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DETERMINING MINERAL PROJECT VALUE — THE PREFERENCE THEORY

1. Introduction

Monte Carlo simulation is a computational method that takes into account uncertainty of input parameter — each uncertain variable consists of a distribution of values. Among many possible applications, the method is used in economic evaluation of mining projects (DCF technique) — it can be regarded as an extension of a traditional scenario analysis, which takes into consideration almost all possible scenarios of a project. In each scenario the project values are randomly selected and combined to calculate the value of the project for that combination. A computer program for the Monte Carlo method can be used to generate hundreds of variations of cash flows, giving the same number of possible NPVs of the project. These NPVs, when displayed graphically, yield an expected project value and an explicit demonstration of the uncertainty surrounding that value. So that we can say that Monte Carlo simulation yields results for multiple attributes: amount of maximum loss, amount of maximum gain, and amount of expected gain. Thus stochastic simulations are shown to be useful in several aspects of project evaluation.

The probability distributions, used in Monte Carlo simulation, reflect the uncertainty of some variables and risk incorporated in project analysis. That's why the risk component in a risk-adjusted discount rate decreases in proportion to the amount of risk expressed in the probabilistic range of input values — if all risks are totally incorporated, a riskless discount rate must be used.

Because of this fact, there is a misunderstanding concerning the expected value of the resultant NPV distribution in Monte Carlo simulation — is it the value of the project or not? Indeed — using low (or risk-free) discount rate in a simulation the expected value of the distribution of the NPVs cannot be the price the investor would be willing to pay considering all project's risks, because it is simply too high. It is because at this stage the risk preferences of the decision-maker have not yet been considered.

Attempts to avoid some of the confusion associated with the use of DCF feasibility measures in undertaking stochastic simulation lead to decision model called preference or

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expected utility theory (PT). This approach is an extension of the expected value concept. In preference-analysis the manager's attitudes about risk-taking are applied into a quantitative decision model. As a result he receives a more realistic measure of value among comparable projects characterized by uncertainty and risk. Decision-analysis methods provide a mechanism for measuring a managers' risk tolerance and ability to make formal comparisons between various projects.

2. Preference theory

The first work about quantifying an individual's feelings about money and risks was written by Bernoulli in 1738 [1]. In 1944 von Neumann and Morgenstern developed a mathematical theory, which was published in 1953 [3]. Various axioms of it were stated alternatively in Savage [5] and Pratt *et al*. [4]. The theory is based on some very fundamental and reasonable concepts of rational decision-making.

The theory assumes that the attractiveness of alternatives depends on:

— the probability of the possible results of each alternative,

— the preferences of the decision-makers for those results.

As Torries [6] writes, preference theory concepts apply particularly well to mineral project evaluation.

Before explaining the theory, it is important to say some words about the manager's attitude towards risk. Distinction between risk neutrality and risk aversion influences on the way an investor values projects. The basis for modern finance theory and the classical methods of project evaluation is risk-neutral decision making. According to the theory, they should be indifferent to investments with the same expected NPVs even if they represent different degrees of risk. As quoted Walls and Eggert [7] investors are — in fact — risk averse. This is why they will select less risky projects regardless of the same level of NPV.

The PT is just a method, which measures different degrees of risk aversion and applies the observed risk preferences into decision-making process.

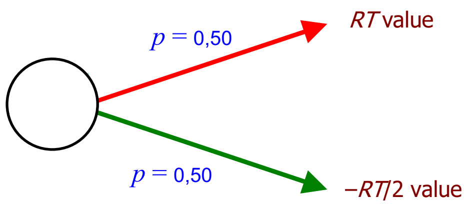
Starting point in the PT is a utility function, which measures risk preferences of a person or company. It represents the investor's risk attitudes across a range of financial outcomes. The most common form (but not only one) of it is exponential:

$$u\,(x) = a - be^{-x/RT} \tag{1}$$

where:

- u — utility as a function of variable of interest,
- RT — the risk tolerance coefficient,
- x — the monetary variable,
- a, b — the formula's constants,
- e — the exponential constant.

The *RT* coefficient is a measure of the investor's risk attitude. It represents an amount as to which the investor will just be indifferent when faced with a 50–50 chance of gaining that sum and losing half of that sum (an example is shown on Figure 1). The *RT* coefficient is obtained through a survey of decision-makers.



Case	***RT*** **value [Mill CU]**	**–*RT*/2 value [Mill CU]**	**Decision**
1	10	5	accept
2	15	7,5	indifferent
3	20	10	reject

Fig. 1. Risk tolerance (*RT*) concept [7]
(CU — currency units)

Having obtained *RT* coefficient it is possible to provide a risk-adjusted valuation for any risky project. The formula (based on the exponential utility function) is given as follow:

$$CE_x = -\,RT \ln\left(\sum_{i=1}^{n} p_i e^{x_i/RT}\right) \tag{2}$$

where:

CE_x — the certainty equivalent,
p_i — the probability of outcome i,
x_i — the value of outcome i,
RT — the risk tolerance coefficient,
e — the exponential constant,
n — number of outcomes.

The certainty equivalent is a risk-adjusted value of the investment. The CE_x is equal to the expected value less a risk discount. This discount is determined by the investor's risk tolerance and the risk characteristics of the project, that is probability distribution of possible outcomes. The formula quoted above is calibrated for discrete probability function.

The decision criterion under the CE_x approach is to accept the project with the highest certainty equivalent. An example of comparison between projects is provided in Torries' book [6] (Fig. 2). Let's consider that an investor faces a choice between project A and B. Both of them have the same expected value 12 million. Risk-neutral managers cannot choose between the projects, because both have the same expected value of NPV.

Risk-averse decision-makers, however, would make decision, depending on their *RT* (according to formula (2)), for example:

1) if *RT* = 100 million they would prefer project B,
2) if *RT* = 33 million (i.e. more risk averse investors) — they would consider project B to be riskier than project A (because of project B's large downside risk compared to project A).

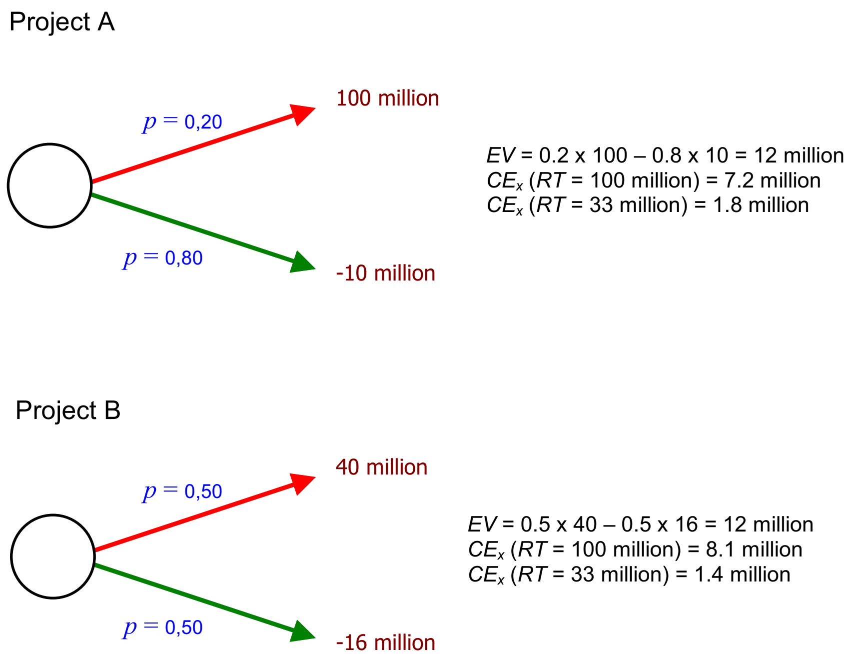


Fig. 2. Expected value vs. certainty equivalent decision criteria [7, 6]

It is apparent that the *RT* measure has a considerable effect on the valuation of a risky investment. The C_x measure explicitly considers the relative magnitudes of capital being exposed to the chance of loss for each of the projects and the investor's relative strength of preference toward the uncertain financial consequences.

3. Obtaining the value of mining project from probabilistic DCF

Let us consider project A, with probabilistic distribution of NPV obtained in a Monte Carlo simulation (Fig. 3). The project was calculated at the "risk free" discount rate (5%, real). The expected value of the NPVs distribution is 81.301 million. On the other hand we know, that the "expected" NPV calculated for a base case scenario in a deterministic approach, using a 14% risk-adjusted discount rate is only 2.595 million. The question is:

which is the proper value of the project? Because a Monte Carlo simulation assumes risk neutrality of an investor whilst the deterministic approach — risk aversion, we can expect the latter value to be correct. The problem is, how to calculate the value of project A having Monte Carlo outcomes?

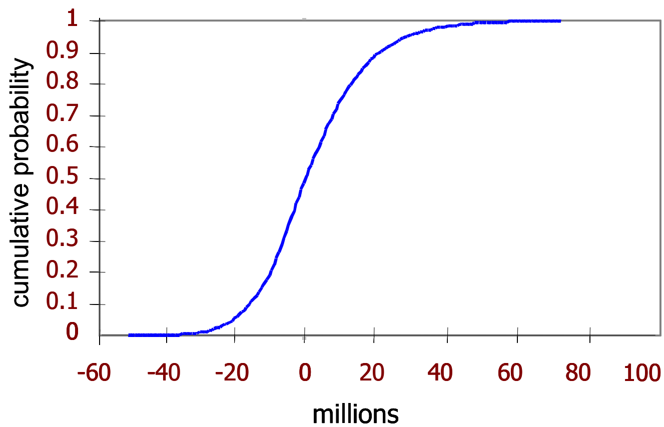


Fig. 3. Cumulative distribution of NPV of project A

As the probability distribution of NPV shows, the most we can get from project A is 80 million whereas the maximum possible loss is 60 million. Our aim is to determine what is the value (i.e. C_x) of the project. The answer depends on the risk preference of the investor. Some investors (who are averse to the risk associated with losing a given sum of money, regardless of the potential benefits) will reject the project but some with greater risk tolerance undertake it. Monte Carlo simulation provides the same information to two different investors, who assess merits of the project in different ways because of their different attitude towards the involved risk.

In order to evaluate project A let's consider the use of the certainty equivalent and the risk tolerance coefficient of a manager. Because the formula (2) assumes discrete probability function the resulting distribution must be converted to discrete probabilities so that C_x for the project can be determined for any value of RT. For this reason an approach known as the equal probable method should be used [6] after [2]. Thus the histogram of NPV project is divided into five intervals (as claims Torries five is usually an appropriate number to estimate the discrete values). To compute the certainty equivalent for the project each of the intervals has a single representative point, which is used along with the probability of each interval occurring (p = 0.20 for five intervals). The discrete probabilities of values for project A are shown in Table 1. These discrete values and probabilities are then used to calculate C_x values (formula (2)) for project A for different values of RT.

The conclusion is that only an investor with a risk tolerance of 173.7 million will place the same value of the project as one obtained through deterministic analysis. It means that an investor who has selected 14% risk-adjusted discount rate in deterministic approach demonstrates the very level of risk tolerance for this project.

TABLE 1

Discrete equal-probability intervals and representative points for project A

Interval	Representative point	Probability
(60)–(5)	(8)	0.20
(5)–(1)	(3)	0.20
(1)–5	3	0.20
5–8	6	0.20
8–80	28	0.20

4. Summary

There has been some confusion over the outcomes of probabilistic analysis — especially in the aspect of the project's value. Because of the fact that most of the risk inherent in the project is reflected in the range of input variables (probability distributions), the Monte Carlo simulation applies in the DCF worksheet a risk-free discount rate. The use of risk-free discount rates results in too high — in comparison to the deterministic analysis, which applies risk-adjusted discount rates — expected values of projects. The difference between deterministic and probabilistic expected values results from the fact that the former approach takes into account an investors attitude to risk-taking whilst the latter does not. The way, which helps determining the project's value in a Monte Carlo simulation, is the preference theory (PT). This theory, taking into account an ability of a decision-maker to take risk (risk tolerance, *RT*), consequently enables to determine the amount, which the investor would be willing to invest in this project.

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