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SUBOPTIMAL MODEL PREDICTIVE CONTROL FOR MIMO ACTIVE SUSPENSION

SUMMARY

The paper presents nonlinear Model Predictive Control algorithm applied to full-active four-dimensional magnetic bearing system MBC500 being produced by Magnetic Moments, USA. The system is nonlinear due to the measurement units and the current amplifiers (actuators). Nonlinear model of the system has been established basing on the phenomenological description with parameters identified. The model is used to predict response of the suspension system. The control law bases on this prediction as it is in the linear case. However, in the nonlinear case this control law is suboptimal because it assures only that the states and controls are feasible. The feasibility means that the rotating shaft-ends deviation form the nominal position is below the limit as well as the control signals do not exceed constraints. The performance of the active suspension control system is compared with lead compensators being build in the MBC500 system as well as with weighted minimumvariance control and linear quadratic regulator with observer. It is shown that nonlinear MIMO predictive control assures significant improvement of the control quality.

***Keywords**: active suspension, model-based predictive control, nonlinear models, identification*

SUBOPTYMALNY ALGORYTM STEROWANIA PREDYKCYJNEGO W WIELOWYMIAROWYM UKŁADZIE ZAWIESZENIA AKTYWNEGO

W pracy przedstawiony jest nieliniowy algorytm sterowania predykcyjnego zastosowanego do aktywnego tłumienia drgań w zawieszeniu magnetycznym laboratoryjnego modelu maszyny wirnikowej MBC500 firmy Magnetic Moments. Sterowany obiekt jest nieliniowy z uwagi na hallotronowy pomiar położenia końców wirującego wałka oraz nieliniową charakterystykę wzmacniaczy prądowych zasilających układ elektromagnetyczny. Parametry nieliniowego modelu fenomenologicznego zostały zidentyfikowane specjalną procedurą optymalizacyjną. Za pomocą modelu wyznacza się predykcję wyjścia układu drgającego podczas wirowania wału. Predykcję wykorzystuje się do obliczania sterowania. Nieliniowość powoduje jednak, że tak sformułowane prawo sterowania nie jest optymalne ze względu na kwadratowy wskaźnik jakości. Jednakże dopuszczalność rozwiązania w sensie spełnienia ograniczeń na przemieszczenia końców wałka zapewnia stabilność układu. Jego działanie porównane jest w pracy z działaniem wbudowanych regulatorów PD, regulatora minimalizującego wariancję oraz regulatora liniowo-kwadratowego.

1. INTRODUCTION

The essential control problem in rotary machines technology is rejection of periodical disturbances of rotating shaft position. The disturbances are generated by eccentric centrifugal forces with frequency following from angular speed. The active contraction system usually uses specially designed electromagnetic units generating reactive forces. In so-called fully active suspension systems the magnetic bearings are used. Nevertheless the technological solutions, the key issue of active positioning system performance is the control algorithms. The paper presents new predictive approach to control system design. The presentation concerns magnetic levitated system but the approach can be used for any active vibration system. The motivation for the choice of predictive methodology is a straightforward possibility of inclusion of nonlinear models with constraints into the design procedure.

Magnetic levitated turbine MBC500 [1] is the laboratory model of rotary machine with magnetic bearings, assembled by Magnetic Moments LLC (USA) – see Figure 1. Different control problems concerns the MBC500 model as rotating shaft-ends stabilization, reduction of shaft vibration, angular velocity control, optimal energy consumption by drifting system, vibration control while resonance approaching etc.

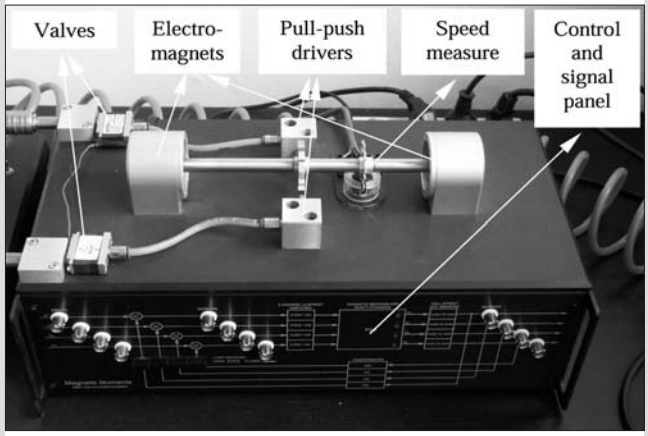

Fig. 1. MBC500 levitating system with angular velocity engine and control

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This model serves as the example of a complex system with several control problems to be solved. The paper addresses mainly the problem of shaft-ends position stabilization while shaft rotates. Here, fully active suspension system using magnetic bearings is applied.

The unit consists of two active radial magnetic bearings and a supported rotor, mounted on a special case. The shaft is actively positioned in the radial directions at the shaft ends, providing 4 degrees of freedom (x_1, x_2, y_1, y_2) – see Figure 2. The position is measured in 4 axes using hallotrons. The system includes four linear current amplifiers and four linear lead-lag compensators, which control the radial bearing axes (PD compensators). MBC500 has been enhanced with pneumatic push-pull driver and angular velocity control unit. The built-in controllers can be replaced by the outer ones using connectors mounted on the front panel (Fig. 1).

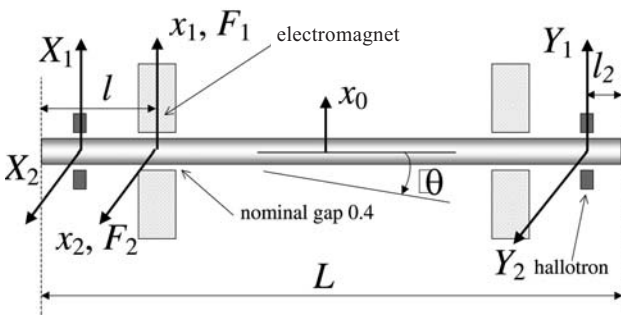

Fig. 2. Schematic diagram of the research unit MBC500

The block diagram of the MBC500 unit is presented in Figure 3. The electromagnet and measure unit are two nonlinear blocks. Other parts of the system can be described as linear. Detailed description of the first principle models is given in [1] (or [2]). The essential difficulty of the control algorithm design is non-linearity, inner feedback from distance x to electromagnetic force F (see Fig. 3), and hard constraints (x can not reach its limits, see Figure 2 – nominal gap is 0.4 mm). An *ad hoc* controllers, PD compensators, shaping dynamics of measure voltage v_s into control voltage v_c, being built in the unit provide stabilization of the shaft-ends position within the nominal gap for angular speed up to about 200 rps. For a higher speed shaft-ends reach limits. This is very danger and certainly has to be avoided due to possible damage. The situation can be even more difficult if eccentrics are greater due to shaft bias – this can reduce maximal speed much below the limit of 200 rps.

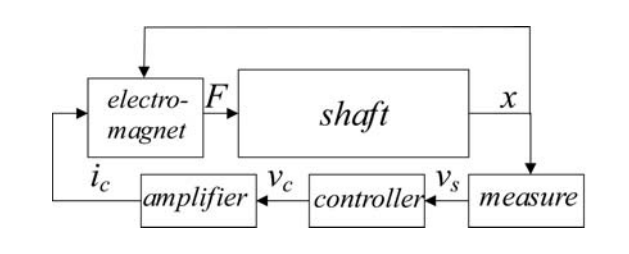

Fig. 3. Dynamical elements of MBC500

Probably the most important disadvantage of the four autonomous PD compensators is neglecting inter-channel couplings. This is yet another difficulty of the control algorithm design. To make the speed possibly greater the control algorithms should be improved.

2. MBC500 MODEL AND LINEAR CONTROL STRATEGIES

Frequency analysis of the first principle model given in [1] allows the lead compensator design. The compensator can be interpreted as limited structure controllers. Indeed, the transfer function of the controller is as follows

$$v_c(s) = \frac{1.41\left(1+8.9\cdot 10^{-4}s\right)}{\left(1+3.3\cdot 10^{-4}s\right)\left(1+2.2\cdot 10^{-5}s\right)} v_s(s) \tag{1}$$

The range of dynamics shaping using the relation (1) is much limited because of only four parameters tunable. On the other hand increasing the order of the controller makes it much dependent upon the unit model. Thus the problem of robustness arises. It is shown in [1] that further improvement of disturbances rejection can be achieved by special design of filters followed from frequency resonance analysis. The sensitivity of the filtering follows from its notch form. It is shown in [3] that resonant frequency depends much on parameters of the unit. It becomes obvious that model of the unit is the key issue in design a control algorithm.

The first trial of the controller improvement is done in [4]. Weighted Minimum Variance (WMV) control is applied. To derive controller polynomials a linear model of the control path is necessary. However, no straightforward identification experiment is possible. The open loop system is unstable, thus a stabilizing control is necessary. The only in-build PD controller can be used. This is however continuous control. The identification problem of discrete model of the control path being looped with continuous feedback is detailed is [4]. The model obtained serves as initial for WMV controller synthesis. To improve the performance adaptation of the model has been imposed. Updated parameters of the controllers improves the overall performance of the system. The most important disadvantage of the WMV approach is the orders of the controller polynomials which was almost 20. This followed form the necessity of retrieving respect transfer functions from the closing loop formulae which increases the order. All order-reduction trials deteriorated the model quality and as well as the resulting system performance.

The second approach is Linear Quadratic Regulator (LQR), a classical linear methodology. It is well known that LQR is the most effective control method for linear systems. The essential advantage of LQR is that closed loop stability is assured. On the other hand all states has to be measured. The system structure (Fig. 3) shows that direct state measurement is not possible and using an observer is necessary. The proper choice of the observer is very difficult.

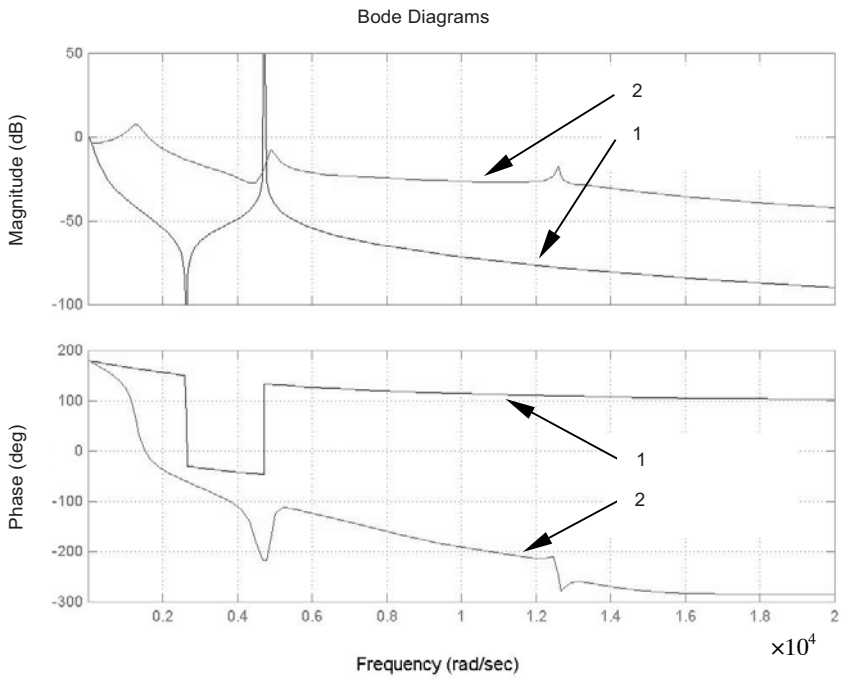

Fig. 4. Frequency responses: 1 – linear model; 2 – phenomenological models

Even if one neglects couplings between states x_i and y_i (it can be easily proved that such assumption does not deteriorates modeling [2]) and leave couplings between x_1 and x_2 (y_1 and y_2 respectively) then number of states for x axes is 10: two states of two first order inertia of amplifiers and eight states of the two degree of freedom shaft. The first demand is stability of the observer. Ten degree of the observer matrix makes the design difficult. But more important is the influence of the observer dynamics on the system performance. Analysis of the system performance is extremely difficult. This difficulty follows from the fact that linear approximation is valid only in the vicinity of the origin. Two nonlinear block of the structure significantly influence the system performance. To stress a difference between linear and first principle model compare frequency response of both models shown in Figure 4.

3. NONLINEARITIES AND PREDICTIVE CONTROL

Nonlinearities are included in electromagnet

$$F_1(t)=\left[\frac{\left(i_c(t)+0.5\right)^2}{\left(x_1(t)-0.0004\right)^2}-\frac{\left(i_c(t)-0.5\right)^2}{\left(x_1(t)-0.0004\right)^2}\right]\cdot 2.8\cdot 10^{-8}\left[\frac{\mathrm{Nm}^2}{\mathrm{A}^2}\right] \tag{2}$$

and measurement element

$$V_s(t)=5\left[\frac{\mathrm{V}}{\mathrm{mm}}\right]X_1(t)+24\left[\frac{\mathrm{V}}{\mathrm{mm}}\right]X_1^3(t) \tag{3}$$

There are also sources of nonlinearities hidden in the shaft dynamic description following from angle q, but range of the angle values is small enough to approximate trigonometric function with linear one. Linear approximation of (2) and (3) can not be done separately due to inner feedback presented in Figure 3. This constitutes yet another source of linear model mismatch leading to deterioration of the system performance discussed in the previous section.

Parameters of the nonlinear part of the model influence the system behaviour significantly. Figure 5 shows the comparison between frequency responses of the simulated and real system [3]. It becomes obvious that first principle model with parameters given in the source literature [1] can not be directly used.

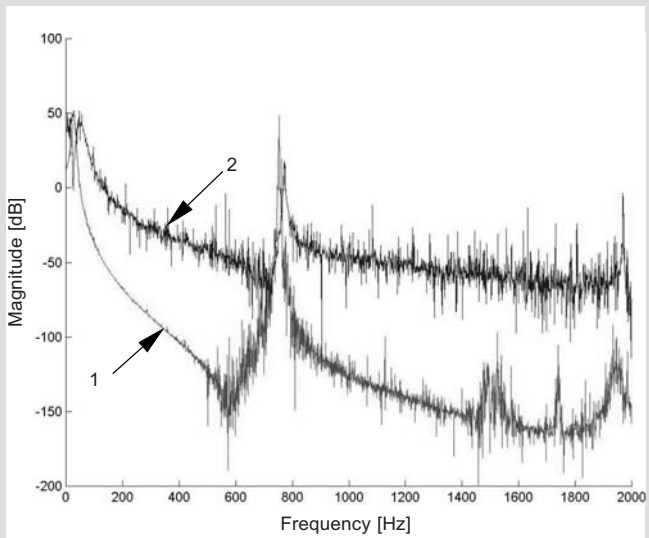

Fig. 5. Frequency responses: 1– nonlinear model; 2 – real unit

Nonlinear identification theory offers no general methods. Usually a sort of nonlinear optimization is used. In [5] an interesting searching procedure is proposed. This procedure uses gradients of error function (squared difference between model and the unit outputs) calculated according to unknown parameters. 'Unknown' means here these parameters on which the model is mostly sensitive. The gradients determine directions of searching as it is done in artificial neural networks (ANN) learning. However, no back-propagation is necessary here due to number of parameters is much smaller. Some ideas of learning are taken form ANN theory as adaptation of learning rate and epochs of learning.

Improved nonlinear model is used in control predictive algorithm firstly proposed in [6]. The most important theoretical achievement in the field of nonlinear model predictive control (MPC) was formulations of sufficient conditions for the closed-loop stability of the MPC systems. However, the major problems arise directly form the non--linearity of the plant namely necessity of on-line solution of usually non-convex optimal control task. If the control system can not respond to this need within every sampling period the MPC strategy can not be used any longer. This is the case discussed in this paper. It has been reported in [3] and [4] that sampling time has to be chosen here les then 1 ms. Clearly, algorithms proposed in the last decade are intractable in this case due to necessity of the optimal solution.

The main goal of theoretical analysis of the non-linear MPC addresses stability of the nominal system. The most common approach is to use cost function as a Lyapunov function. Ensuring stability by Lyapunov theory assumes that predicted control minimizing the cost function is known. This means that the solution of (non-convex) nonlinear program is obtainable what is not possible in MBC500 case. Fortunately it has been shown, that under mild assumptions the system remains stable even the optimality is lost [7]. This lead to a new approach of using sub-optimal stable MPC. Examples of such algorithms has been already reported (e.g. [7, 8]).

A new formulation directly using nonlinear model is given in [6]. To formulate the algorithm assume that general discrete nonlinear model of the plant to be controlled is given by:

$$z_{k+1} = f(z_k, u_k), \quad x_k = g(z_k) \tag{4}$$

where $z_k \in R^n$, $u_k \in R^m$ denote the state and control vectors at discrete time k and $f(\cdot): R^n \times R^m \to R^n$ is assumed to be continuous at the origin with $f(0, 0) = 0$, $x_k \in R^p$ is the output of the system. The objective is to regulate the output x to the origin. At each instant k a finite horizon (N) control sequence is determined as the following vector

$$\pi_k = [v_{k|k}, v_{k+1|k}, ..., v_{k+N-1|k}] \tag{5}$$

The vector (5) is calculated according to constraints: every control $v \in U$. Let the state that generates (5) according to (4) are

$$\chi_{k+1} = [x_{k+1|k}, x_{k+2|k}, ..., x_{k+N|k}] \tag{6}$$

with $x_{k/k} = x_k$. The algorithm that calculates (5) has to fulfil output constraints: $x \in X$. The current control action u_k is chosen as the first element of (5) i.e. $u_k = v_{k/k}$ (receding horizon). The feasibility is assumed through the paper that means the set U and X makes it possible to determine stable control for all initial conditions.

Note, that the above description suits very well to discrete counterpart of continuous model of MBC500 unit. Discretization can be done with Euler method due to short sampling time assumed. Constraints with respect to shaft-ends position can be also clearly stated as well as the input one. However, input constraints are soft.

It is shown in [6] that keeping solution (5) within the constraints the closed loop system is stable and, that the following algorithm can parameterise any feasible solution. Denote the predicted control sequence built on the sequence π_{k-1} determined in the previous instant as

$$\pi^p_{k-1} = [v_{k|k-1}, ..., v_{k+N-2|k-1}, v_{k+N-2|k-1}] \tag{7}$$

and the corresponding output profile as

$$\chi^p_{k+1} = [x_{k+1|k}, x_{k+2|k}, ..., x_{k+N|k}] \tag{8}$$

Note, that u_{k-1} has been already set and to calculate (8) the model (4) is utilized thus the current measurement x_k is also used. The control algorithm is defined as

$$\pi_k = \mathbf{M}_k \chi^p_{k+1} + \pi^p_{k-1} \tag{9}$$

where matrix M_k is chosen in order to fulfil input and state constraints. In the case of MBC500 axes x and y has been controlled separately. Note, that still the control is of multi-input multi-output (MIMO) due to two degree of freedom in every axes. Control horizon has been chosen as $N = 3$, sampling time 0.2 ms. To propose the matrix linear approximation can be used. In fact, if the system (4) is linear, control law (9) is optimal in the mean-square sense [6]. Thus, it is reasonably to find first matrix $\mathbf{M}_k$ according to linear approximation and (if necessary) retune its elements ($\mathbf{M}_k$ is the controller parameter). Because of two input and two output and $N = 3$ order of matrix $\mathbf{M}_k$ is 6×6 (identity matrix has been used as control weighting matrix).

4. APPLICATION TO MBC500 SYSTEM

Four algorithm has been tested for MBC500 system namely: build-in PD compensators, WMV controllers, LQR control with state observer and non-linear MPC algorithm. Due

Table 1. Amplitude [%] of shaft-end displacement for different controller and angular speed

Angular speed [rps]	5	20	40	60	80
PD	39.3	50.2	61	66.7	72.9
WMV	17.4	26.7	35.1	38.9	42.2
LQR	11.8	22.2	35.2	46.7	54.5
MPC	10.3	17.4	25.1	39.9	33.2

to nonlinear character of the system it became not possible to provide frequency analysis and sort of experimentation has to be established. The experiments has been organized as follows. Constant angular speed has been kept. This could be possible thanks to additional control system of pressure in drifting unit opening of pneumatic valves (see Fig. 1). This system has measured angular speed and corrected it with the valves. All algorithms has been implemented in DSP card DS1104 of dSpace. The amplitude of shaft-ends position has been measured and compared for different angular speed. Results are given in Table 1 (as % of the maximal amplitude).

5. CONCLUSION

The precise model of the system to be controlled not always provides expected performance in nonlinear case. The reason follows from difficulty generated by non-linearities and possible constraints on the system outputs and inputs. Nonlinear model predictive control directly uses nonlinear model and can easily handle with the constraints. The paper proposes nonlinear MPC algorithm for MBC500 system to reduce periodical disturbances influencing position of shaft-ends while rotating. The performance of the control system is significantly better than in the case of linear controller. Three of them have been compared in the paper. The price paid for this result is the nonlinear model accuracy. In other words the results presented in Table 1 have been obtained for the nominal model. To achieve such disturbance reduction heavy computational efforts has to be taken to increase nonlinear model accuracy. One can expect significant deterioration of the results if the system remains time-varying. The future research will address this (robustness) problem. The problem can be stated as follows: how great can be model inaccuracy to provide disturbances reduction on the pre-specified level for the given range of frequency.

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