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The logarithmic ACD model: The microstructure of the German and Polish stock markets[1]

1. Introduction

The timing of transactions (i.e., the quantity purchased in a period of time) is often a key economic variable; as such, it should be modeled or forecasted. The microstructure of financial markets is investigated using transaction-by-transaction data. The timing of these transactions can be very important in order to understand market participant behavior from the point of view of economic theory.

In recent years, there have been a lot of contributions dealing with the financial market microstructure. These contributions have focused both on theoretical models and empirical findings. Nowadays, most exchanges (NYSE, NASDAQ, Paris Bourse, Frankfurter Boerse) and even smaller exchanges like Vienna and Warsaw compile databases of tick-by-tick data which, depending on the exchange, give information on the trade process (time of the trade, price, volume) and the bid-ask quote process (time of quotes, bid and ask quotes, depths) or the state of the order book. Researchers can now work in new empirical and theoretical areas. However, in order to use high-frequency data, new econometric tools are necessary.

High-frequency data about investor activity typically arrives at irregular time intervals. Classic standard econometric techniques were good for the treatment of fixed time intervals. Typically, researchers aggregated market data to some fixed time intervals. The most-frequently used data in the case of consumption was monthly or yearly data. However, stock market transactions are very often

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conducted within a fraction of a second. It is plausible that, in the case of a short-time interval, one can observe many intervals with no new information, so the data becomes heteroskedastic. On the other hand, we cannot analyze the micro-structure properties of the data in the case of a long interval. The consequence of multiple transactions is the averaging of the timing and characteristics of particular transactions, meaning that the researcher can lose certain valuable information.

The problems of market microstructure are complex, because the rate of arrival of transactions may exhibit calendar or seasonal effects (dependent on the day, week, or year), which implies problems with the determination of an appropriate length-of-time interval. The frequency may depend not only on a news release but also sometimes on an unobservable factor that is to some extent deterministic but also partly stochastic. In this case, one says that there is a stochastic process underlying trading activity. The breaking point in research into the microstructure of financial markets was provided by Engle and Russel (1997) and (1998).

In our contribution, we try to compare the microstructure of selected stocks comprised from the DAX30 and WIG20 using models of the ACD type under the assumption of generalized gamma and Burr distributions of durations.

The remaining part of our paper is scheduled as follows: Section 2 contains a short overview of existing models in the literature and the methods used in studies of microstructure. In Section 3, the methodology is presented; and in Section 4, the results of our computations are shown and discussed in detail. Finally, Section 5 presents the conclusions of the paper.

In the next section, we conduct a brief literature review on the topic.

2. Literature overview

Early contributors on market microstructure (for example Hafner, 1996; Eddelbutel and McCurdy, 1998; and Guillaume et al., 1997) concentrated on high-frequency data modeled by the so-called *"fixed interval"* econometric models. These models include the stochastic volatility- and GARCH-type models. The common feature of this kind of modeling is that the data is regularly sampled at a very high frequency. However, one main drawback of these models is that they do not take into account the irregular spacing of data. The contributors tried to conduct an alternative to regular sampling, using time transformation techniques. The aim was to transform irregularly spaced data into fixed-interval data.

Engle and Russel (1997) and (1998) treated arrival times as random variables that follow a stochastic point process. Arrival time depends on (financial) random variables such as volume, bid ask spread, or price. The authors derived a new model for dependent-point processes. The set of parameters typical of the stochastic process consisted of past events in order to reflect the transaction process. The most-important application of the model was to the measurement and forecasting of the intensity of transaction arrivals. The contributors parameterized the conditional intensity as a function of the time between past events. In addition, they allowed some natural extensions; e.g., taking into account the characteristics of past transactions. Also, outside influence cannot be excluded. The authors, assuming the dependence of the conditional intensity on past durations, called their model the *Autoregressive Conditional Duration* (ACD) model. The ACD model is the counterpart of the GARCH model.

This model for the durations between two successive market events (such as the buying or selling of a security) takes into account a clustering effect in the durations. In the model, short (long) durations tend to be followed by short (long) durations. This effect resembles that which is found in the volatility of many financial time series. The contributors applied the model to the modeling of the foreign exchange market and the IBM stock. In a subsequent paper, Engle (2000) linked the ACD-duration model with a GARCH model for returns. This combination allowed the modeling of irregularly timed data. The following extensions or counterparts were developed by a number of contributors.

Ghysels and Jasiak (1998) suggested the stochastic volatility duration (SVD) model. Gramming and Maurer (2000) derived an ACD model based on the Burr distribution (an alternative to the Weibull distribution). Jasiak (1998) extended ACD to the fractionally integrated ACD model. This model enables the modelling of long-range dependence in the durations.

Bauwens and Veredas (2004) defined a class of models for the analysis of durations, known as stochastic conditional duration (SCD) models. These models are based on the assumption that the durations are generated by a dynamic stochastic latent variable. The model yields a wide range of shapes hazard functions. The estimation of the parameters is conducted by quasi-maximum likelihood and by using the Kalman filter. The model is applied to trade, price, and volume durations of stocks traded at the NYSE. The authors also investigate the relation between price durations, spread, trade intensity, and trading volume.

The most-recent research tries to reflect the new realities on financial markets implied by the introduction of new technology and high-frequency trading, especially after the crisis of 2008.

Based on order-level data from 2008, Hasbrouck and Saar (2013) found that some traders on the NASDAQ could respond to events such as changes in the limit order book in 2–3 milliseconds. Ye et al. (2013) proved that trading could be conducted at even faster speeds. They suggested that high-frequency traders' need for speed depends on the particular strategies that they follow.

According to Jones (2012), Brogaard et al. (2012), and Carrion (2013), most market participants and researchers are convinced that HFT market trading enhances market quality by reducing spreads and raising informational efficiency. Some contributions (e.g., Kirilenko et al., 2011; Easley et al., 2011; 2012a, and Madhavan, 2013) express concerns that HFT market trading can induce market instability. However, Brogaard et al. (2011), O'Hara (2011), and O'Hara et al. (2013) stress that the bulk of liquidity provision in many markets is provided by high-frequency traders.

Angel et al. (2011) stress that, when retail orders do go to the NYSE, they often benefit from liquidity provided by DMMs (designated marker makers) and SLPs (strategic liquidity providers), many of which are actually high-frequency trading firms. Another observation is that the trading costs of retail traders have been falling over the past 30 years. In addition, this decline has sped up in recent years. Using data from the Toronto Stock Exchange, Malinova and Park (2013) show empirically that retail trading costs have fallen because of the presence of HFTs. Hendershott et al. (2011) provide evidence that algorithmic trading particularly improved market quality with respect to improved liquidity and the enhanced informativeness of quotes. Boehmer et al. (2015) supported this prediction on the basis of data from 39 markets.

HFTs are looking for the fastest way to trade. Technological innovations are key factors in making a decision to trade or not. For exchanges and markets, provision of these innovations is a key factor in their competitiveness (and survival). The profitability of the new fast technology of trading is discussed in Brogaard et al. (2014), Cespa and Vives (2013), Pagnotta and Philippon (2011), and Biais et al. (2015). According to Laughlin et al. (2014), the speeding up of communication to about three milliseconds between Chicago and New York markets increased costs significantly. Haldane (2011) stresses the role of the speed of high-frequency trading in these words: "Adverse selection today has taken on a different shape. In a high speed, co-located world, being informed means seeing and acting on market prices sooner than competitors. Today, it pays to be faster than the average bear, not smarter. To be uninformed is to be slow".

Hasbrouck and Saar (2009) underline that technology allows orders to be submitted (and cancelled) instantaneously. The optimum strategies use this option in order to apply complex trading strategies.

One of the most-important questions is whether one can actually link "buy" and "sell" trades with upcoming information (Easley et al., 2012a, 2012b, 2013). In their opinion, the active side of the trade is oriented more to the spread than the actual content of the released information.

O'Hara (2015) stresses that a fundamental change in how traders trade and how markets operate can be observed in recent years. In her opinion, the high-frequency algorithms operate across the market and use the power of technology to forecast price movements of securities. The forecasts take into account the behavior of correlated assets. Thus, the starting point of empirical analyses should be to assess the predictive power of market variables, both within and across markets. The main focus in the future should be oriented towards understanding the changing nature of the market, including understanding the changing nature of market data.

More complete surveys of HFT topics may be found in reviews by Biais and Wooley (2011), Angel et al. (2011), Jones (2012), and Goldstein et al. (2014).

3. Methodology

We shall consider the dynamic parametrization of the conditional mean function (Engle and Russell, 1998):

$$\psi_i := \psi_i(\theta) = E[x_i \mid \mathcal{F}_i; \theta]$$

where $\mathcal{F}_i$ denotes the information set up to observation t_{i-1} (beginning of i-th duration x_i between two events occurred at times t_{i-1} and t_{i-1}) and θ is the vector of parameters.

It is assumed that standardized durations:

$$\varepsilon_i = \frac{x_i}{\psi_i}$$

are independent and identically distributed random variables with $E[\varepsilon_i] = 1$. Variation in autoregressive conditional duration models arise from different choices of functional form for the conditional mean function and choices of distribution of standardized durations.

The most basic specification assumes linear parametrization of the conditional mean function (Engle and Russell, 1998):

$$\psi_i = \omega + \sum_{j=1}^{P} \alpha_j x_{i-j} + \sum_{j=1}^{Q} \beta_j \psi_{i-j}$$

where $\omega > 0$, $\alpha_j \geq 0$, $\beta_j \geq 0$ for all j and $\sum_{j=1}^{P} \alpha_j + \sum_{j=1}^{Q} \beta_j < 1$. The first three constraints ensure that conditional durations are positive, whereas the last inequality ensures the existence of an unconditional mean of duration. Bauwens and Giot (2000) propose two extensions of the linear ACD model. Models called logarithmic ACD are of the forms:

$$\ln\psi_i = \omega + \sum_{j=1}^{P}\alpha_j \ln\varepsilon_{i-j} + \sum_{j=1}^{Q}\beta_j\psi_{i-j}$$

and

$$\ln\psi_i = \omega + \sum_{j=1}^{P}\alpha_j\varepsilon_{i-j} + \sum_{j=1}^{Q}\beta_j\psi_{i-j}$$

We refer to the different specifications as $LACD_1$ and $LACD_2$ respectively. There are no sign restrictions on parameters to ensure the positivity of conditional duration.

For each cited specification, researchers have to choose a distribution for standardized durations. In their seminal paper, Engle and Russel (1998) study exponential and Weibull distributions (it is worth mentioning that the former is used in quasi maximum likelihood estimation).

In this paper, we try to fit generalized gamma and Burr distributions. The density of the generalized gamma distribution (Lunde, 2000) is given as:

$$f(\varepsilon) = \frac{\gamma\varepsilon^{\kappa\gamma-1}}{\theta^{\kappa\gamma}\Gamma(\kappa)}\exp\left\{-\left(\frac{\varepsilon}{\theta}\right)^{\gamma}\right\}$$

with $\theta = \frac{\Gamma(\kappa)}{\Gamma\left(\kappa+\frac{1}{\gamma}\right)}$ and $\kappa, \gamma > 0$. The generalized gamma distribution includes the Weibull distribution for $\kappa = \gamma = 1$ and the exponential distribution if $\kappa = 1$.

Gramming and Maurer (2000) examine the Burr distribution, whose density is as follows:

$$f(\varepsilon) = \frac{\theta\kappa\varepsilon^{\kappa-1}}{\left(1+\sigma^2\theta\varepsilon\kappa\right)^{\frac{1}{\sigma^2}+1}}$$

where $\theta = \sigma^{2\left(1+\frac{1}{\kappa}\right)}\frac{\Gamma\left(\frac{1}{\sigma^2}+1\right)}{\Gamma\left(\frac{1}{\kappa}+1\right)\Gamma\left(\frac{1}{\sigma^2}-\frac{1}{\kappa}\right)}$ with $\sigma > 0$ and κ such that $-\kappa < 1 < \frac{\kappa}{\sigma^2}$.

We get the Weibull distribution if $\sigma \to 0$ and exponential distribution if, additionally, $\kappa = 1$.

A concept that is often used in duration analysis is the hazard function. Assuming that duration X is a continuous random variable, the hazard function is defined as (see, for example, Bauwens and Giot, 2001):

$$h(x) = \lim_{dx \to 0} \frac{P[x \leq X < x + dx \mid X \geq x]}{dx}$$

In the formula above, the numerator is the probability that the event occurs in the interval $[x, x + dx)$, given that it has not occurred before, while the denominator is the width of the interval. The fraction represents the rate of event occurrence per second. Taking the limit as dx goes to 0 we obtain an instantaneous rate of occurrence. This leads to an alternative definition of hazard function of the form:

$$h(x) = \frac{f(x)}{S(x)} = \frac{-dS(x)}{dx}$$

Where $f(x)$ and $S(x)$ are the density and survival functions of random variable X, respectively. It can be shown that the hazard functions of an exponential distribution is flat, while the hazard function of Weibull can be either flat or monotone (increasing and decreasing), depending on the values of the parameters. In the case of Burr and the generalized gamma, the hazard function can take many different shapes (including non-monotone cases), depending on parameter values.

4. Empirical results

We consider tick-by-tick transactions of DAX30 and WIG20 companies. The first dataset contains the prices of 25 companies from 2013.03.22 to 2013.05.17 (37 trading days). The second dataset contains observations from 2013.09.02 to 2013.10.18 (35 trading days) for 12 actively traded Polish companies. To compute price durations, one needs to set up a price threshold. Taking into account the large number of time series, it is not possible to choose one universal price threshold. In addition, the tick sizes of companies are very different (based on upper and lower price bands). To avoid incomparable either short or long duration series we set a price threshold based on the tick size of each company. For most DAX30 companies we set the price threshold as 0.1 euro (which equals a 20 tick size) and different values for Polish companies (this is the result of huge differences in the number of trades and scale of prices). We remove overnight durations and durations corresponding to events recorded outside regular opening hours (9:00 to 17:30 for German and 9:30 to 16:50 for Polish companies).

In Table 1 we present the main descriptive statistics of plain-price durations along with the result of Ljung-Box test with 15 lags.

Table 1

Descriptive statistics of raw-price durations (number of observations (*N*), mean, standard deviation, minimum, quantiles, maximum, and Ljung-Box test statistic)

DAX30					
Statistic	**Min**	**0.25q**	**Median**	**0.75q**	**Max**
N	1626	2772	3071	3570	5802
Mean	198.72	320.89	372.62	410.55	695.58
S.D.	267.2	440.4	537.16	613.92	955.03
Min	1	1	1	1	1
0.25q	36	63	69	83	147
Median	106.5	175.5	196	216.5	381.5
0.75q	252	394	474.25	518	873.75
Max	2778	5194	6332	8514	22975
LB(15)	326.31	672.72	776.62	1111.87	2565.88
WIG20					
Statistic	**Min**	**0.25q**	**Median**	**0.75q**	**Max**
N	1544	1975.75	2316	3382	4470
Mean	218.75	291.68	418.28	491.43	623.31
S.D.	426.03	627.67	894.34	1116.63	1513.92
min	1	1	1	1	1
0.25q	16	17.5	23	35.625	57
Median	65	80	102.5	133.75	226
0.75q	229	280.38	364.63	440.31	674.25
Max	6537	9434.75	12068.5	16952.5	21037
LB(15)	72.23	193.86	293.73	613.96	1507.94

The computation results confirm the stylized facts observed in the duration data. There is an overdispersion and autocorrelation in all series under study. On average, Polish companies exhibit weaker serial correlation yet the highest overdispersion. In both cases, the series of price durations exhibit a diurnal pattern (which may be different for each day of the week). In a way similar to Bauwens and Giot (2000), we take into account the time of day and the day of the week. We use cubic splines with nodes set every 60 minutes with two additional nodes: 10 minutes after the market opens and 10 minutes before the end of the session.

The table 2 presents the descriptive statistics of diurnally adjusted price durations (plain durations divided by seasonal component).

Table 2

Descriptive statistics of adjusted price durations (number of observations, mean, standard deviation, minimum, quantiles, maximum, and Ljung-Box test statistic)

DAX30					
Statistic	**Min**	**0.25q**	**Median**	**0.75q**	**Max**
N	1626	2772	3071	3749,25	5802
Mean	1.017	1.027	1.036	1.042	1.064
S.D.	1.096	1.186	1.214	1.248	1.634
min	0.001	0.001	0.002	0.002	0.003
0.25q	0.194	0.241	0.258	0.272	0.299
Median	0.521	0.622	0.636	0.653	0.7
0.75q	1.193	1.347	1.364	1.375	1.422
Max	8.817	11.74	13.689	15.131	25.356
LB(15)	257.79	410.408	458.694	667.238	3730.499
WIG20					
Statistic	**Min**	**0.25q**	**Median**	**0.75q**	**Max**
N	1544	1975.75	2316	3382	4470
Mean	1.01	1.022	1.034	1.042	1.08
S.D.	1.745	1.928	2.005	2.121	2.42
min	0.001	0.001	0.001	0.002	0.003
0.25q	0.051	0.065	0.087	0.11	0.14
Median	0.253	0.306	0.35	0.408	0.459
0.75q	0.983	1.104	1.152	1.215	1.292
Max	19.666	28.698	32.101	36.076	70.971
LB(15)	107.421	112.962	357.678	511.005	1224.436

With this procedure, the mean of adjusted durations is close to 1. It can be seen that seasonal adjustment reduces overdispersion and autocorrelation.

We estimate (by the maximum likelihood estimation method) models being combinations of the parametrization of conditional mean functions and distributions (with a total number of 12 different models, the models are restricted with lag order of $P = Q = 1$)[2]. We select a model that best fits in several ways. First, we restrict our attention to models that "remove" autocorrelation (we apply the Ljung-Box test to residuals and their squares). Denoting by $f_i(x_i \mid \mathcal{F}_i)$ the sequence of one-step-ahead density forecasts, we calculate the probability integral transform PIT (Diebold et. al, 1998):

$$z_i = \int_{-\infty}^{x_i} f(u)\,du$$

and apply the Anderson-Darling and Cramer von Mises GOF test. Finally, we choose a model associated with the smallest BIC and "significant" parameters (details of estimation results are available from the authors upon request). In Tables 3 and 4, we present the estimation results and the testing for "best" autoregressive conditional duration models for all companies under study.

Restriction $P = Q = 1$ in all specifications is sufficient to describe clustering in price duration series. It follows from the values of the Ljung-Box test statistics for residuals (with lowest p-values of 0.09 [Thyssen] and 0.14 [JSW]), and the results of this test applied to squared residuals (lowest p-values: 0.06 [Allianz] and 0.73 [KGHM]). In most cases, $LACD_1$ models fit the best. The $LACD_1$ model implies that the concave news impact curve (relationship between ε_{i-1} and x_i) is asymmetric and that the difference in the impact of innovations with $\varepsilon_i < 1$ is larger than with $\varepsilon_i > 1$ (Hautsch, 2003). In one case (BASF), the outcome is the opposite. Sum $\alpha + \beta$ for linear parametrization (four cases) equals at least 0.93 and confirms the clustering of durations (this implies a slowly decreasing autocorrelation function). For logarithmic parametrization, clustering increases with parameter β. Comparing results (using quartiles for parameters β), we can conclude that these values are a little higher for Polish companies.

Given the results of Anderson-Darling and Cramer von Mises testing, we conclude that the assumed distributions are correct, with a strong rejection of both exponential and Weibull distribution.

In about two-thirds of the cases, the Burr distribution fits better than the generalized gamma distribution.

Regarding the parameters of these distributions, in only two cases (Linde and Kernel) is the hazard function monotone (decreasing starting at ∞). In all of the remaining cases, the hazard function has an inverted-U shape. Analyzing parameters of generalized gamma distributions, we find one case that corresponds to a U-shaped hazard function (PGNiG), starting at ∞, and tending to ∞ as ε tends to ∞ (Bauwens and Giot, 2001). In all of the remaining cases, the hazard function has an inverted-U shape.

[2] We use R environment and package ACDm for all computations (https://cran.r-project.org/web/packages/ACDm/index.html)

Table 3

Estimation and testing results for DAX30 companies

Company	Model	Dist.	LB(15)	LB2(15)	A-D	CvM	ω	α	β	κ	μ
Adidas	LACD1	Burr	12.66	7.09	0.60	0.59	0.09	0.16	0.92	1.13	0.29
Allianz	LACD1	Burr	16.87	24.12	0.91	0.95	0.09	0.16	0.86	1.07	0.19
BASF	LACD2	GG	19.88	20.32	0.12	0.17	–0.16	0.16	0.94	1.48	0.75
BAYER	LACD1	Burr	16.68	10.55	0.89	0.86	0.11	0.20	0.89	1.15	0.27
Beiersdorf	LACD1	GG	15.98	7.79	0.11	0.13	0.07	0.12	0.91	1.28	0.81
BMW	LACD1	GG	6.11	2.94	0.80	0.84	0.09	0.17	0.85	2.54	0.59
Continental	ACD	Burr	18.54	4.75	0.92	0.92	0.07	0.17	0.76	1.05	0.14
Daimler	LACD1	Burr	11.80	4.65	0.86	0.79	0.12	0.23	0.77	1.13	0.22
Deutsche Bank	LACD1	Burr	13.23	16.19	0.50	0.59	0.09	0.22	0.84	1.23	0.23
Deutsche Börse	LACD1	GG	11.22	7.63	0.08	0.10	0.08	0.14	0.87	1.48	0.74
Deutsche Post	LACD1	Burr	7.66	1.43	0.80	0.86	0.10	0.17	0.85	1.08	0.21
Deutsche Telekom	LACD1	Burr	13.21	9.36	0.74	0.70	0.10	0.19	0.90	1.13	0.24
Deutsche_Lufthansa	LACD1	GG	17.58	2.79	0.17	0.20	0.10	0.18	0.87	2.20	0.61
EON	LACD1	GG	19.74	6.45	0.50	0.43	0.11	0.19	0.95	3.55	0.46
Fresenius SE	LACD1	GG	21.51	16.56	0.19	0.20	0.08	0.13	0.88	1.23	0.84
Fresenius Medical Care	LACD1	Burr	19.07	5.29	0.68	0.63	0.09	0.15	0.93	1.06	0.21
Heidelberg Cement	LACD1	GG	22.56	13.59	0.15	0.20	0.06	0.12	0.90	1.41	0.79
Henkel	LACD1	Burr	13.57	5.22	0.76	0.83	0.10	0.18	0.85	1.13	0.25
Infineon	LACD1	Burr	15.31	7.03	0.79	0.76	0.13	0.23	0.80	1.09	0.25
K&S	ACD	Burr	15.90	12.03	0.77	0.73	0.07	0.20	0.73	1.07	0.19
Lanxess	LACD1	Burr	10.11	14.50	0.76	0.75	0.10	0.18	0.86	1.09	0.25

Table 3 cont.

Company	Model	Dist.	LB(15)	LB2(15)	A-D	CvM	ω	α	β	κ	μ
Linde	LACD1	Burr	20.27	10.50	0.87	0.97	0.09	0.13	0.91	0.97	0.16
SAP	LACD1	Burr	16.82	4.31	0.96	0.92	0.13	0.24	0.89	1.13	0.28
Siemens	LACD1	Burr	14.49	5.42	0.22	0.22	0.11	0.19	0.91	1.11	0.25
Thyssen	LACD1	Burr	22.60	3.76	0.45	0.40	0.08	0.14	0.92	1.10	0.26

LB(15) denotes the value of the Ljung-Box test statistics applied to residuals, while LB2(15) that applied to squared residuals, A-D and CvM are p-values in GOF testing, parameter μ refers to γ for generalized gamma distribution, and σ^2 for Burr distribution.

Table 4

Estimation and testing results for WIG20 companies

Company	Model	dist.	LB(15)	LB2(15)	A-D	CvM	ω	α	β	κ	μ
PKOBP	LACD1	GG	20.77	1.23	0.62	0.51	–0.17	0.16	0.92	3.42	0.36
PZU	LACD1	GG	4.89	0.49	0.11	0.10	0.35	0.29	0.85	15.02	0.15
KGHM	LACD1	GG	20.94	11.33	0.22	0.32	0.15	0.19	0.91	3.54	0.39
PEKAO	LACD1	GG	20.61	4.44	0.47	0.50	0.20	0.21	0.91	3.29	0.36
PKNORLEN	LACD1	GG	12.66	0.41	0.12	0.08	0.15	0.14	0.93	3.81	0.32
PGE	LACD1	GG	17.88	0.67	0.10	0.10	0.28	0.27	0.84	5.63	0.26
PGNIG	ACD	GG	9.13	2.41	0.10	0.10	0.06	0.28	0.70	2.24	0.38
TPSA	LACD1	GG	15.43	4.72	0.09	0.12	0.23	0.20	0.89	4.39	0.28
ASSECOPOL	LACD1	GG	11.06	2.83	0.18	0.18	0.10	0.11	0.95	1.45	0.56
JSW	ACD	GG	20.98	5.76	0.14	0.13	0.03	0.12	0.86	2.87	0.41
KERNEL	LACD1	Burr	8.50	1.46	0.16	0.20	0.10	0.10	0.94	0.77	0.12
LOTOS	LACD1	GG	12.73	1.76	0.19	0.22	0.12	0.12	0.97	2.71	0.39

LB(15) denotes the value of the Ljung-Box test statistics applied to residuals, while LB2(15) that applied to squared residuals, A-D and CvM are p-values in GOF testing, parameter μ refers to γ for generalized gamma distribution, and σ^2 for Burr distribution.

5. Conclusions

To conclude, logarithmic ACD models are useful tools for describing transaction processes on the Frankfurt and Warsaw Stock Exchanges.

The conducted empirical analysis of raw price duration for selected companies listed on the DAX30 and WIG20 shows that, on average, Polish companies exhibit weaker serial correlation yet higher overdispersion than German companies. Both statistics depend on quantiles. For higher quantiles they become larger. This dependence is weaker in the case of adjusted price durations with respect to overdispersion, and is more visible in autocorrelation. In addition, the dependence of autocorrelation on the quantiles of raw data is much more pronounced in Polish than in German price durations.

The fitted ACD model for price durations for almost all companies listed on both stock markets under study is LACD1. While for most German companies on the DAX30, the Burr distribution fits better than generalized gamma distribution, the latter distribution fits well in the case of Polish blue chips. Analyzing series by hazard function, we note a similarity of hazard functions for companies from both markets, the functions in general displaying a U-shaped pattern.

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