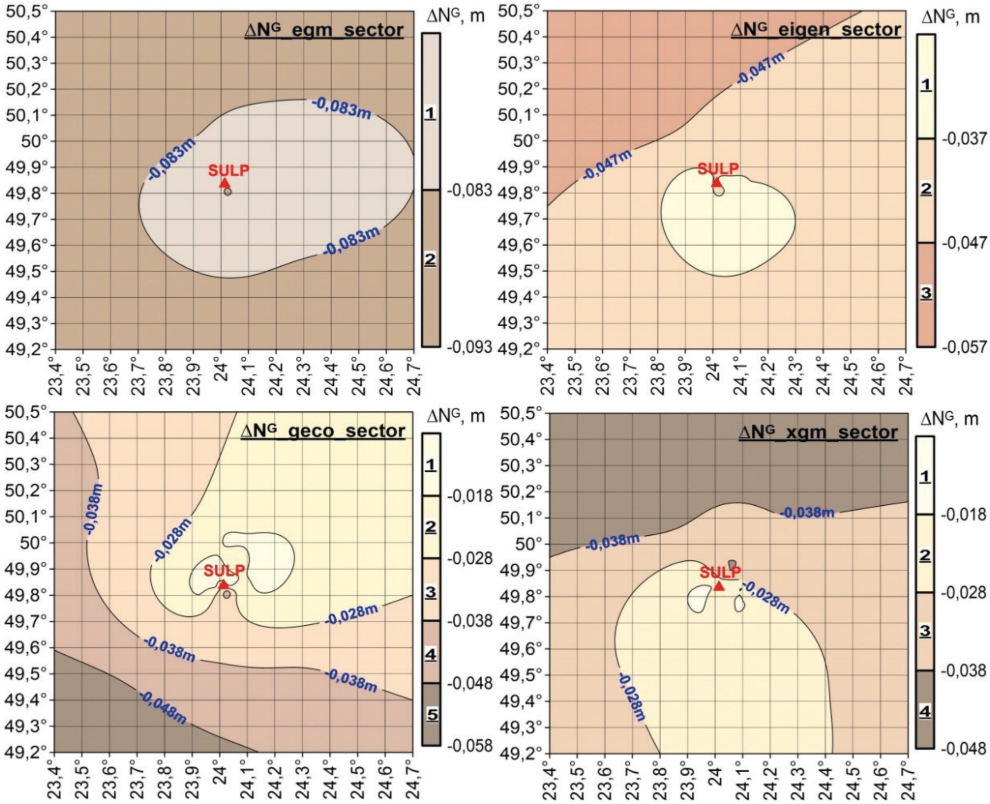

Fig. 5. Contour maps of averaged errors of geoid models EGM08, EIGEN-6C4, GECO and XGM2019e obtained by sector analysis

Figure 5 shows that the EGM08 contour error map consists of two sections of mean errors, the EIGEN-6C4 of three, the GECO of five, and the XGM2019e of four sections. Each section corresponds to 1 cm of the spatial change of the average error.

Calculated by means of formula (5), the corrected heights of the geoid models were compared with the heights of the calculated geoid. Statistical characteristics of residual errors of the corrected heights of the geoid models are presented in Table 5.

Table 5. Statistical characteristics of residual errors of the geoid models heights after the correcting by the sector analysis method

Statistical characteristic	ΔN_{egm}^{sec} [cm]	ΔN_{eigen}^{sec} [cm]	ΔN_{geco}^{sec} [cm]	ΔN_{xgm}^{sec} [cm]
Average (Δ^*)	0.4	0.4	0.1	0.3
Minimum (Δ_{min})	−3.8	−3.4	−4.2	−3.7
Maximum (Δ_{max})	3.6	3.5	3.8	3.3
Range (R)	7.4	6.9	8.0	7.0
Standard deviation (σ)	1.8	1.9	2.5	2.0
Root mean square (RMS)	1.9	2.0	2.5	2.0

From Table 5 we can see that the standard deviations of the residual errors of the EGM08, EIGEN-6C4, GECO, and XGM2019e models are 1.8 cm, 1.9 cm, 2.5 cm, and 2.0 cm. Such a result is 7–22% better than the initial error deviations. The root mean square values of the residual errors after corrected heights of the four geoid models are 1.9 cm, 2.0 cm, 2.5 cm, and 2.0 cm, respectively for each model, which is 42–78% better than the first data (see Tab. 4).

However, after correcting the model heights by this method, it was found that the values of the residual errors of some points exceed the limits of the accuracy of standard deviations. Thus, from the eighteen values tested, four points for EGM08, seven for EIGEN-6C4, seven for GECO and eight for XGM2019e were obtained with the worst accuracy, which is 22%, 39%, 39%, and 44% of the total values. This situation requires a more detailed analysis to identify the factors that influenced the accuracy of the model values of geoid heights for these points.

4. Conclusions

A comparative analysis of the global model heights of the geoid EGM08, EIGEN-6C4, GECO, and XGM2019e determined relative to the reference ellipsoids of the WGS84 and GRS80 systems was performed. The difference in model heights between WGS84 and GRS80 was equal to 93 cm. The heights determined by the ICGEM site utility for the WGS84 system should be reduced by −41 cm, and the heights determined for the GRS80 ellipsoid should be increased by +52 cm. These values correspond to the "zero-degree term" for the two reference systems. The undulation value must be taken into account to bring the heights of the geoid model to the scale of the heights of the calculated geoid from GNSS observations.

Spatial analysis of geoid models in the relative system showed that the standard deviation of geoid heights in the northern direction is 13.6 cm, and in the south it is 36.5 cm. The range of values also differs significantly and is 0.43 m for the northern part and 1.23 m for the south. These results indicate a sharp increase in the surface roughness of the geoid for the southern research area compared to the northern. The height errors of the models in the relative system were estimated by the standard deviation of 2.9 cm for the northern part and 2.3 cm for the southern part. No sharp spatial change in the errors identified in the relative system of reference was detected. Also, the analysis of data in the relative system made it possible to identify five points with gross errors.

A comparative analysis of the errors of geoid models determined by the results of GNSS leveling shows that the standard deviations of the models EGM08, EIGEN-6C4, GECO, and XGM2019e are 2.0 cm, 2.2 cm, 3.2 cm, and 2.4 cm, and the root mean square values are 8.6 cm, 4.6 cm, 4.4 cm, and 3.8 cm, respectively. From this is we can conclude that the XGM2019e model is the most accurate for the research area in comparison with the average standard value. The errors of all considered models tend to change from the northeast to the southwest.

The paper presents a method of adjusting the values of the heights of global geoid models to increase the accuracy of the GNSS leveling method on the example of the research area. The sector analysis used in this work allows us to conclude that the construction of contour maps of the spatial distribution of the average errors of geoid models makes it possible to perform height correcting EGM08, EIGEN-6C4, GECO, and XGM2019e at the level of standart deviation of 1.8 cm, 1.9 cm, 2.5 cm, and 2.0 cm, respectively. Also, in this case, the root mean square values of the residual errors can be reduced to 1.9 cm, 2.0 cm, 2.5 cm, and 2.0 cm for the respective models. Thus, according to the sector analysis, the heights of the global geoid models can be improved and used to apply the GNSS leveling method to local areas.

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