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IDENTIFICATION OF FLEXIBLE STRUCTURE MODEL FOR PIEZOELECTRIC VIBRATION CONTROL

SUMMARY

Modern structures are light and compliance. Therefore, they are strongly influenced by external and internal perturbations. To reduce the structure vibrations the active vibration controls systems are designed. When active control of flexible structures is sought, three items must be individually and simultaneously considered.

1) The creation of reduced-order models.
2) Placement of sensors and actuators.
3) Countermeasures for spillover.

All these problems are particularly important in the case of active system with parameter-distributed piezoelectric sensors and actuators. The glued piezoelectrics and their amplifiers change the structure parameters and should be taken into account in the model of the open-loop system.
In the paper we consider an identification method which leads directly to the reduced-order model of the open-loop system. Analytical and experimental investigations were carried out for flexible beam with fixed one end and with opposite free end. Piezoelectric strips were glued on both sides of the beam: one works as a sensor and second one as an actuator. It means that we have designed single input single output system.
It is appeared that used amplifier works as a low-pass filter which cuts-off the high-frequency dynamic effects. The frequency response function (FRF) was recorded and model of open-loop system was adjusted to experimental results. Taking into account the theory of reduced-order model of flexible structures we can state that resonances are connected with poles while anti-resonances with zeros of system transfer function. The resonance and anti-resonance frequencies were used directly in the model while damping coefficients were calculated by using of the least-square sum method.
In the next step some control laws were used to damp the vibration of the beam. The proper control law was chosen with help of computer simulations. The active control loop rapidly accelerates the vanishing of the impulse-excited transient vibrations of the structure. Experimental results confirmed the analytical considerations.

***Keywords**: vibration control, piezo-electrics, identifications, control laws*

IDENTYFIKACJA MODELU PODATNEJ KONSTRUKCJI MECHANICZNEJ NA POTRZEBY PIEZOELEKTRYCZNEGO UKŁADU STEROWANIA DRGANIAMI

W artykule opisano badania eksperymentalne aktywnego układu tłumienia drgań na przykładzie konstrukcji modelowanej jako belka. Elementem pomiarowym i elementem wykonawczym są wstęgi piezoelektryczne. W wyniku zastosowanej procedury identyfikacyjnej otrzymano model matematyczny układu otwartego. Różni się on w sposób istotny od znanych modeli analitycznych. Zaprojektowano dwa prawa sterowania. Dla jednego z nich zbadano eksperymentalnie działanie aktywnego układu tłumienia drgań. Wykazano, że poprawnie przeprowadzony proces identyfikacji dynamiki obiektu w znakomity sposób upraszcza proces projektowania układu sterowania drganiami.

1. INTRODUCTION

Present-day, the light and flexible structures are realized to save materials and energy and to increase the efficiency of the devices. Unfortunately, such structures are open to influence of external and internal excitations what results in vibrations of the structure. The vibrations deteriorate the operation performance and at the extreme end they can lead to the catastrophe. It is a cause why many specialists during last years were looked for vibration active control systems of such structures as: robots, vehicles, rotating machinery, tall buildings, bridges, antennas and solar batteries in space satellites. So more, such control systems are widely applied since now on the market we have the reliable control system components: sensors, actuators and digital controllers. Particularly, the development of actuators: pneumo- hydro- electro- servomechanisms, magnetic bearings, piezoelectric, end so on, was important.

The design procedure of active vibration control system consists of a six stages.

1) Eduction of mathematical model of flexible structure with help of analytical or numerical methods (for example – finite element method).
2) Analysis of the structure dynamics (the calculation of eigen frequencies, modes, nodal points, stability, time and frequency responses).
3) Reduction of the structure model to obtain the simplified model of the open-loop system.
4) Designing of the control law which shapes the proper dynamics of the reduced closed-loop system.
5) Computer simulation of closed-loop system with full model.
6) Experimental verification of the closed-loop system.

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The main problem that goes with reduced-order modeling of a structure with controlled vibrations is the spillover problem described by Balas [1]. It is the phenomenon of interference between the controlled modes and the omitted uncontrolled modes. While injecting control energy to the controlled modes, the actuator can also excite higher modes that are not modeled for control. This is called control spillover. On the other hand, sensors can pick up information related to both controlled and uncontrolled modes – it is so called measurement spillover.

In the literature we can find the following methods of the model reduction.

- Modal filtration methods [2, 3], which need of a state estimation.
- Co-location of sensors and actuators [3, 4]. In such case we usually design local (single input single output) control systems basing on input-output models.
- Robust control methods [5], where the omitted dynamics of the structure is considered as a disturbance model took into account during robust controller design.
- Reduction methods of physical model [6, 7]. In this case we design the model of lamped masses connected by flexible-damping elements. The control law is also designed in the physical space where physical states are directly measured.

In the paper we consider a modified version of the last method. The reduced model will be obtained during the identification procedure of the open-loop system. As a plant we will use an uniform beam with one side fixed and with free second end.

2. MODAL REDUCTION OF THE STRUCTURE MODEL

Classic motion equations of flexible or mass-lumped structure with a finite number of degrees of freedom are as follows

$$\mathbf{M}\ddot{x} + \mathbf{C}\dot{x} + \mathbf{K}x = f \tag{1}$$

where:

x and f – the vectors of generalized displacements (translations and displacements) and forces (point forces and torques),

$\mathbf{M}$, $\mathbf{K}$ and $\mathbf{C}$ – respectively the mass, stiffness, and damping matrices, they are symmetric and semi positive definite.

$\mathbf{M}$ and $\mathbf{K}$ arise from the discretization of the structure, usually with finite elements. A lumped mass system has diagonal mass matrix while the finite element method usually leads to non-diagonal mass matrix. In the complex machine tools the lamped mass system also have non-diagonal mass matrix [10].

The damping matrix $\mathbf{C}$ represents the various dissipation mechanisms in the structure, which are usually poorly known. Therefore, it is customary to assume hypothesis about Rayleigh damping: $\mathbf{C} = \alpha\mathbf{M} + \beta\mathbf{K}$, where coefficients α, β are selected to fit the structure under consideration.

Let us introduce modal transformation: $x = \Phi z$, where z is the vector of modal coordinates. The transformation leads to decoupled modal equations

$$\ddot{z} + 2\xi\Omega\dot{z} + \Omega^2 = \mu^{-1}\Phi^T f \tag{2}$$

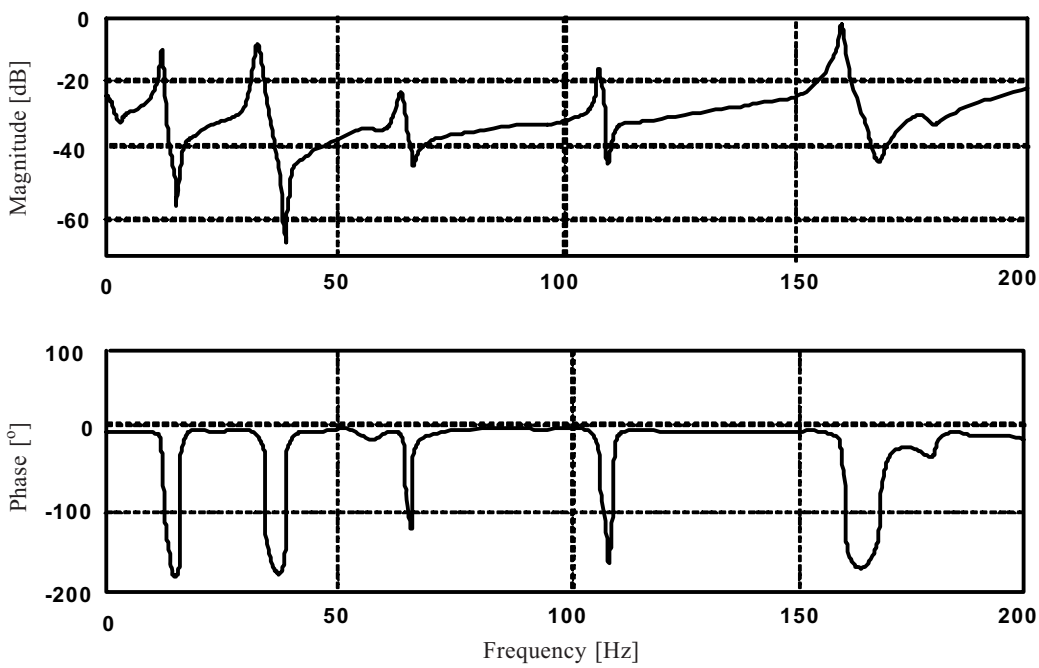

Fig. 1. Typical frequency characteristics of a mechanical structure

where:

$\Omega = \mathrm{diag}(\omega_i)$ – matrix of natural frequencies,

$\mu = \mathrm{diag}(\mu_i)$ – matrix of modal masses,

$\xi = \mathrm{diag}\left(\xi_i = \frac{1}{2}\left[\frac{\alpha}{\omega_i} + \beta\omega_i\right]\right)$ – matrix of model damping.

The transfer function between the force f as an input and z as an output is in the following form

$$G(j\omega) = \left[-\omega^2\mathbf{M} + j\omega\mathbf{C} + \mathbf{K}\right]^{-1} = \\ = \sum_{i=1}^{n} \frac{\phi_i\phi_i^T}{\mu_i\left(\omega_i^2 - \omega^2 + 2j\xi_i\omega_i\omega\right)} \tag{3}$$

Typical Bode characteristics of the transfer function (3) are shown in the Figure 1.

For control purposes we usually reduce the model to m lowest modes

$$G(\omega) = \sum_{i=0}^{m} \frac{\phi_i^2(k)}{\mu_i\left(\omega_i^2 - \omega^2\right)} + R \tag{4}$$

and residual modes are

$$R = \sum_{m+1}^{n} \frac{\phi_i\phi_i^T}{\mu_i\omega_i} \tag{5}$$

Residual modes in the form (5) cause that transfer function (4) is non-proper function [8]. It leads to big differences between zeros of transfer functions (3) and (4).

For $\omega = 0$ we obtain from (3) the modal expansion of the static flexibility matrix

$$G(0) = \mathbf{K}^{-1} = \sum_{i=1}^{n} \frac{\phi_i\phi_i^T}{\mu_i\omega_i} = \Phi\left(\Phi^T\mathbf{K}\Phi\right)^{-1}\Phi^T \tag{6}$$

One of the aims of vibration control system of machine tools is to make the static flexible matrix elements as small as possible.

3. PIEZOELECTRIC VIBRATION CONTROL SYSTEM

The main problem in structure vibration control is the availability of the actuators with sufficiently high control force and wide bandwidth. For vibration control of rotating parts (rotors) we use usually the magnetic bearings or linear piezoelectric actuators. For non-rotating structure to vibration control we use "structure borne" actuators as: reaction wheels, control moment gyros, proof-mass actuators, piezo strips, etc. For the further considerations we take into account the piezoelectric strip as an actuator and as a measurement unit.

Consider cantilever beam as a model of a flexible structure with piezoelectric actuator and sensor as in Figures 2 [9].

The beam with actuator and sensor forms an open-loop system. The open-loop system was identified in the frequency domain. The amplitude-frequency characteristics are shown in Figure 3. We can notice the resonances for frequencies: 10, 60, 180 [Hz] and the anti-resonances for frequencies: 1.3, 20 i 150 [Hz]. We adjusted mathematical model (Fig. 4) by proper choice of the damping coefficients ξ. The lead member of the transfer function is connected with dynamics of the amplifier used to supply the piezoelectric actuator.

The Bode characteristics of the transfer function from Figure 4 are presented in Figure 5. We used MATLAB and SISO Design Tool to consider the influence of different controllers on the dynamics of closed-loop system. The characteristics of closed-loop system with inertial controller (Fig. 7) are presented in Figure 6. Comparing Figures 5 and 6 we can notice that the reduction of the highest peak of amplitude approach 30 [dB].

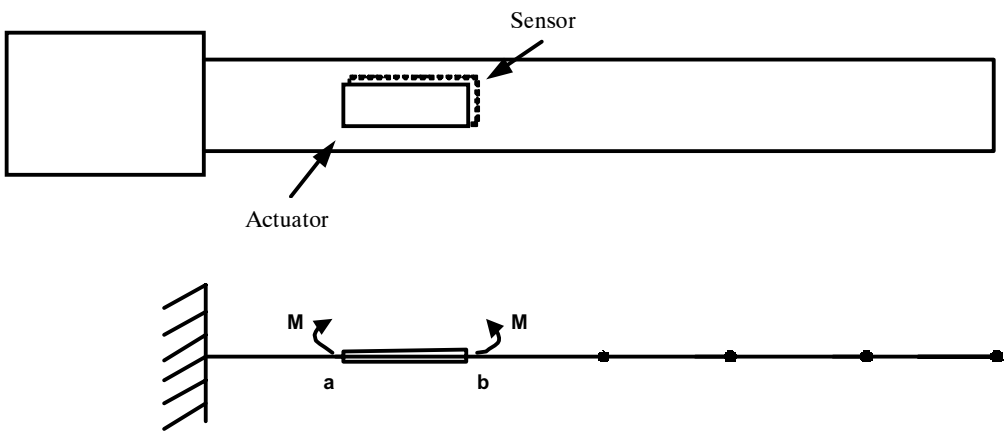

Fig. 2. Beam with piezoelectric actuator and sensor

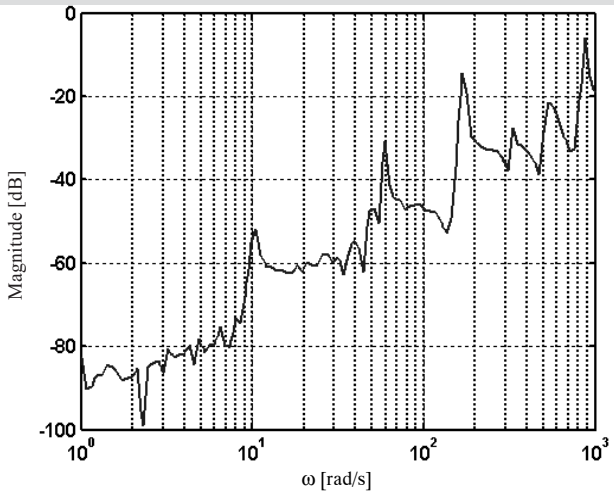

Fig. 3. Recorded amplitude-frequency characteristics of the open-loop system

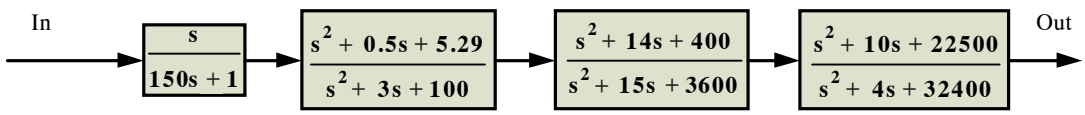

Fig. 4. Transfer function of the open-loop system

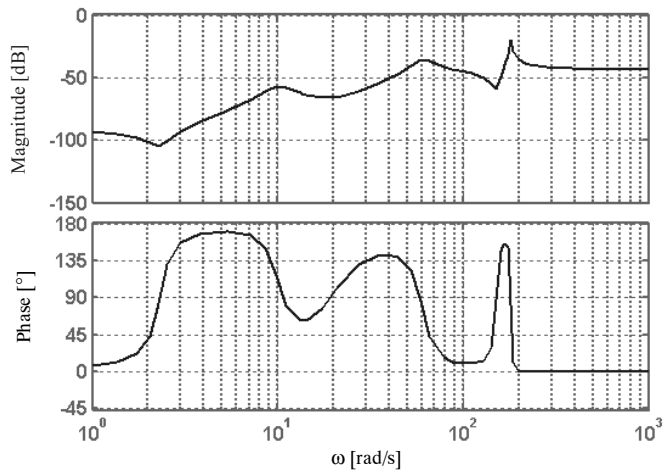

Fig. 5. Bode characteristics of the identified model of open-loop system

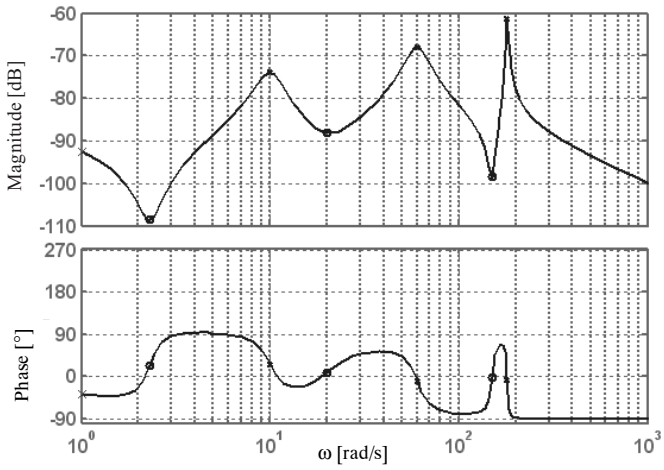

Fig. 6. Bode characteristics of the system with inertial controller

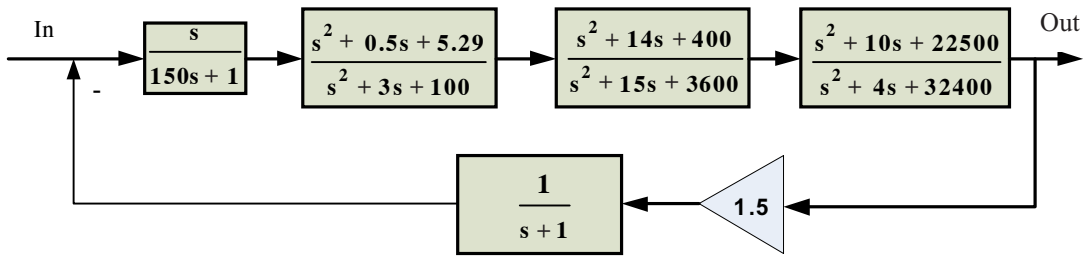

Fig. 7. Closed-loop system with inertial controller

For comparison the second-order filter was used as a controller (Fig. 8). The Bode characteristics of the closed-loop system with such controller are given in Figure 9.

The second-order filter in the feedback loop assures stronger damping and bigger roll-off (–40 dB/dek.) than in the case of inertial controller. But its drawback is connected with the increasing sensitivity of the system stability to the parameters of identified model. We can notice it by the comparison of root locus plots for both controllers: inertial (see Fig. 10) and filter (see Fig. 11).

The closed-loop system with inertial controller is always stable since branches of plots are in the left half--plane for all controller gains. System with second-order filter becomes unstable for some controller gains. For non-correct model the system designed as stable one can appear unstable.

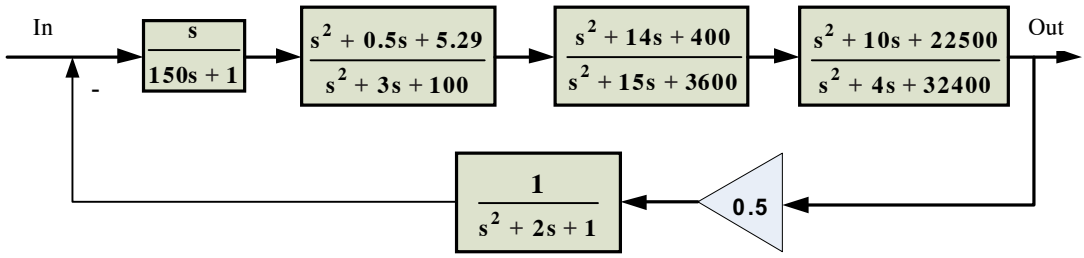

Fig. 8. Model of system with second-order filter as the controller

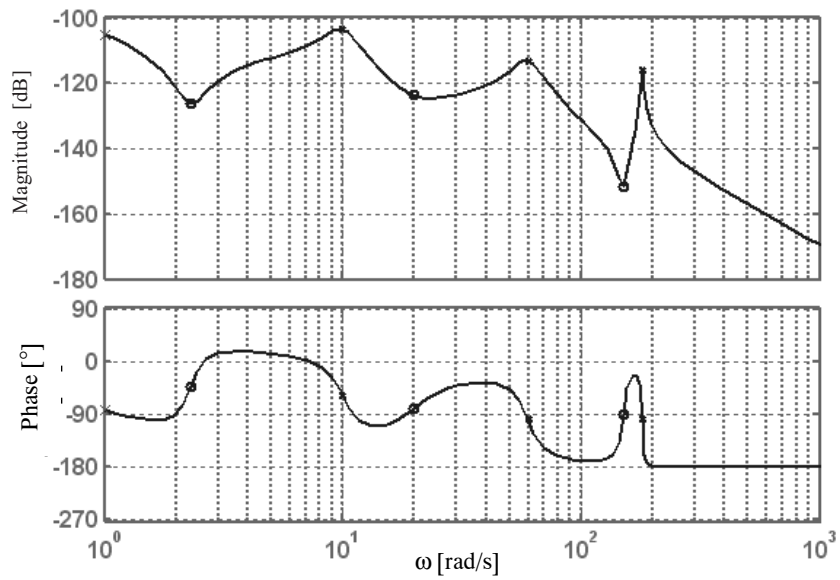

Fig. 9. Bode characteristics of the closed-loop system with second-order filter

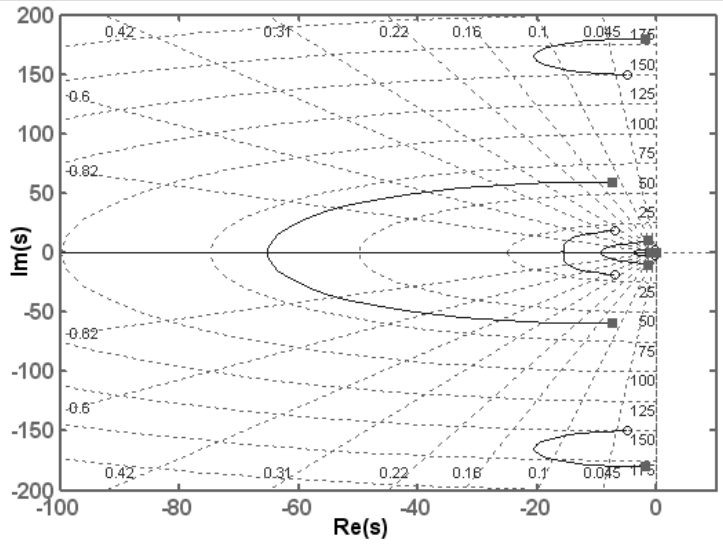

Fig. 10. Root locus for the system with inertial controller

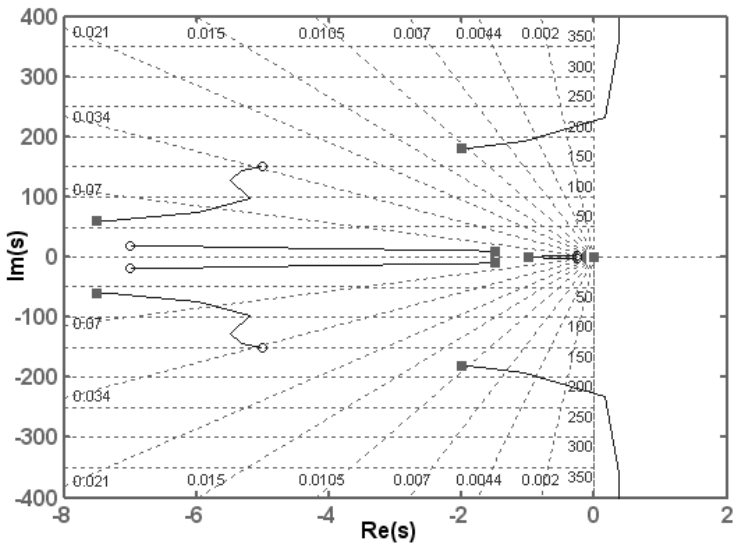

Fig. 11. Root lotus of the system with second-order filter

4. EXPERIMENTAL RESULTS

The inertial controller was verified in experimental way by using of it in the vibration control system of beam from Figure 2. The control system was lock-in during transient response and the results are seen in Figure 12. It was calculated that logarithmic decrement of damping increased from $\delta = 0.2$ to $\delta = 0.51$.

The experimental amplitude-frequency characteristics are presented in Figure 13. We see the high level of the damping in the whole range of frequency. The characteristics are similar to the respective characteristics of the simulated closed-loop system from Figure 6.

5. CONCLUSIONS

In the paper we have described the investigations of the vibration control system applied to the uniform beam. Piezo strips were used as a sensor and as an actuator. There are many causes which influence on the transfer function of the open-loop system. Reduced model strongly changes zeros of the transfer function. The piezo strips increase the stiffness of the beam. The amplifier works as an integral member and introduces the lead component to the transfer function. So more, the amplifier damp the higher frequency components of the Bode characteristics. As a result, the true transfer function is far from its analytical model.

Therefore, it is much better to use the identification procedure to obtain the correct mathematical model of the open-loop system. We can use the measured resonances and anti-resonances directly in the model as poles and zeros, respectively, while damping coefficients can be manually adjusted or calculated by the least square sum method. Such identification procedure is very simple and gives perfect results.

Simple inertial controller is robust against the parameter changes of the open-loop system.

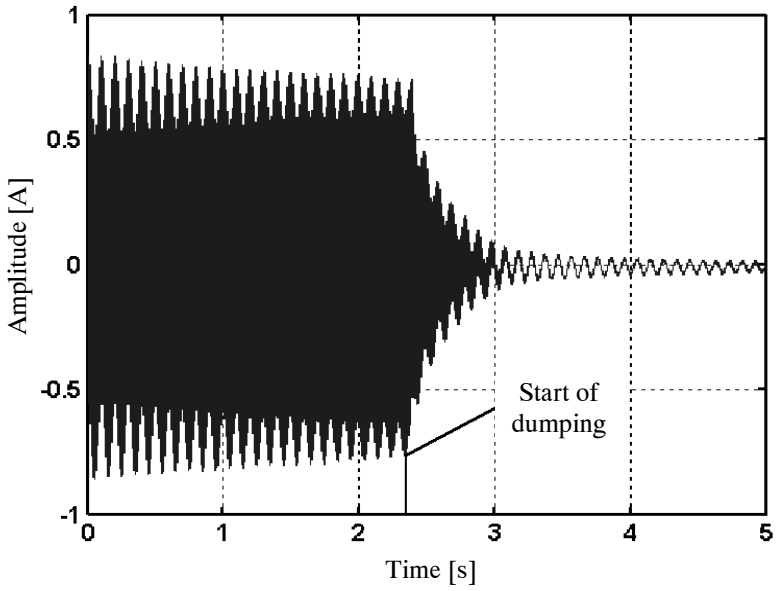

Fig. 12. An impulse response of experimental beam.
The control system starts after 2.4 s

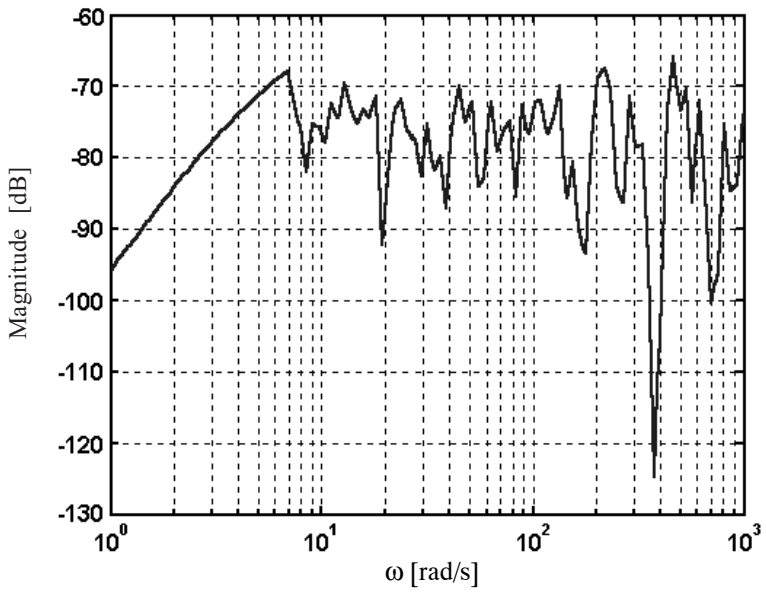

Fig. 13. Amplitude-frequency characteristics of the closed-loop system with inertial controller recorded on the lab stand

References

[1] Balas M.J.: *Feedback Control of Flexible Systems.* IEEE Transactions on Automatic Control, vol. AC-23, No. 4, 1978, 673–679

[2] Gosiewski Z.: *Aktywne sterowanie drganiami wirników.* Monografia nr 27, Koszalin, Wydawnictwa Uczelniane WSInż. 1989, 133

[3] Merowitch L.: *Dynamics and Control of Structures.* New York, Wiley 1990

[4] Balas M.J.: *Direct Velocity Feedback Control of Large Space Structures.* J. of Guidance, Control and Dynamics, 2–3, 1979, 252–253

[5] Dorato P. *et al.*: *Robust Control for Unstructured Perturbations.* An Introduction Lecture Notes in Control and Information Sciences, 168, 19, Springer-Verlag, 1992

[6] Seto K., Mitsuta S.: *A new method for making a reduced order model of flexible structures using unobservability and uncontrollability and its application in vibration control.* Proceedings 1st MOVIC, vol. 1, Tokyo, Japan, 1992, 152–158

[7] Seto K.: *Modeling Strategy Suited to Motion and Vibration Control for Flexible Structure.* Proceedings 7th MOVIC, St. Louis, USA, 2004, CD paper#25

[8] Preumount A.: *Vibration Control of Active Structures*

[9] Zientalski K.: *Piezoelektryczny układ sterowania drganiami belki.* Praca magisterska wykonana pod kier. Z. Gosiewskiego, Warszawa, WAT 2004

[10] Gosiewski Z., Szafarczyk M.: *Mechatronic Approach to the Machine Tool Design.* Proc. of 4th Int. Seminar and Workshop „Engineering Design in Integrated Product Development", Rydzyna, Poland, 2004, 165–172