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APPLICATION OF LQR THEORY TO OPTIMIZATION OF VIBRATION ISOLATION OF SITTING HUMAN BODY

SUMMARY

In the paper the linear quadratic regulator (LQR) theory was applied to optimization of vibration isolation system of sitting human operator body subjected to vertical excitations. The optimization approach was applied to system composed of active human body model (AHBM) and passive, linear vibration isolation system (VIS). Synthesis of optimal LQR for the given system was done for two existing, active biodynamical models "back-off" and "back-on" of sitting human body. Optimization was done for chosen form of vibration isolation criterion. Analytical formula of that form, corresponding to present standards concerning the human body expositions to vibration, was presented. Influence of the LQR on dynamical behavior of the system composed of AHBM and VIS was estimated on the basis of numerical calculations, discussed and graphically presented.

Keywords: *LQR, vibration isolation, biomechanical models*

ZASTOSOWANIE TEORII REGULATORA LQR DO OPTYMALIZACJI WIBROIZOLACJI CIAŁA SIEDZĄCEGO CZŁOWIEKA

W pracy przedstawiono zastosowanie liniowego regulatora kwadratowego (LQR) do optymalizacji układu wibroizolacji ciała siedzącego człowieka – operatora poddanego drganiom pionowym. Dobór optymalnego regulatora przeprowadzono dla dwóch wersji aktywnego modelu ciała siedzącego człowieka, modelu z oparciem i modelu bez oparcia. Regulator LQR był zastosowany do układu złożonego z aktywnego modelu ciała człowieka (AHBM) i liniowego układu wibroizolacji (VIS). Optymalizacja została przeprowadzona dla założonej postaci kryterium, odpowiadającej obecnym normom dotyczącym ekspozycji ciała człowieka na wibracje. Wpływ regulatora LQR na zachowanie się układu AHBM-VIS przedstawiono w postaci graficznej, na podstawie wyników przeprowadzonych obliczeń numerycznych.

Słowa kluczowe: *biomechaniczne modele, wibroizolacja, LQR*

1. INTRODUCTIONS

Human body can be considered as a very complex, active biomechanical system. However, majority of existing biomechanical models of sitting human operator body were built as passive, lumped parameter systems (Coermann 1962, Gierke 1971, Hopkins, 1971, Muskian 1976, Książek 1979, Potiemkin 1979, ISO 7962, Kiene 1989, Smith 1992, Mansfield 1996, Yoshimura 1996 and Wei 1998). The structures of these models were a priori assumed and the parameters of these structures were identified by comparison of the corresponding theoretical and experimental quantities such as transmissibility functions, impedances or apparent masses.

The first active human biomechanical models (AHBM) of the sitting human body were proposed and presented in (Książek 1997a, 1997b, 1998, 1999a, 1999b). In the presented paper these models were adopted as the objects subjected to vibration. Investigation concerning an aspect of synthesis of optimal vibration isolation for such models was target of the work.

The optimization of vibration isolation of the active models was already considered in different aspects in (Książek 2003, 2004, 2005). In the presented paperthe optimization of vibration isolation based on the linear quadratic regulator (LQR) theory (Bryson 1981, 2002, Mathworks 2001), was applied as the first design step in seeking a state – feedback law, that minimizes chosen a priori cost function named further the criterion of optimization of vibration isolation system. The form of this function was assumed taking into account structure of AHBM-VIS system and existing international standards concerning human body expositions to vibration (ISO 2631, PN-91/N-01354 and Engel 1993).

2. PROBLEM PRESENTATION AND ASSUMPTIONS

In the Figure 1 the structure of the dynamical system composed of active model of sitting human body (AHBM) and vibration isolation system (VIS) was presented. Structure and values of parameters of the AHBM were adopted from (Książek 1999b, 2004). The active human body model (AHBM) is composed of masses m_1, m_2, dampers with damping coefficients: α_1, α_1, springs with stiffness coefficients k_1, k_1 and actuator A developing active force F_a acting between masses m_1 and m_2. Vibration isolation system (VIS) is represented by seat of mass m_3, damper with damping coefficient α_3 and spring with stiffness coefficient k_3. Force F_{st} is the force developed by regulator LQR.

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3. EQUATIONS OF MOTION OF THE SYSTEM AHBM-VIS

The differential equations of motion for the system illustrated in the Figure 1 can be written in the form (3.1) taking into account the descriptions mentioned in paragraph 2.

$$\begin{cases} m_1\ddot{x}_1 + k_1(x_1 - x_3) + \alpha_1(\dot{x}_1 - \dot{x}_3) - F_a = 0 \\ m_2\ddot{x}_2 + k_2(x_2 - x_3) + \alpha_2(\dot{x}_2 - \dot{x}_3) + F_a = 0 \\ m_3\ddot{x}_3 + k_1(x_1 - x_3) - \alpha_1(\dot{x}_1 - \dot{x}_3) - k_2(x_2 - x_3) - \alpha_2(\dot{x}_2 - \dot{x}_3) + F_{st} + F = 0 \end{cases} \tag{3.1}$$

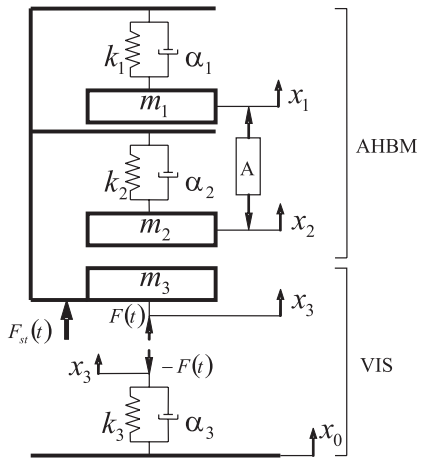

Fig. 1. Structure of Active Human Body Models (AHBM) and Vibration Isolation System (VIS)

In (Książek 1999a 1999b) was shown that the force F_a has different form for the position "back-off" and position "back-on" of the sitting human body. These two different forms are marked as $F_a^{\text{back-off}}$ and $F_a^{\text{back-on}}$. The analytical expressions for these forces are presented by the formulae (3.2) and (3.3).

$$F_a^{back-off} = -k_{11}(x_1 - x_3) - k_{12}(\dot{x}_1 - \dot{x}_3) - k_{13}(x_2 + x_3) - k_{14}(\dot{x}_2 + \dot{x}_3) \tag{3.2}$$

$$F_a^{back-on} = -k_{11}(x_1 + x_3) - k_{12}(\dot{x}_1 - \dot{x}_3) - k_{13}(x_2 + x_3) - k_{14}(\dot{x}_2 - \dot{x}_3) \tag{3.3}$$

Force developed by passive part of VIS is given by the formula (3.4)

$$F = k_3(x_3 - x_0) + \alpha_3(\dot{x}_3 - \dot{x}_0) \tag{3.4}$$

The equations (3.1) can be rewritten in the following form

$$\begin{cases} \ddot{x}_1 = -\left(\dfrac{1}{m_1}\right)(k_1(x_1 - x_3) + \alpha_1(\dot{x}_1 - \dot{x}_3) - F_a) \\ \ddot{x}_2 = -\left(\dfrac{1}{m_2}\right)(k_2(x_2 - x_3) + \alpha_2(\dot{x}_2 - \dot{x}_3) + F_a) \\ \ddot{x}_3 = -\left(\dfrac{1}{m_3}\right)(-k_1(x_1 - x_3) - \alpha_1(\dot{x}_1 - \dot{x}_3) - k_2(x_2 - x_3) - \alpha_2(\dot{x}_2 - \dot{x}_3) + F_{st} + F) \end{cases} \tag{3.5}$$

Introducing new variables

$$y = [x_1 \quad x_2 \quad x_3 \quad \dot{x}_1 \quad \dot{x}_2 \quad \dot{x}_3]^T \tag{3.6}$$

$$w = [x_0 \quad \dot{x}_0]^T \tag{3.7}$$

$$u = [F_{st}] \tag{3.8}$$

where:

$w = [x_0 \quad \dot{x}_0]^T$ – vector of generalized forcing forces,

$u = F_{st}$ – vector of generalized control forces, equations (3.5) take form

$$\begin{cases} \dot{y}_1 = y_4 \\ \dot{y}_2 = y_5 \\ \dot{y}_3 = y_6 \\ \dot{y}_4 = -\left(\dfrac{1}{m_1}\right)\left(k_1(y_1 - y_3) + \alpha_1(y_4 - y_6) - F_a\right) \\ \dot{y}_5 = -\left(\dfrac{1}{m_2}\right)\left(k_2(y_2 - y_3) + \alpha_2(y_5 - y_6) + F_a\right) \\ \dot{y}_6 = -\left(\dfrac{1}{m_3}\right)\left(-k_1(y_1 - y_3) - \alpha_1(y_4 - y_6) - k_2(y_2 - y_3) - \alpha_2(y_5 - y_6) + u + F\right) \end{cases} \tag{3.9}$$

In matrix form we have

$$\dot{y} = Ay + Bu + Dw \tag{3.10}$$

The corresponding matrices for "back-off" and "back-on" models have the following forms

$$A^{back-off} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ \dfrac{-k_1 - k_{11}}{m_1} & -\dfrac{k_{13}}{m_1} & \dfrac{k_1 + k_{11} - k_{13}}{m_1} & -\dfrac{k_{12} - \alpha_1}{m_1} & -\dfrac{k_{14}}{m_1} & \dfrac{k_{12} - k_{14} + \alpha_1}{m_1} \\ \dfrac{k_{11}}{m_2} & \dfrac{k_{13} - k_2}{m_2} & \dfrac{k_2 - k_{11} + k_{13}}{m_2} & \dfrac{k_{12}}{m_2} & \dfrac{k_{14} - \alpha_2}{m_2} & \dfrac{k_{14} - k_{12} + \alpha_2}{m_2} \\ \dfrac{k_1}{m_3} & \dfrac{k_2}{m_3} & \dfrac{-k_1 - k_2 - k_3}{m_3} & \dfrac{\alpha_1}{m_3} & \dfrac{\alpha_2}{m_3} & \dfrac{-\alpha_1 - \alpha_2 - \alpha_3}{m_3} \end{bmatrix} \tag{3.11}$$

$$A^{back-on} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ \dfrac{-k_1 - k_{11}}{m_1} & -\dfrac{k_{13}}{m_1} & \dfrac{k_1 - k_{11} - k_{13}}{m_1} & \dfrac{-k_{12} - \alpha_1}{m_1} & -\dfrac{k_{14}}{m_1} & \dfrac{k_{12} + k_{14} + \alpha_1}{m_1} \\ \dfrac{k_{11}}{m_2} & \dfrac{k_{13} - k_2}{m_2} & \dfrac{k_2 + k_{11} + k_{13}}{m_2} & \dfrac{k_{12}}{m_2} & \dfrac{k_{14} - \alpha_2}{m_2} & \dfrac{\alpha_2 - k_{14} - k_{12}}{m_2} \\ \dfrac{k_1}{m_3} & \dfrac{k_2}{m_3} & \dfrac{-k_1 - k_2 - k_3}{m_3} & \dfrac{\alpha_1}{m_3} & \dfrac{\alpha_2}{m_3} & \dfrac{-\alpha_1 - \alpha_2 - \alpha_3}{m_3} \end{bmatrix} \tag{3.12}$$

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & -\dfrac{1}{m_3} \end{bmatrix}^T, \quad D = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & \dfrac{k_3}{m_3} \\ 0 & 0 & 0 & 0 & 0 & \dfrac{\alpha_3}{m_3} \end{bmatrix}^T$$

4. APPLICATION OF LQR METHOD

4.1. Theoretical background

The LQR method for the problem presented in (3.10) can be applied under assumption that we have all full-state feedback (i.e. that we can measure all six states of (3.9)). In that case we can find the minimizing gain matrix K, which determines the feedback control law.

The gain K is obtained by solving an Riccati equation and is called the LQ-optimal gain. The procedure of getting of the gain K can be presented in the following steps:

1. Writing the Hamiltonian of the form

$$H(y,\lambda,t)=\frac{1}{2}\begin{bmatrix} y^T & u^T \end{bmatrix}\begin{bmatrix} Q & N \\ N^T & R \end{bmatrix}\begin{bmatrix} y \\ u \end{bmatrix}+\lambda^T\,[A \quad B]\begin{bmatrix} y \\ u \end{bmatrix} \tag{4.1}$$

2. Construction of the following differential equations

$$\dot{y}=\frac{\partial H(y,\lambda,t)}{\partial\lambda},\quad y(0)=y_0 \tag{4.2}$$

$$\dot{\lambda}=-\frac{\partial H(y,\lambda,t)}{\partial y},\quad \lambda(T)=0 \tag{4.3}$$

$$\frac{\partial H(y,\lambda,t)}{\partial u}=0 \tag{4.4}$$

3. Getting, after some conformal mathematical transformation, equation of Riccati

$$-\dot{S}=Q+SA+A^T S-(SB+N)R^{-1}(B^T S+N^T),\quad S(T)=0 \tag{4.5}$$

The solution of the equation (4.5) is time – varying gain $K(t) = B^T S(t) + N^T$. In this case the controller takes the form $u(t) = -K(t)y(t)$. When T tends to constant value as the time-to-go becomes large, the controller effectively becomes regulator, i.e. a feedback controller

$$u(t)=-Ky(t) \tag{4.6}$$

with constant gains K =const, which must be calculated from the following relation

$$K=B^T S+N^T \tag{4.7}$$

In that case the differential equation (4.5) transforms to algebraic one

$$Q+SA+A^T S-(SB+N)R^{-1}(B^T S+N^T)=0 \tag{4.8}$$

The matrices Q, R, N,called the weighting matrices, are the components of the criterion of optimization which can be, in general form, written as follows

$$J=\frac{1}{2}\int_0^T\left(\begin{bmatrix} y^T & u^T \end{bmatrix}\begin{bmatrix} Q & N \\ N^T & R \end{bmatrix}\begin{bmatrix} y \\ u \end{bmatrix}\right) \tag{4.9}$$

4.2. Application of LQR theory to AHBM-VIS model

For the system AHBM-VIS the criterion (4.9) was assumed in the following form

$$J=\frac{1}{2}\int_0^T(\lambda_1\dot{y}_4^2+\lambda_2\dot{y}_5^2+\lambda_3\dot{y}_6^2+\lambda_4 y_4^2+\lambda_5 y_5^2+\lambda_6 y_6^2+\lambda_7(y_1-y_3)^2+\lambda_8(y_2-y_3)^2+\lambda_9 y_3^2)\,dt \tag{4.10}$$

where λ_i (i = 1...9) are the weighting multipliers in criterion (4.10). These multipliers can be transformed as follows

$$\lambda_i=\lambda_{i0}\frac{\zeta_i}{1-\zeta_i},\quad \lambda_{i0}=1,\ i=1\ldots9 \tag{4.11}$$

Comparing criterion (4.9) with (4.10) and taking into account that $Q = Q^T$, components of the matrix Q can be presented by columns of its lower triangle as follows

$$Q = \begin{bmatrix} Q_{(1,1)} & - & - & - & - & - \\ Q_{(2,1)} & Q_{(2,2)} & - & - & - & - \\ Q_{(3,1)} & Q_{(3,2)} & Q_{(3,3)} & - & - & - \\ Q_{(4,1)} & Q_{(4,2)} & Q_{(4,3)} & Q_{(4,4)} & - & - \\ Q_{(5,1)} & Q_{(5,2)} & Q_{(5,3)} & Q_{(5,4)} & Q_{(5,5)} & - \\ Q_{(6,1)} & Q_{(6,2)} & Q_{(6,3)} & Q_{(6,4)} & Q_{(6,5)} & Q_{(6,6)} \end{bmatrix}$$

where:

$$Q_{(i,1)} = \begin{bmatrix} \frac{2k_1 k_2 \zeta_1}{m_1^2(1-\zeta_1)} + k_{11}^2\left(\frac{\zeta_1}{m_1^2(1-\zeta_1)} + \frac{\lambda_2}{m_2^2(1-\zeta_2)}\right) + k_1^2\left(\frac{\zeta_1}{m_1^2(1-\zeta_1)} + \frac{\zeta_3}{m_3^2(1-\zeta_3)}\right) + \frac{\zeta_7}{1-\zeta_7} \\ \frac{(k_1+k_{11})k_{13}\zeta_1}{m_1^2(1-\zeta_1)} + \frac{(k_{13}-k_2)k_{11}\zeta_2}{m_2^2(1-\zeta_2)} + \frac{k_1 k_2 \zeta_3}{m_3^2(1-\zeta_3)} \\ \frac{(k_{13}-k_1-k_{11})(k_1-k_{11})\zeta_1}{m_1^2(1-\zeta_1)} + \frac{(k_2-k_{11}+k_{13})k_{11}\zeta_2}{m_2^2(1-\zeta_2)} - \frac{(k_1+k_2+k_3)k_1\zeta_3}{m_3^2(1-\zeta_3)} - \frac{\zeta_7}{1-\zeta_7} \\ \frac{(k_1+k_{11})(k_{12}+\alpha_2)\zeta_1}{m_1^2(1-\zeta_1)} + \frac{k_{11}k_{12}\zeta_2}{m_2^2(1-\zeta_2)} + \frac{k_1\alpha_1\zeta_3}{m_3^2(1-\zeta_3)} \\ \frac{(k_1+k_{11})k_{14}\zeta_1}{m_1^2(1-\zeta_1)} + \frac{(k_{14}-\alpha_2)k_{11}\zeta_2}{m_2^2(1-\zeta_2)} + \frac{k_1\alpha_2\zeta_3}{m_3^2(1-\zeta_3)} \\ \frac{(k_1+k_{11})(k_{14}-k_{12}-\alpha_1)\zeta_1}{m_1^2(1-\zeta_1)} + \frac{(k_{14}-k_{12}+\alpha_2)k_{11}\zeta_2}{m_2^2(1-\zeta_2)} + \frac{(\alpha_1+\alpha_2+\alpha_3)k_1\zeta_3}{m_3^2(1-\zeta_3)} \end{bmatrix} \tag{4.12}$$

$$Q_{(i,2)} = \begin{bmatrix} - \\ k_{13}^2\left(\frac{\zeta_1}{m_1^2(1-\zeta_1)} + \frac{\zeta_2}{m_2^2(1-\zeta_2)}\right) + k_2^2\left(\frac{\zeta_2}{m_2^2(1-\zeta_2)} + \frac{\zeta_3}{m_3^2(1-\zeta_3)}\right) - \frac{2k_2 k_{13}\zeta_2}{m_2^2(1-\zeta_2)} + \frac{\zeta_8}{1-\zeta_8} \\ \frac{(k_{13}-k_1-k_{11})k_{13}\zeta_1}{m_1^2(1-\zeta_1)} + \frac{(k_{13}-k_2)(k_2-k_{11}+k_{13})\zeta_2}{m_2^2(1-\zeta_2)} - \frac{(k_1+k_2+k_3)k_2\zeta_3}{m_3^2(1-\zeta_3)} - \frac{\zeta_8}{1-\zeta_8} \\ \frac{(k_{12}+\alpha_1)k_{13}\zeta_1}{m_1^2(1-\zeta_1)} + \frac{(k_{13}-k_2)k_{12}\zeta_2}{m_2^2(1-\zeta_2)} + \frac{k_2\alpha_1\zeta_3}{m_3^2(1-\zeta_3)} \\ \frac{k_{13}k_{14}\zeta_1}{m_1^2(1-\zeta_1)} + \frac{(k_{13}-k_2)(k_{14}-\alpha_2)\zeta_2}{m_2^2(1-\zeta_2)} + \frac{k_2\alpha_2\zeta_3}{m_3^2(1-\zeta_3)} \\ \frac{(k_{14}-k_{12}-\alpha_1)k_{13}\zeta_1}{m_1^2(1-\zeta_1)} + \frac{(k_{13}-k_2)(k_{14}-k_{12}+\alpha_2)\zeta_2}{m_2^2(1-\zeta_2)} - \frac{(\alpha_1+\alpha_2+\alpha_3)k_2\zeta_3}{m_3^2(1-\zeta_3)} \end{bmatrix}$$

$$Q_{(i,3)} = \begin{bmatrix} - \\ - \\ \dfrac{m_3^2\left((k_1+k_{11}-k_{13})^2 m_2^2 \dfrac{\zeta_1}{(1-\zeta_1)} + (k_{11}-k_{13})(k_{11}-2k_2-k_{13}) m_1^2 \dfrac{\zeta_2}{(1-\zeta_2)}\right) + (k_1+k_2+k_3)^2 m_1^2 m_2^2 \dfrac{\zeta_3}{(1-\zeta_3)}}{m_1^2 m_2^2 m_3^2} \\ + \dfrac{k_2^2\zeta_2}{m_2^2(1-\zeta_2)} + \dfrac{\zeta_7}{1-\zeta_7} + \dfrac{\zeta_8}{1-\zeta_8} + \dfrac{\zeta_9}{1-\zeta_9} \\ \dfrac{(k_{13}-k_1-k_{11})(k_{12}+\alpha_1)\zeta_1}{m_1^2(1-\zeta_1)} + \dfrac{(k_2-k_{11}+k_{13})k_{12}\zeta_2}{m_2^2(1-\zeta_2)} - \dfrac{(k_1+k_2+k_3)\alpha_1\zeta_3}{m_3^2(1-\zeta_3)} \\ \dfrac{(k_{13}-k_1-k_{11})k_{14}\zeta_1}{m_1^2(1-\zeta_1)} + \dfrac{(k_{14}-\alpha_2)(k_2-k_{11}+k_{13})\zeta_2}{m_2^2(1-\zeta_2)} - \dfrac{(k_1+k_2+k_3)\alpha_2\zeta_3}{m_3^2(1-\zeta_3)} \\ \dfrac{(k_1+k_{11}-k_{13})(k_{12}-k_{14}+\alpha_1)\zeta_1}{m_1^2(1-\zeta_1)} + \dfrac{(k_2-k_{11}-k_{13})(k_{14}-k_{12}+\alpha_2)\zeta_2}{m_2^2(1-\zeta_2)} + \dfrac{(k_1+k_2+k_3)(\alpha_1+\alpha_2+\alpha_3)\zeta_3}{m_3^2(1-\zeta_3)} \end{bmatrix}$$

$$Q_{(i,4)} = \begin{bmatrix} - \\ - \\ - \\ \dfrac{2k_{12}\alpha_1\zeta_1}{m_1^2(1-\zeta_1)} + k_{12}^2\left(\dfrac{\zeta_1}{m_1^2(1-\zeta_1)} + \dfrac{\zeta_2}{m_2^2(1-\zeta_2)}\right) + \alpha_1^2\left(\dfrac{\zeta_1}{m_1^2(1-\zeta_1)} + \dfrac{\zeta_3}{m_3^2(1-\zeta_3)}\right) + \dfrac{\zeta_4}{1-\zeta_4} \\ \dfrac{(k_{12}+\alpha_1)k_{14}\zeta_1}{m_1^2(1-\zeta_1)} + \dfrac{(k_{14}-\alpha_2)k_{12}\zeta_2}{m_2^2(1-\zeta_2)} + \dfrac{\alpha_1\alpha_2\zeta_3}{m_3^2(1-\zeta_3)} \\ \dfrac{(k_{12}+\alpha_1)(k_{14}-k_{12}-\alpha_1)\zeta_1}{m_1^2(1-\zeta_1)} + \dfrac{(k_{14}-k_{12}+\alpha_2)k_{12}\zeta_2}{m_2^2(1-\zeta_2)} - \dfrac{(\alpha_1+\alpha_2+\alpha_3)\alpha_1\zeta_3}{m_3^2(1-\zeta_3)} \end{bmatrix}$$

$$Q_{(i,5)} = \begin{bmatrix} - \\ - \\ - \\ - \\ k_{14}^2\left(\dfrac{\zeta_1}{m_1^2(1-\zeta_1)} + \dfrac{\zeta_2}{m_2^2(1-\zeta_2)}\right) + \alpha_2^2\left(\dfrac{\zeta_2}{m_2^2(1-\zeta_2)} + \dfrac{\zeta_3}{m_3^2(1-\zeta_3)}\right) - \dfrac{2k_{12}\alpha_2\zeta_2}{m_1^2(1-\zeta_2)} + \dfrac{\zeta_5}{1-\zeta_5} \\ \dfrac{(k_{14}-k_{12}-\alpha_1)k_{14}\zeta_1}{m_1^2(1-\zeta_1)} + \dfrac{(k_{14}-\alpha_2)(k_{14}-k_{12}+\alpha_2)\zeta_2}{m_2^2(1-\zeta_2)} - \dfrac{(\alpha_1+\alpha_2+\alpha_3)\alpha_2\zeta_3}{m_3^2(1-\zeta_3)} \end{bmatrix}$$

$$Q_{(i,6)} = \begin{bmatrix} - \\ - \\ - \\ - \\ - \\ \dfrac{m_3^2\left((k_{12}-k_{14}-\alpha_1)^2 m_2^2 \dfrac{\zeta_1}{(1-\zeta_1)} + (k_{14}-k_{12}-\alpha_2)^2 m_1^2 \dfrac{\zeta_2}{(1-\zeta_2)}\right) + (\alpha_1+\alpha_2-\alpha_3)^2 m_1^2 m_2^2 \dfrac{\zeta_3}{(1-\zeta_3)}}{m_1^2 m_2^2 m_3^2} \\ + \dfrac{\zeta_6}{(1-\zeta_6)} \end{bmatrix}$$

Matrix N has the form

$$N^T = \left[-\frac{k_1 \zeta_3}{m_3^2 (1-\zeta_3)} - \frac{k_2 \zeta_3}{m_3^2 (1-\zeta_3)} \frac{(k_1 + k_2 + k_3)\zeta_3}{m_3^2 (1-\zeta_3)} - \frac{\alpha_1 \zeta_3}{m_3^2 (1-\zeta_3)} - \frac{\alpha_2 \zeta_3}{m_3^2 (1-\zeta_3)} \frac{(\alpha_1 + \alpha_2 + \alpha_3)\zeta_3}{m_3^2 (1-\zeta_3)} \right] \tag{4.13}$$

and R is one component matrix

$$R = \left[\frac{\zeta_3}{m_3^2 (1-\zeta_3)} \right] \tag{4.14}$$

5. NUMERICAL SOLUTIONS

5.1. Numerical data

The numerical values of the parameters of the AHBM and VIS models were taken from (Bryson 1981) and presented in Table 5.1.

In numerical calculations considered system was subjected to harmonic excitation described by the following variables

$$w_1 = \frac{1}{4\pi^2 f_0^2} \cos(2\pi f_0 t), \qquad w_2 = \frac{1}{2\pi f_0^2} \sin(2\pi f_0 t) \tag{5.1}$$

It was assumed that acceleration $\ddot{x}$ is a harmonic function with frequency f_0 and its amplitude is 1 m/s^2.

Where f_0 corresponds to first resonance frequency of the system.

Table 5.1
Numerical values of the parameters of AHBM and VIS model (Bryson 1981)

Parameters of models	"Back-off" $m_1 + m_2 = 70.8$ kg	Parameters of models	"Back-on" $m_1 + m_2 = 70.8$ kg
m_1 [kg]	9.1	m_1 [kg]	66
m_2 [kg]	61.7	m_2 [kg]	4.8
m_3 [kg]	35	m_3 [kg]	35
k_1 [N/m]	11972.5557	k_1 [N/m]	51189.32
k_2 [N/m]	224456.7485	k_2 [N/m]	63335.50
k_3 [N/m]	9950	k_3 [N/m]	9950
α_1 [Ns/m]	3251.9783	α_1 [Ns/m]	1704.17
α_2 [Ns/m]	519.0440	α_2 [Ns/m]	1262.59
α_3 [Ns/m]	860	α_3 [Ns/m]	860
k_{11} [N/m]	97323.2354	k_{11} [N/m]	123251.32
k_{12} [Ns/m]	–2226.0653	k_{12} [Ns/m]	–1781.04
k_{13} [N/m]	–1960.5176	k_{13} [N/m]	–104227.69
k_{14} [Ns/m]	1164.3525	k_{14} [Ns/m]	759.69
f_0 [Hz]	1.55	f_0 [Hz]	1.58

5.2. Results of numerical calculations

Numerical calculations were presented for $\zeta_i = 0.5$, $i = 1\ldots9$ For such values of ζ_i the values of K are given as follows

$$K^{back\text{-}off} = [-82588.9 \quad -30800.6 \quad 91303.8 \quad -2426.04 \quad -1986.87 \quad 1580.32]^T$$

$$K^{back\text{-}on} = [-74591. \quad 903462. \quad -441635. \quad 10062.7 \quad -400.684 \quad -12902.7]^T$$

Numerical simulations were conducted assuming different types of excitations but presented results correspond to sinusoidal excitation with frequency f_0.

Exemplary time histories of the most important variables of the models were shown in Figures 2–11. In the Figures 2 and 3 the relative displacements with and without LQR regulator were compared.

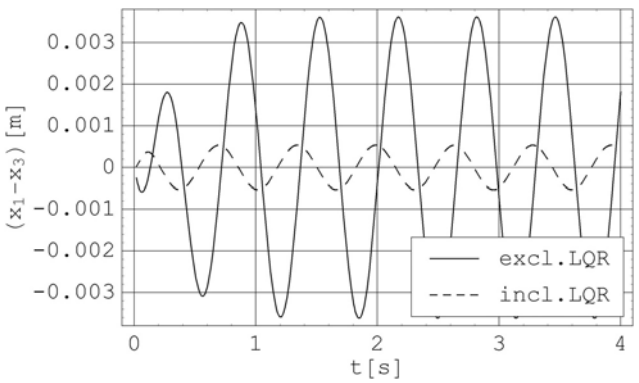

Fig. 2. Time history of relative displacement $x_1 - x_3(y_1 - y_3)$ for "back-off" model

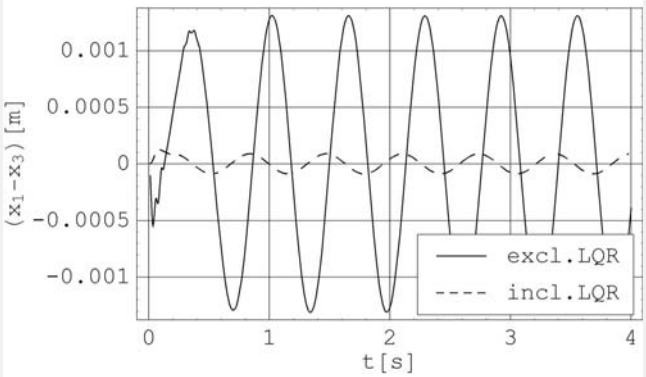

Fig. 3. Time history of relative displacement $x_1 - x_3(y_1 - y_3)$ for "back-on" model

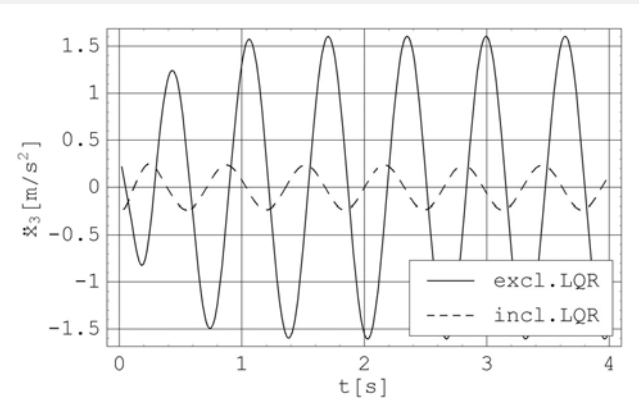

Fig. 4. Time history of acceleration $\ddot{x}_3$ ($\ddot{y}_6$) for "back-off" model

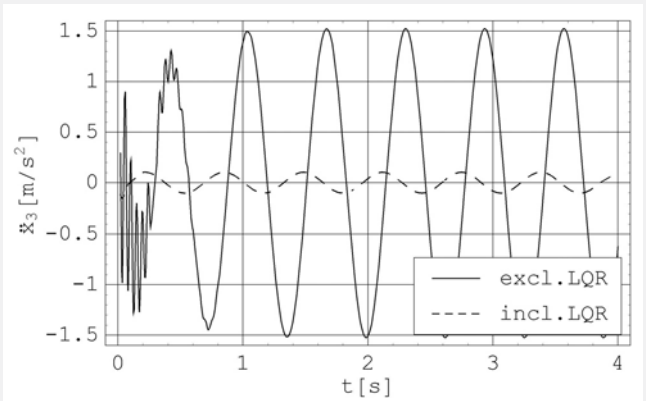

Fig. 5. Time history of acceleration $\ddot{x}_3$ ($\ddot{y}_6$) for "back-on" model

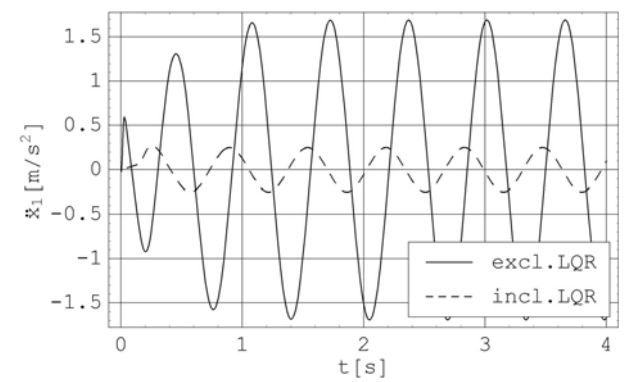

Fig. 6. Time history of acceleration $\ddot{x}_1$ ($\ddot{y}_3$) for "back-off" model

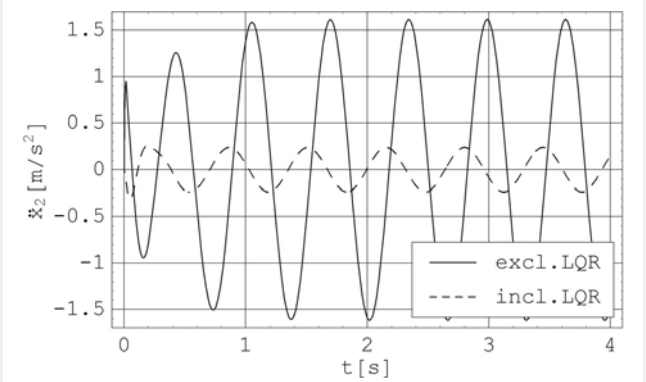

Fig. 7. Time history of acceleration $\ddot{x}_2$ ($\ddot{y}_4$) for "back-off" model

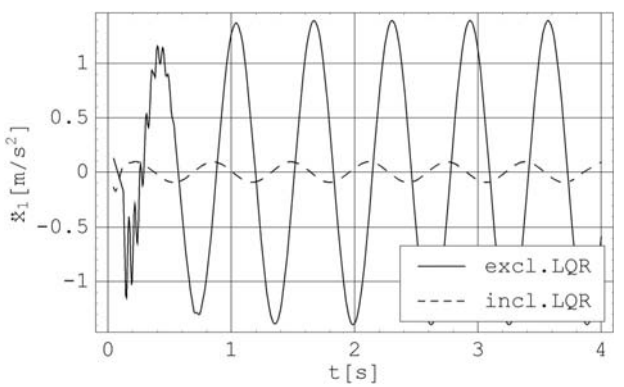

Fig. 8. Time history of acceleration $\ddot{x}_1$ ($\dot{y}_3$) for "back-on" model

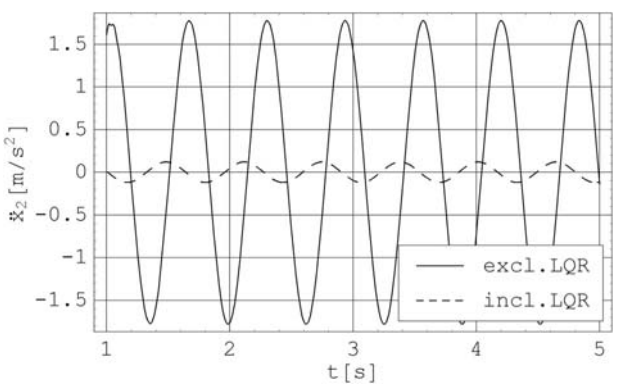

Fig. 9. Time history of acceleration $\ddot{x}_2$ ($\dot{y}_4$) for "back- on" model

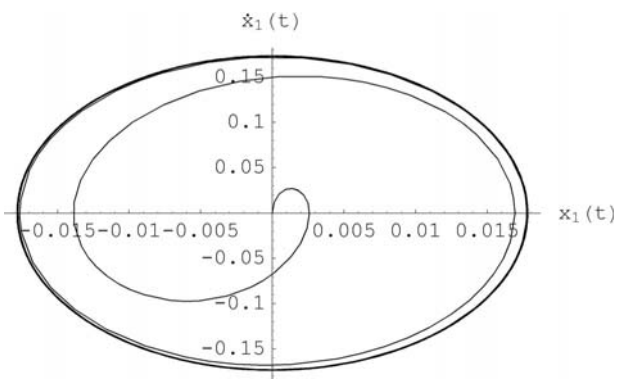

Fig. 10. Phase portrait of $\dot{x}_1(t)$ ($y_3(t)$) and $x_1(t)$($y_1(t)$) for the model "back-off" without LQR, t = 1...15 s

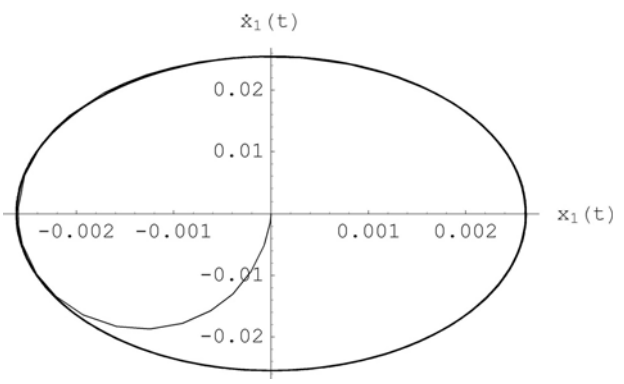

Fig. 11. Phase portrait of $\dot{x}_1(t)$ ($y_3(t)$) and $x_1(t)$($y_1(t)$) for the model "back-off" with LQR, t = 1...15 s

Figures 2 and 3 show big influence of application of control force on the values of relative displacements of the masses of the model. The displacements for the models with application of LQR are about ten times lower than the same displacements without regulator. There are however slight differences between the "back-off" and "back-on" models. This is the results of bigger stiffness of "back-on" model. Analogical results, but not presented in the paper, were confirmed by calculations concerning the relative displacements $x_2 - x_3(y_2 - y_3)$, and $x_3 - x_0(y_3 - y_0)$.

In Figures 4 and 5 the time histories of accelerations of mass m_3 for "back-off" and "back-on" models were presented. In this particular case, decrease of acceleration of mass of seat for the system with LQR is almost eight times lower than for the system without LQR.

The relation between the accelerations of masses m_1 and m_2 for the model "back-off" were shown in Figures 6 and 7.

The relation between the accelerations of masses m_1 and m_2 for the model "back-on" were shown in Figures 8 and 9.

In Figures 10 and 11 the chosen results of numerical simulation were presented on the phase plane. Such approach, as a direct presentation of transformation (3.6) can be very useful in interpretation of initial, non-stationary state of the AHBM-VIS system. Exemplary results were shown for mass m_1 of "back-off" model without and with LQR.

The calculation to this work has been completed on the computer and by means of the software made by ACK and available in Cyfronet (MNISW/Sun6800/PK/155/2006).

6. CONCLUDED REMARKS

Investigations concerning the vibration isolation of human body operators, at present state of knowledge, are founded on rather simple structures of biomechanical models and simple, sometimes one parameter criteria. Such approaches yield final results which are very simplified and very distant from the expectations. It seems obvious that more complex, but realistic biomechanical models and suitable criteria of vibration isolation must be applied to ameliorate comfort of the operator. In the presented paper the complex, active biomechanical models of sitting human operator body and criteria composed of nine components (composed of accelerations and relative displacements) were applied in the LQR procedure of optimization of considered system. The accelerations can be considered as the indicators of the forces acting on the masses of models. The relative displacements can be considered as indicators of stresses or strains in elements connecting particular parts of the human body.

In the paper the accelerations of masses m_1, m_2, m_3 and relative displacements x_1–x_3 (y_1–y_3), x_2–x_3 (y_2–y_3), x_3–x_0 (y_3–y_0), were assumed as measurable quantities and were the base for the construction of criterion of optimization of vibration isolation system. Procedure based on LQR theory can be considered as the first step to full synthesis of optimal vibration isolation system by LQG regulator.

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