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THE THEORETICAL BACKGROUND FOR CALCULATION OF GEOMETRICAL PARAMETERS OF THE STABLE MELTING ZONE IN SINGLE-ROW COKE CUPOLAS

1. INTRODUCTION

The attempts made still back in the 20th century to formulate an equation which would serve in calculation of the height of the melting zone failed to give positive results. The obstacle were difficulties in providing a mathematical description of the individual volumes and surface areas of the melting pieces of metallic charge and in determining their number and total surface area. The total surface area of the melting pieces of metallic charge, otherwise called the surface of the melting zone development, should be characterized (in a stable process) by constant value which, in turn, means constant melting rate and continuous process of coke feeding to the combustion zone. In the studies undertaken back in the 20th century with a view to create a mathematical or verbal only description of the melting process and coke combustion effects it was usually assumed that portions of the coke burden are fed to the combustion zone "stepwise" when the metal volume directly adjacent to the zone upper boundary had melted down. This assumption made the model of metal melting and coke combustion ambiguous, and the solution of the problem very difficult.

A new approach to the problem of calculating the height of the melting zone was described in [1, 2], where the term of a mean integral volume and mean integral surface of the metal pieces melting in the zone was adopted. The following equation was derived for calculation of the melting zone height

$$H_t = \frac{100 p_F \bar{r}_m K_{\rho,t} L_f \rho_m}{K_w L_k \alpha_2 \Delta T_{g,2} \rho_{n,m}} \ln \frac{T_{g,2} - T_{m,f}}{T_{g,3} - T_{m,f}} \tag{1}$$

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where:

H_t – the height of the melting zone, m,

p_F – the blast air volume (under standard conditions), $m^3/(m^2 \cdot s)$,

$\overline{r}_m$ – the mean integral modulus of the pieces of metallic charge, m,

$K_{\rho,t}$ – the dimensionless coefficient allowing for a ratio between the melting zone volume (bulk volume of metal and coke) and bulk volume of metal pieces in this zone,

L_f – the melting heat of metal, the value increased by the melting heat of slag and by the heat of metal and slag overheating, J/kg,

ρ_m – the mass density of the pieces of metallic charge, kg/m^3,

K_w – the charge coke consumption rate, $\dfrac{\text{kg coke}}{\text{100 kg Fe}}$ or mass %,

L_k – the air blast volume consumed in the process of combustion of a unit coke mass under standard conditions, m^3/kg,

α_2 – the heat transfer coefficient in the melting zone between the gas and the surface area of the melting pieces of metallic charge, $W/(m^2 \cdot K)$,

$\Delta T_{g,2}$ – the gas temperature drop in the melting zone caused by the melting process,

$\rho_{n,m}$ – the bulk density of metal, kg/m^3,

$T_{g,2}$ – the gas temperature on an inlet to the melting zone, °C,

$T_{g,3}$ – the gas temperature on an outlet from the melting zone, °C,

$T_{m,f}$ – the temperature of metal melting, °C.

Equation (1) refers to the pieces of metallic charge of the same shape and mass, and of the same and constant melting temperature.

For calculation of the mean integral modulus of the melting pieces of metallic charge the following relationships were derived, valid for the pieces in the form of plates, prisms, cubes, and spheres:

$$\varphi_v = \frac{1}{2} - \frac{1}{6m_b} - \frac{1}{6m_c} - \frac{1}{12 m_b m_c} \tag{2}$$

$$\varphi_f = \frac{m_b m_c}{m_b + m_c + m_b m_c} \tag{3}$$

when

$$\overline{r}_m = \frac{v_{m,o}}{f_{m,o}} \frac{\varphi_v}{\varphi_f} = \frac{\overline{v}_m}{\overline{f}_m} \tag{4}$$

where:

$$m_b = \frac{b}{a},$$

$$m_c = \frac{c}{a},$$

a, b, c – the thickness, width and length of plates, respectively,

$v_{m,o}$ – the starting volume of the pieces of metallic charge, m^3,

$f_{m,o}$ – the starting surface area of the pieces of metallic charge, m^2,

$$\varphi_v = \frac{\overline{v}_m}{v_{m,o}},$$

$$\varphi_f = \frac{\overline{f}_m}{f_{m,o}},$$

$\overline{v}_m$ – the mean integral volume of the melting pieces of metal, m^3,

$\overline{f}_m$ – the mean integral surface area of the melting pieces of metal, m^2.

The equations for calculation of the coefficients φ_v and φ_f were obtained from the following expressions determining the mean integral values

$$\varphi_v = \frac{\int_0^1 \left[1 - X\left(1 + \frac{1}{m_b} + \frac{1}{m_c}\right) + X^2\left(\frac{1}{m_b} + \frac{1}{m_c} + \frac{1}{m_b m_c}\right) - \frac{X^3}{m_b m_c} \right] dX}{\int_0^1 dX} \tag{5}$$

$$\varphi_f = \frac{\int_0^1 \left[1 - \frac{2(1 + m_b + m_c)X - 3X^2}{m_b + m_c + m_b m_c} \right] dX}{\int_0^1 dX} \tag{6}$$

where:

$$X = \frac{\tau}{\tau_c},$$

τ – the melting time,

τ_c – the total melting time (for thickness a).

The application of the mean integral values implies the following two problems:

1) Is the use of mean integral values capable of ensuring the sufficiently accurate results of calculations?
2) How are we supposed to calculate the true volume and surface area of the melting pieces of metallic charge?

It is the aim of the present study to give an answer to the questions posed above.

2. CALCULATION OF THE NUMBER OF THE MELTING PIECES OF METAL IN THE MELTING ZONE, OF THE NUMBER OF THE MELTING SEQUENCES OF THE PIECES, AND OF THE NUMBER OF PIECES IN EACH SEQUENCE

Let us first calculate the metal mass in the melting zone using the following equation for the balance of bulk volumes of the metal and coke

$$F_c H_t = \frac{M_{m,t}}{\rho_{n,m}} + \frac{M_{k,t}}{\rho_{n,k}} = \frac{M_{m,t}}{\rho_{n,m}} K_{\rho,t} \tag{7}$$

where:

F_c – the area of the internal cupola section in the melting zone, m^2,
$M_{m,t}$ – the metal mass in the melting zone, kg,
$M_{k,t}$ – the coke mass in the melting zone, kg,
$\rho_{n,k}$ – the bulk density of coke, kg/m^3.

From equation (7) we calculate $M_{m,t}$

$$M_{m,t} = \frac{F_c H_t \rho_{n,m}}{K_{\rho,t}} \tag{8}$$

when:

$$K_{\rho,t} = 1 + \frac{\overline{K}}{100} \frac{\rho_{n,m}}{\rho_{n,k}} \tag{9}$$

$$\overline{K} = \frac{K_w}{\varphi_v} \tag{10}$$

where $\overline{K}$ – an average coke content in the melting zone, %.

Using the term of a mean integral volume of the melting pieces of metallic charge, we calculate their number in the melting zone from equation

$$n_m = \frac{M_{m,t}}{\rho_m \overline{v}_m} \tag{11}$$

where n_m – the number of the melting pieces of metal in the melting zone.

Let us assume that the melting pieces of metallic charge in a number n_m are forming along the zone height identical sequences of pieces decreasing towards the lower zone boundary; the sequences are distributed at random in the coke. Let us assume further that the number of such sequences is equal to a number of the pieces of metallic charge characterised by the starting mean dimension X_m, and that the pieces of this metallic charge are moving along with coke through the internal cupola section from the upper boundary of the melting zone.

Thus, the number of the sequences will be calculated from the relationship

$$N_c = \frac{F_c}{X_m^2 K_\rho} \tag{12}$$

when:

$$X_m = \frac{a+b+c}{3} \tag{13}$$

$$K_\rho = 1 + \frac{K_w}{100} \frac{\rho_{n,m}}{\rho_{n,k}} \tag{14}$$

The physical sense of parameter K_ρ is analogical to that of parameter $K_{\rho,t}$, and it expresses a relationship between the volume of preheating zone (bulk volume of metal and coke) and the bulk volume of metal in this zone.

The value of N_c, calculated from equation (12), is rounded off to the value of the nearest natural number.

Knowing the values of n_m and N_c, we calculate the number of the melting pieces of metal n in each sequence

$$n = \frac{n_m}{N_c} \tag{15}$$

The total surface area of the melting pieces of metallic charge is called the surface of development of the melting zone. In a stable process (i.e. the one characterised by, among others, constant melting rate), this surface should have a constant value, which means that any decrease in this value due to the melting process will be automatically compensated by pieces of metallic charge moving from the preheating zone to the melting zone.

The surface of development is calculated from the following equation

$$F_t = n_m \overline{f}_m = \frac{M_{m,t}}{\rho_m \overline{r}_m} \tag{16}$$

where F_t – the surface of development of the melting zone, m^2.

So far, the discussion was based on the use of mean integral quantities, i.e. $\overline{r}_m, \overline{v}_m, \overline{f}_m$. Let us now proceed to the most important task of this study – that is to the development of a theoretical background for calculation of the mass and surface distribution in the melting pieces of metal in individual sequences.

3. CALCULATION OF THE VOLUME OF SEQUENCES

Let us assume that in a sequence of the melting plates the difference in basic dimensions of the neighbouring plates is the same and amounts to $2z$. This means that the volume of the i-th melting plate can be expressed by equation

$$V_{m,i} = (a - 2zi)\cdot(b - 2zi)\cdot(c - 2zi) \tag{17}$$

Using (17), we shall write down the total volume of the plates melting in one sequence as $V'_{c,n}$ or $V''_{c,n}$

$$V'_{c,n} = \sum_{i=1}^{i=n} (a-2zi)(b-2zi)(c-2zi) \tag{18}$$

or as

$$V''_{c,n} = \sum_{i=0}^{i=n} (a-2zi)(b-2zi)(c-2zi) \tag{19}$$

From (18) it follows that the sequence of pieces starts with the piece whose basic dimensions have been reduced by $2z$; while from (19) it follows that the sequence starts with a piece of metal of the starting volume.

In equations (18) and (19) we shall factor out the quantities a, b and c and obtain the following relationships:

$$V'_{c,n} = \mathrm{v}_{m,o} \sum_{i=1}^{i=n} (1-\delta_i)\left(1-\frac{\delta}{m_b}i\right)\left(1-\frac{\delta}{m_c}i\right) \tag{20}$$

$$V''_{c,n} = \mathrm{v}_{m,o} \sum_{i=0}^{i=n} (1-\delta_i)\left(1-\frac{\delta}{m_b}i\right)\left(1-\frac{\delta}{m_c}i\right) \tag{21}$$

where $\delta = \frac{2z}{a}$.

To make further calculations easier, let us denote the sums in equations (20) and (21) by R_1 and R_2:

$$V'_{c,n} = \mathrm{v}_{m,o} R_1 \tag{22}$$

$$V''_{c,n} = \mathrm{v}_{m,o} R_2 \tag{23}$$

Now the sums R_1 and R_2 will be successively calculated.

Calculation of the sum R_1

Let us multiply the terms in sum R_1 obtaining

$$R_1 = 1 - k_1\delta i + k_2\delta^2 i^2 - k_3\delta^3 i^3 \tag{24}$$

where:

$$k_1 = 1 + \frac{1}{m_b} + \frac{1}{m_c},$$

$$k_2 = \frac{1}{m_b} + \frac{1}{m_c} + \frac{1}{m_b m_c},$$

$$k_3 = \frac{1}{m_b m_c}.$$

Let us write down in a developed form several terms from (24) for $i = 1, 2, 3, ..., n$:

$i = 1 \qquad 1 - k_1\delta 1 + k_2\delta^2 1^2 - k_3\delta^3 1^3$ (a)

$i = 2 \qquad 1 - k_1\delta 2 + k_2\delta^2 2^2 - k_3\delta^3 2^3$ (b)

$i = 3 \qquad 1 - k_1\delta 3 + k_2\delta^2 3^2 - k_3\delta^3 3^3$ (c)

....

$i = n \qquad 1 - k_1\delta\nu + k_2\delta^2 n^2 - k_3\delta^3 n^3$ (d)

Let us sum up the columns in equations (a)–(d):

- first column: $1+1+1+\ldots+1 = n$ (e)
- second column: $-k_1\delta(1+2+3+...+n) = -k_1\delta\dfrac{n(n+1)}{2}$ (f)
- third column: $k_2\delta^2\left(1^2+2^2+3^2+...+n^2\right) = k_2\delta^2\dfrac{n(2n^2+3n+1)}{6}$ (g)
- fourth column: $-k_3\delta^3\left(1^3+2^3+3^3+...+n^3\right) = -k_3\delta^3\dfrac{n^2(n^2+2n+1)}{4}$ (h)

To write down the sums (f), (g) and (h), the equations given in [3] will be used.

Let us add the sums from (e) to (h)

$$R_1 = n - k_1\delta\frac{n(n+1)}{2} + k_2\delta^2\frac{n\left(2n^2+3n+1\right)}{6} - k_3\delta^3\frac{n^2\left(n^2+2n+1\right)}{4} \tag{25}$$

Let it be assumed that the number of the melting pieces of metal in a given sequence equals dimension a divided by $2z$, that is

$$n = \frac{a}{2z} \tag{26}$$

Thus, from the definition of δ, the following relationship follows

$$\delta = \frac{1}{\frac{a}{2z}} = \frac{1}{n} \tag{27}$$

Let us substitute (27) to (25) obtaining after reductions

$$R_1 = \left(1 - \frac{1}{2}k_1 + \frac{1}{3}k_2 - \frac{1}{4}k_3\right)n + \frac{1}{2}\left(k_2 - k_1 - k_3\right) + \frac{2k_2 - 3k_3}{12n} \tag{28}$$

Having introduced to (28) the relationships existing between k_1, k_2 and k_3, we obtain the relationship

$$R_1 = \varphi_v n - \frac{1}{2} + \frac{2m_b + 2m_c - 1}{12m_b m_c n} \tag{29}$$

where φ_v is given by equation (2).

Now let us substitute (29) to (22) and obtain the searched formula for calculation of the volume of a sequence of the melting pieces of metal

$$V'_{c,n} = \mathrm{v}_{m,o}\left(\varphi_v n - \frac{1}{2} + \frac{2m_b + 2m_c - 1}{12m_b m_c n}\right) \tag{30}$$

To obtain a formula for calculation of a mean volume of one piece in a given sequence of the melting pieces, we shall divide $V'_{c,n}$ by n, obtaining from (30) the following relationship

$$\mathrm{v}'_{m,n} = \frac{V'_{c,n}}{n} = \mathrm{v}_{m,o}\varphi'_{v,n} \tag{31}$$

when

$$\varphi'_{v,n} = \varphi_v - \frac{1}{2n} + \frac{2m_b + 2m_c - 1}{12 m_b m_c n^2} \tag{32}$$

Let us now calculate the sum R_2 from equation (21)

Calculation of the sum R_2

For $i = 0$ the first term of the sum in equation (21) is equal to 1, and hence the calculated sum R_2 will be larger by 1 than the sum R_1

$$R_2 = R_1 + 1 \tag{33}$$

or, having allowed for (29), we shall obtain

$$R_2 = \varphi_v n + \frac{1}{2} + \frac{2m_b + 2m_c - 1}{12 m_b m_c n^2} \tag{34}$$

Let us substitute (34) to (23), thus obtaining the second formula for calculation of the volume of a sequence of the melting pieces of metal

$$V''_{c,n} = \mathrm{v}_{m,o} \left(\varphi_v n + \frac{1}{2} + \frac{2m_b + 2m_c - 1}{12 m_b m_c n} \right) \tag{35}$$

After dividing (35) by n, we obtain the following formula for calculation of a mean volume of the pieces of metal melting in a given sequence

$$\mathrm{v}''_{m,n} = \frac{V''_{c,n}}{n} = \mathrm{v}_{m,o} \varphi''_{v,n} \tag{36}$$

when

$$\varphi''_{v,n} = \varphi_v + \frac{1}{2n} + \frac{2m_b + 2m_c - 1}{12 m_b m_c n^2} \tag{37}$$

Analysis of the derived equations

From the comparison of equations (30) and (35) it follows that there is an inequality $V'_{c,n} < V''_{c,n}$ and that the calculated volumes of the sequences of the melting pieces of metal differ by a value $\mathrm{v}_{m,o}$. An analogical inequality exists between the coefficients $\varphi'_{v,n}$ and

$\varphi''_{v,n}$, which differ from each other by a value $1/n$. In further discussion, equations (30) and (35) as well as (31) and (36) will be substituted by their respective arithmetic means:

$$V_{c,n} = \frac{V'_{c,n} + V''_{c,n}}{2} = \mathrm{v}_{m,o}\left(\varphi_v n + \frac{2m_b + 2m_c - 1}{12m_b m_c n^2}\right) \tag{38}$$

$$\varphi_{v,n} = \frac{\varphi'_{v,n} + \varphi''_{v,n}}{2} = \varphi_v + \frac{2m_b + 2m_c - 1}{12m_b m_c n^2} \tag{39}$$

For $n = \infty$, from equation (38) we obtain $V_{c,n} = \infty$, while equation (39) is reduced to a form given in (2). So, the question now arises whether equation (2) is valid in the case of the melting zone with a finite number of pieces in each sequence. On specific numerical examples we shall prove that equation (2) can be used instead of equation (39) when the number of the pieces of metal exceeds certain minimal value, i.e. when it is comprised within a range of 3 to 7 pieces, which is exactly what happens in a cupola process. Let us calculate the value of coefficients φ_v and $\varphi_{v,n}$ for pieces of metal characterised by different values of m_b and m_c.

Calculations

a) $m_b = m_c = 10$, $\varphi_v = 0.4675$, $\varphi_{v,3} = 0.4711$, $\varphi_{v,5} = 0.4688$.

The differences between the values of $\varphi_{v,n}$ and φ_v are +0.77 and +0.28%, respectively.

b) $m_b = m_c = 5$, $\varphi_v = 0.437$, $\varphi_{v,3} = 0.444$, difference +1.6%,

c) $m_b = 5$, $m_c = 10$, $\varphi_v = 0.452$, $\varphi_{v,3} = 0.457$, difference +1.2%,

d) $m_b = 1$, $m_c = 5$, $\varphi_v = 0.317$, $\varphi_{v,5} = 0.3243$, difference +2.3%,

e) $m_b = m_c = 1$, $\varphi_v = 0.25$, $\varphi_{v,7} = 0.255$, difference +2%.

From the calculations given above it follows that with increasing n the values of $\varphi_{v,n}$ approach very quickly the values of φ_v, and when the sequence of the melting pieces includes several pieces, a very convenient equation (2) still holds good. It does not require an *a priori* knowledge of the value of n, which is calculated from (2) (calculation of H_t).

From the above discussion it follows that instead of equation (38) we can use the equation in a form reduced to

$$V_c = v_{m,o}\, \varphi_v n \tag{40}$$

where n – is calculated from (15).

So, the conclusion is that the application of the term of a mean integral volume is an allowable approximation, rendering only a very small error in calculations.

4. CALCULATION OF THE SURFACE AREA OF SEQUENCES

Like in the case of the volume of sequences, let us assume that the difference in basic dimensions of the melting plates is $2z$. Basing on this assumption, the melting surface area of the i-th plate can be written down in the form of equation

$$f_{m,i} = 2(a - 2zi)(b - 2zi)+2(a - 2zi)(c - 2zi)+2(b - 2zi)(c - 2zi) \tag{41}$$

Using (41) we shall write down the total surface area of the plates melting in a given sequence as:

$$F'_{c,n} = 2\sum_{i=1}^{i=n} (a-2zi)(b-2zi)+(a-2zi)(c-2zi)+(b-2zi)(c-2zi) \tag{42}$$

$$F''_{c,n} = 2\sum_{i=0}^{i=n} (a-2zi)(b-2zi)+(a-2zi)(c-2zi)+(b-2zi)(c-2zi) \tag{43}$$

Equation (42) allows for the surface of development of the sequence of plates in which the first plate has dimensions smaller by $2z$ than the starting dimensions, while according to equation (43), the first plate has the starting dimensions.

Let us rearrange equation (41) to the following form

$$f_{m,i} = f_{m,o}(1 - k_4\delta i + k_5\delta^2 i^2) \tag{44}$$

where:

$$k_4 = \frac{2(1+m_b+m_c)}{m_b+m_c+m_b m_c},$$

$$k_5 = \frac{3}{m_b+m_c+m_b m_c},$$

$f_{m,o} = 2(ab+ac+bc) = 2(m_b+m_c+m_b m_c)a^2$ – starting surface area of the plate,

$$\delta = \frac{2z}{a}.$$

Let us substitute (44) to (42) and (43):

$$F'_{c,n} = f_{m,o}\sum_{i=1}^{i=n}\left(1-k_4\delta i+k_5\delta^2 i^2\right) \tag{45}$$

$$F''_{c,n} = f_{m,o}\sum_{i=0}^{i=n}\left(1-k_4\delta i+k_5\delta^2 i^2\right) \tag{46}$$

To make further calculations easier, let the sums in equations (45) and (46) be denoted by S_1 and S_2:

$$F'_{c,n} = f_{m,o} S_1 \tag{47}$$

$$F''_{c,n} = f_{m,o} S_2 \tag{48}$$

Let us calculate successively the sums S_1 and S_2.

Calculation of the sum S_1

Let us write down in a developed form the terms in the sum S_1 for i = 1, 2, 3 and n:

$i = 1$ $\quad 1 - k_4\delta 1 + k_5\delta^2 1^2$ (a)

$i = 2$ $\quad 1 - k_4\delta 2 + k_5\delta^2 2^2$ (b)

$i = 3$ $\quad 1 - k_4\delta 3 + k_5\delta^2 3^2$ (c)

....

$i = n$ $\quad 1 - k_4\delta\nu + k_5\delta^2 n^2$ (d)

Let us sum up the columns in equations (a)–(d):

– first column: $1+1+1+\ldots+1= n$ (e)

– second column: $-k_4\delta(1+2+3+\ldots+n) = -k_4\delta\dfrac{n(n+1)}{2}$ (f)

– third column: $k_5\delta^2(1^2+2^2+3^2+\ldots+n^2) = k_5\delta^2\dfrac{n(2n+3n+1)}{6}$ (g)

Let us add the sums (e), (f) and (g)

$$S_1 = n - k_4\delta\frac{n(n+1)}{2} + k_5\delta^2\frac{n(2n^2+3n+1)}{6} \tag{49}$$

By substituting relation (27) to (49) we obtain

$$S_1 = n - k_4\frac{n+1}{2} + k_5\frac{2n^2+3n+1}{6} \tag{50}$$

Let us substitute to (50) the relationships existing between k_4 and k_5

$$S_1 = \frac{m_b m_c n}{M} + \frac{1-2m_b-2m_c}{2M} + \frac{1}{2Mn} \tag{51}$$

where $M = m_b + m_c + m_b m_c$.

Using (3), equation (51) will be written down as

$$S_1 = \varphi_f \cdot n + \frac{1-2m_b-2m_c}{2M} + \frac{1}{2Mn} \tag{52}$$

By substituting (52) to (47) we obtain the searched formula for calculation of the surface area of a sequence of the melting plates

$$F'_{c,n} = f_{m,o}\left(\varphi_f n + \frac{1-2m_b-2m_c}{2M} + \frac{1}{2Mn}\right) \tag{53}$$

To derive an equation for calculation of a mean surface area of the plate, let us further divide $F'_{c,n}$ by n, obtaining from (53) the following relationship

$$f'_{m,n} = \frac{F'_{c,n}}{n} = f_{m,o}\varphi'_{f,n} \tag{54}$$

when $\varphi'_{f,n} = \varphi_f + \frac{1-2m_b-2m_c}{2Mn} + \frac{1}{2Mn^2}$ (55)

Let us now calculate the sum S_2 in equation (48).

Calculation of the sum S_2

For $i = 0$, the first term of the sum in equation (46) is equal to 1, and hence the calculated sum S_2 will be larger by 1 than the sum S_1, which means that

$$S_2 = S_1 + 1 \tag{56}$$

or, having allowed for (52), we obtain

$$S_2 = \varphi_f n + \frac{1+2m_b m_c}{2M} + \frac{1}{2Mn} \tag{57}$$

Let us substitute (57) to (48), and thus obtain the second equation for calculation of the surface area of a sequence of the melting plates

$$F''_{c,n} = f_{m,o}\left(\varphi_f n + \frac{1+2m_b m_c}{2M} + \frac{1}{2Mn}\right) \tag{58}$$

To obtain an equation for calculation of a mean surface area of the pieces in a sequence, let us divide $F''_{c,n}$ by n

$$f''_{c,n} = \frac{F''_{c,n}}{n} = f_{m,o}\varphi''_{f,n} \tag{59}$$

when $\varphi''_{f,n} = \varphi_f + \frac{1+2m_b m_c}{2Mn} + \frac{1}{2Mn^2}$ (60)

Analysis of the derived equations

From the comparison of equations (53) and (58) it follows that there is an inequality $F'_{c,n} < F''_{c,n}$ which is quite obvious in view of the adopted summation limits in equations (45) and (46) (the middle term in equation (53) has a negative value). The same inequality exists between the coefficients $\varphi'_{v,n}$ and $\varphi''_{v,n}$.

In further discussion, equations (53) and (58) as well as (55) and (60) will be substituted by their respective arithmetic means:

$$F_{c,n} = \frac{F'_{c,n} + F''_{c,n}}{2} = f_{m,o}\left(\varphi_f n + \frac{1+m_b m_c - m_b - m_c}{2M} + \frac{1}{2Mn}\right) \tag{61}$$

$$\varphi_{f,n} = \frac{\varphi'_{f,n} + \varphi''_{f,n}}{2} = \varphi_f + \frac{1+m_b m_c - m_b - m_c}{2Mn} + \frac{1}{2Mn^2} \tag{62}$$

Let us now calculate the values of the coefficient $\varphi_{v,n}$ for several values of m_b, m_c and n.

Calculations

a) $m_b = m_c = 10$, $\varphi_f = \frac{100}{120} = 0.833$, $\varphi_{f,5} = 0.9$, the difference in the values of $\varphi_{f,5}$ and φ_f is +8%. For $n = 10$, $\varphi_{f,10} = 0.867$, and the difference in the values is +4,1%.

b) $m_b = m_c = 5$, $\varphi_f = \frac{25}{35} = 0.714$, $\varphi_{f,5} = 0.76$, the difference is +6,5%, $\varphi_{f,10} = 0.737$, the difference is +3.22%.

From the calculations given above it follows that to obtain a few percent difference in the values of $\varphi_{v,n}$ and φ_v, $n = 10$ has to be adopted (for square plates). The lower is the value of the product $m_b m_c$, the lower value can be adopted for n. From the above discussion it follows that instead of equation (61), we can use the equation in a form reduced to

$$F_c = f_{m,o}\, \varphi_f\, n \tag{63}$$

where n is calculated from (15).

The conclusion is that the application of the term of a mean integral surface area is an allowable approximation, rendering an admissible and calculable error.

5. EXAMPLE OF CALCULATION OF THE GEOMETRICAL PARAMETERS OF A MELTING ZONE

The geometrical parameters of a melting zone include: H_t, n_m, N_c, n, F_t as well as a volume and surface area of the pieces of metallic charge in individual sequences.

For calculations the following data will be adopted: p_F = 1.6 m/s, K_w = 12%, $T_{m,f}$ = 1150°C, cold blast, a = 0.5 m, b = 0.2 m, c = 0.3 m, α_2 = 200 W/(m^2·K) (estimated value), C_k = 0.86, L_f = 300 000 J/kg, d_c= 0.8m (cupola diameter), ρ_m = 7000 kg/m^3, $\rho_{n,m}$=2500 kg/m^3, $\rho_{n,k}$ = 500 kg/m^3, η_v = 0.525 (acc. to H. Jungbluth formula – combustion ratio for calculation of L_k).

Supporting calculations:

$v_{m,o} = 3{\cdot}10^{-3}$ m^3, $f_{m,o}$= 0.17 m^2, φ_v = 0.434, φ_f= 0.706, $\overline{v}_m$ = 1.3·10^{-3} m^3, $\overline{f}_m$ = 0.12 m^2, $\overline{r}_m$ = 0.011 m, $\overline{K}$ = 27.6%, $K_{\rho,t}$ = 2.38, $T_{g,2}$ = 1620°C (acc. to 2), $\Delta T_{g,2}$ = 234 K (acc. to [1]), $T_{g,3}$ = 1620 – 234 = 1386°C, L_k = 5.83 (acc. to [1]).

Calculation of H_t

$$H_t = \frac{100 \cdot 1.6 \cdot 0.011 \cdot 2.38 \cdot 300\,000 \cdot 7000}{12 \cdot 5.84 \cdot 200 \cdot 234 \cdot 2500} \ln \frac{1620-1150}{1386-1150} = 0.74 \text{ m}$$

Calculation of $M_{m,t}$, n_m, N_c, n:

$$M_{m,t} = \frac{0.5 \cdot 0.74 \cdot 2500}{2.38} = 388.6 \text{ kg}; \quad n_m = \frac{388.6}{7000 \cdot 1.3 \cdot 10^{-3}} = 42{,}7 \text{pieces.}$$

$$N_c = \frac{0.5}{0.183^2 \cdot 1{,}6} = 9.33 = 9 \quad K_\rho = 1.6; \quad X_m = 0.183 \text{ m}, \quad n = \frac{42.7}{9} = 4.74.$$

Calculation of the volume of metal pieces in one sequence:
from (18) and (19) the following relationship (arithmetic mean) follows

$$V_c = \frac{1}{2}\mathrm{v}_{m,o} + \mathrm{v}_{m,1} + \mathrm{v}_{m,2} + \mathrm{v}_{m,3} + \mathrm{v}_{m,4}.$$

where the values of the individual components are equal to:

$$\frac{1}{2}\mathrm{v}_{m,\mathrm{o}} = 0.5 \cdot 3 \cdot 10^{-3} = 1.5 \cdot 10^{-3}\ \mathrm{m}^3,$$

$$\mathrm{v}_{m,1} = (0.3 - 0.0105) \cdot (0.2 - 0.0105) \cdot (0.05 - 0.0105) = 2.167 \cdot 10^{-3}\ \mathrm{m}^3,$$

$$\mathrm{v}_{m,2} = 1.448 \cdot 10^{-3}\ \mathrm{m}^3,\ V_{m,3} = 8.37 \cdot 10^{-4}\ \mathrm{m}^3,\ V_{m,4} = 3.26 \cdot 10^{-3}\ \mathrm{m}^3,$$

when $2z = \frac{0.05}{4.74} = 0.0105$.

The total volume of the pieces in a sequence is $V_c = 6.278 \cdot 10^{-3}$ m^3, while the volume calculated according to the mean integral is: $1.3 \cdot 10^{-3} \cdot 4.74 = 6.162 \cdot 10^{-3}$ m^3.

The mass of metal in the zone calculated by both methods will be: $M_{m,t} = 6.278 \cdot 10^{-3} \cdot 7000 \cdot 9 = 395.5$ kg, $M_{m,t} = 6.162 \cdot 10^{-3} \cdot 7000 \cdot 9 = 388.2$ kg. The obtained results are very close to the value of 388.6 kg.

Calculation of the surface area of metal pieces in one sequence:
from (42) and (43) the following relationship (arithmetic mean) follows

$$F_c = \frac{1}{2} f_{m,o} + f_{m,1} + f_{m,2} + f_{m,3} + f_{m,4},$$

where the values of individual components are equal to:

$$\frac{1}{2} f_{m,o} = 0.5 \cdot 0{,}17 = 0{,}085\ \mathrm{m}^2,$$

$$f_{m,1} = 2(0.3 - 0.0105) \cdot (0.2 - 0.0105) + (0.3 - 0.0105)(0.05 - 0.0105) + (0.2 - 0.0105) \cdot (0.05 - 0.0105) = 0.1476\ \mathrm{m}^2;$$

$$f_{m,2} = 0.1264\ \mathrm{m}^2,$$

$$f_{m,3} = 0.1066\ \mathrm{m}^2,$$

$$f_{m,4} = 0.0882\ \mathrm{m}^2.$$

The total surface area of one sequence is $V_c = 0.5538$ m^2, while that of all the sequences is $F_{m,t} = 0.5538 \cdot 9 = 4.98$ m^2. The surface of development, calculated from the mean integral surface area of a single piece of metal, is $F_t = 0.12 \cdot 42.7 = 5.12$ m^2.

6. FINAL REMARKS

This study describes by means of difference equations the process of decreasing volume and surface area of the melting pieces of metallic charge in the sequences of pieces formed within the melting zone. Next, by summing up the volumes and surface areas of the pieces in each sequence, the relevant equations have been derived and served in calculation of the volume and surface area of the sequences for any arbitrary number of the metal pieces in a sequence. The equations are valid for the pieces of metallic charge in the form of rectangular plates of any arbitrary value of the relative dimensions of their sides. An analysis of the obtained relationships has proved that with increasing n the values of $\varphi_{v,n}$ and $\varphi_{f,n}$ are very rapidly approaching the values of φ_v and φ_f, which means that the use of mean integral values gives a good accuracy of calculations, providing the value of n is not smaller than 5.

The present work can serve for examination of relationships which exist in a melting zone – combustion zone system, very important for formulation of a condition of optimum operation of the single-row coke cupolas.

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