Opuscula Math. 34, no. 1 (2014), 115–122
http://dx.doi.org/10.7494/OpMath.2014.34.1.115 Opuscula Mathematica

A TOTALLY MAGIC CORDIAL LABELING OF ONE-POINT UNION OF n COPIES OF A GRAPH

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Communicated by Dalibor Fronček

Abstract. A graph G is said to have a totally magic cordial (TMC) labeling with constant C if there exists a mapping $f : V(G) \cup E(G) \to \{0,1\}$ such that $f(a)+f(b)+f(ab) \equiv C(\text{mod } 2)$ for all $ab \in E(G)$ and $|n_f(0) - n_f(1)| \leq 1$, where $n_f(i)$ $(i = 0,1)$ is the sum of the number of vertices and edges with label i. In this paper, we establish the totally magic cordial labeling of one-point union of n-copies of cycles, complete graphs and wheels.

Keywords: totally magic cordial labeling, one-point union of graphs.

Mathematics Subject Classification: 05C78.

1. INTRODUCTION

All graphs considered here are finite, simple and undirected. The set of vertices and edges of a graph G is denoted by $V(G)$ and $E(G)$ respectively. Let $p = |V(G)|$ and $q = |E(G)|$. A general reference for graph theoretic ideas can be seen in [3]. The concept of cordial labeling was introduced by Cahit [1]. A binary vertex labeling $f : V(G) \to \{0,1\}$ induces an edge labeling $f^* : E(G) \to \{0,1\}$ defined by $f^*(uv) = |f(u) - f(v)|$. Such labeling is called cordial if the conditions $|v_f(0) - v_f(1)| \leq 1$ and $|e_{f^*}(0) - e_{f^*}(1)| \leq 1$ are satisfied, where $v_f(i)$ and $e_{f^*}(i)$ $(i = 0,1)$ are the number of vertices and edges with label i respectively. A graph is called cordial if it admits a cordial labeling. The cordiality of a one-point union of n copies of graphs is given in [6].

Kotzig and Rosa introduced the concept of edge-magic total labeling in [5]. A bijection $f : V(G) \cup E(G) \to \{1,2,3,\ldots,p+q\}$ is called an edge-magic total labeling of G if $f(x) + f(xy) + f(y)$ is constant (called the magic constant of f) for every edge xy of G. The graph that admits this labeling is called an edge-magic total graph.

The notion of totally magic cordial (TMC) labeling was due to Cahit [2] as a modification of edge magic total labeling and cordial labeling. A graph G is said to have TMC labeling with constant C if there exists a mapping $f : V(G) \cup E(G) \to \{0,1\}$

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such that $f(a)+f(b)+f(ab)\equiv C(\text{mod } 2)$ for all $ab\in E(G)$ and $|n_f(0)-n_f(1)|\le 1$, where $n_f(i)$ $(i=0,1)$ is the sum of the number of vertices and edges with label i.

A rooted graph is a graph in which one vertex is named in a special way so as to distinguish it from other nodes. The special node is called the root of the graph. Let G be a rooted graph. The graph obtained by identifying the roots of n copies of G is called the one-point union of n copies of G and is denoted by $G^{(n)}$.

In this paper, we establish the TMC labeling of a one-point union of n-copies of cycles, complete graphs and wheels.

2. MAIN RESULTS

In this section, we present sufficient conditions for a one-point union of n copies of a rooted graph to be TMC and also obtain conditions under which a one-point union of n copies of graphs such as a cycle, complete graph and wheel are TMC graphs.

We relate the TMC labeling of a one-point union of n copies of a rooted graph to the solution of a system which involves an equation and an inequality.

Theorem 2.1. *Let G be a graph rooted at a vertex u and for $i=1,2,\ldots,k$, $f_i\colon V(G)\cup E(G)\to\{0,1\}$ be such that $f_i(a)+f_i(b)+f_i(ab)\equiv C(\text{mod } 2)$ for all $ab\in E(G)$ and $f_i(u)=0$. Let $n_{f_i}(0)=\alpha_i$, $n_{f_i}(1)=\beta_i$ for $i=1,2,\ldots,k$. Then the one-point union $G^{(n)}$ of n copies of G is TMC if the system* (2.1) *has a nonnegative integral solution for the x_i's:*

$$\left|\sum_{i=1}^{k}(\alpha_i-1)x_i-\sum_{i=1}^{k}\beta_i x_i+1\right|\le 1 \quad \text{and} \quad \sum_{i=1}^{k}x_i=n. \tag{2.1}$$

Proof. Suppose $x_i=\delta_i$, $i=1,2,\ldots,k$, is a nonnegative integral solution of system (2.1). Then we label the δ_i copies of G in $G^{(n)}$ with f_i $(i=1,2,\ldots,k)$. As each of these copies has the property $f_i(a)+f_i(b)+f_i(ab)\equiv C(\text{mod } 2)$ and $f_i(u)=0$ for all $i=1,2,\ldots,k$, $G^{(n)}$ is TMC. □

Corollary 2.2. *Let G be a graph rooted at a vertex u and f be a labeling such that $f(a)+f(b)+f(ab)\equiv C(\text{mod } 2)$ for all $ab\in E(G)$ and $f(u)=0$. If $n_f(0)=n_f(1)+1$, then $G^{(n)}$ is TMC for all $n\ge 1$.*

Example 2.3. One point union of a path is TMC.

Corollary 2.4. *Let G be a graph rooted at u. Let f_i, $i=1,2,3$ be labelings of G such that $f_i(a)+f_i(b)+f_i(ab)\equiv C(\text{mod } 2)$ for all $ab\in E(G)$, $f_i(u)=0$ and $\gamma_i=\alpha_i-\beta_i$.*

1. *If $\gamma_1=-2$ and $\gamma_2=2$, then $G^{(n)}$ is TMC for all $n\not\equiv 1(\text{mod } 4)$.*
2. *If either*
   a) *$\gamma_1=-1$ and $\gamma_2=3$, or*
   b) *$\gamma_1=4$, $\gamma_2=2$ and $\gamma_3=-4$, or*
   c) *$\gamma_1=-3$, $\gamma_2=3$ and $\gamma_3=5$,*
   then $G^{(n)}$ is TMC for all $n\ge 1$.
3. *If $\gamma_1=0$ and $\gamma_2=4$, then $G^{(n)}$ is TMC for all $n\not\equiv 3(\text{mod } 4)$.*

Proof. (1) The system (2.1) in Theorem 2.1 becomes $|-3x_1 + x_2 + 1| \leq 1$, $x_1+x_2 = n$. When $n = 4t$, $x_1 = t$ and $x_2 = 3t$ is the solution. When $n = 4t + 1$, the system has no solution. When $n = 4t + 2$, $x_1 = t + 1$ and $x_2 = 3t + 1$ is the solution. When $n = 4t + 3$, $x_1 = t + 1$ and $x_2 = 3t + 2$ is the solution. Hence, by Theorem 2.1, $G^{(n)}$ is TMC for all $n \not\equiv 1 (\text{mod } 4)$.

(2a). The system (2.1) in Theorem 2.1 becomes $|-2x_1 + 2x_2 + 1| \leq 1$, $x_1+x_2 = n$. When $n = 2t$, $x_1 = t$ and $x_2 = t$ is the solution. When $n = 2t + 1$, $x_1 = t + 1$ and $x_2 = t$ is the solution. Hence, by Theorem 2.1, $G^{(n)}$ is TMC for all $n \geq 1$.

The other parts can similarly be proved. □

3. ONE-POINT UNION OF CYCLES

Let C_m be a cycle of order m. Let

$$V(C_m) = \{v_i | 1 \leq i \leq m\}$$

and

$$E(C_m) = \{v_i v_{i+1} | 1 \leq i < m\} \cup \{v_m v_1\}.$$

We consider C_m as a rooted graph with the vertex v_1 as its root.

Theorem 3.1. *Let $C_m^{(n)}$ be the one-point union of n copies of a cycle C_m. Then $C_m^{(n)}$ is TMC for all $m \geq 3$ and $n \geq 1$.*

Proof. Define the labelings f_1 and f_2 from $V(C_m) \cup E(C_m)$ into $\{0, 1\}$ as follows: $f_1(v_i) = 0$ for $1 \leq i \leq m$, $f_1(v_i v_{i+1}) = 1$ for $1 \leq i < m$, $f_1(v_m v_1) = 1$, $1 \leq i \leq m$ and

$$f_2(v_i) = \begin{cases} 1 & \text{if } i = m, \\ 0 & \text{if } i \neq m, \end{cases} \quad f_2(v_i v_{i+1}) = \begin{cases} 1 & \text{if } 1 \leq i < m - 1, \\ 0 & \text{if } i = m - 1, \end{cases}$$

and $f_2(v_m v_1) = 0$. Then $\alpha_1 = m$, $\beta_1 = m$, $\alpha_2 = m + 1$ and $\beta_2 = m - 1$. Thus system (2.1) in Theorem 2.1 becomes $|-x_1 + x_2 + 1| \leq 1$, $x_1 + x_2 = n$. When $n = 2t$, $x_1 = t$ and $x_2 = t$ is the solution. When $n = 2t + 1$, $x_1 = t + 1$ and $x_2 = t$ is the solution. Hence, by Theorem 2.1, $C_m^{(n)}$ is TMC for all $m \geq 3$ and $n \geq 1$. □

4. ONE-POINT UNION OF COMPLETE GRAPHS

Let K_m be a complete graph of order m. Let

$$V(K_m) = \{v_i | 1 \leq i \leq m\}$$

and

$$E(K_m) = \{v_i v_j | i \neq j, 1 \leq i \leq m, 1 \leq j \leq m\}.$$

We consider K_m as a rooted graph with the vertex v_1 as its root. Let $f : V(K_m) \cup E(K_m) \to \{0, 1\}$ be a TMC labeling of K_m. Without loss of generality, assume $C = 1$.

Then for any edge $e = uv \in E(K_m)$, we have either $f(e) = f(u) = f(v) = 1$ or $f(e) = f(u) = 0$ and $f(v) = 1$ or $f(e) = f(v) = 0$ and $f(u) = 1$ or $f(u) = f(v) = 0$ and $f(e) = 1$. Thus, under the labeling f, the graph K_m can be decomposed as $K_m = K_p \cup K_r \cup K_{p,r}$ where K_p is the sub-complete graph in which all the vertices and edges are labeled with 1, K_r is the sub-complete graph in which all the vertices are labeled with 0 and edges are labeled with 1 and $K_{p,r}$ is the complete bipartite subgraph of K_m with the bipartition $V(K_p) \cup V(K_r)$ and its edges are labeled with 0. Then we find $n_f(0) = r + pr$ and $n_f(1) = \frac{p^2+r^2+p-r}{2}$.

Table 1. Possible values of α_i and β_i for distinct labelings of K_m

i	p	r	α_i	β_i
1	0	m	m	$\frac{m^2-m}{2}$
2	1	m-1	$2 \times (m-1)$	$\frac{m^2-3m+4}{2}$
3	2	m-2	$3 \times (m-2)$	$\frac{m^2-5m+12}{2}$
4	3	m-3	$4 \times (m-3)$	$\frac{m^2-7m+24}{2}$
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
$\lfloor \frac{m+1}{2} \rfloor$	$\lfloor \frac{m-1}{2} \rfloor$	$\lceil \frac{m+1}{2} \rceil$	$\lfloor \frac{m-1}{2} \rfloor \times \lceil \frac{m+1}{2} \rceil$	$\frac{[(\lfloor \frac{m-1}{2} \rfloor)^2 + (\lceil \frac{m+1}{2} \rceil)^2 + \lfloor \frac{m-1}{2} \rfloor + \lceil \frac{m+1}{2} \rceil]}{2}$

Table 1 gives the possible values of α_i and β_i for distinct labelings f_i of K_m such that $f_i(a) + f_i(b) + f_i(ab) \equiv 1(\text{mod } 2)$ for all $ab \in E(K_m)$.

Theorem 4.1. *Let $K_m^{(n)}$ be the one-point union of n copies of a complete graph K_m. If $\sqrt{m-1}$ has an integer value, then $K_m^{(n)}$ is TMC for $m \equiv 1, 2(mod\ 4)$.*

Proof. Let $f : V(K_m) \cup E(K_m) \to \{0, 1\}$ be a TMC labeling of K_m. Under the labeling f, the graph K_m can be decomposed as $K_m = K_p \cup K_r \cup K_{p,r}$. Then we have, $n_f(0) = r + pr$ and $n_f(1) = \frac{p^2+r^2+p-r}{2}$. By Corollary 2.2, $K_m^{(n)}$ is TMC if $n_f(0) = n_f(1) + 1$. Whenever, $n_f(0) = n_f(1) + 1$, $p^2 + p(1 - 2r) + r^2 - 3r + 2 = 0$. This implies that $r = \frac{1}{2}\left[(m + 1) \pm \sqrt{m-1}\right]$ as $p = m - r$. Also, $n_f(0) = n_f(1) + 1$ is possible only when $m \equiv 1, 2(\text{mod } 4)$. Therefore, $K_m^{(n)}$ is TMC for $m \equiv 1, 2(\text{mod } 4)$, if $\sqrt{m-1}$ has an integer value □

Theorem 4.2 ([4])**.** *Let G be an odd graph with $p + q \equiv 2(mod\ 4)$. Then G is not TMC.*

Theorem 4.3. *Let $K_m^{(n)}$ be the one-point union of n copies of a complete graph K_m.*

(i) *If $m \equiv 0(mod\ 8)$, then $K_m^{(n)}$ is not TMC for $n \equiv 3(mod\ 4)$.*
(ii) *If $m \equiv 4(mod\ 8)$, then $K_m^{(n)}$ is not TMC for $n \equiv 1(mod\ 4)$.*

Proof. Clearly, $p = |V(K_m^n)| = n(m-1) + 1$ and $q = |E(K_m^n)| = \frac{nm(m-1)}{2}$ so that $p + q = \frac{n(m-1)(m+2)}{2} + 1$.
Part (i) Assume $m = 8k$ and $n = 4l + 3$. Since the degree of every vertex is odd and

$p+q \equiv 2 \pmod 4$, it follows from Theorem 4.2 that $K_m^{(n)}$ is not TMC.
Part (ii) can similarly be proved. □

Theorem 4.4. *$K_4^{(n)}$ is TMC if and only if $n \not\equiv 1 \pmod 4$.*

Proof. Necessity follows from Theorem 4.3 and for sufficiency we define the labelings f_1 and f_2 as follows: $f_1(v_i) = 0$ for $1 \le i \le 4$, $f_1(v_iv_j) = 1$ for $1 \le i, j \le 4$ and under the labeling f_2 decompose K_4 as $K_1 \cup K_3 \cup K_{1,3}$. From Table 1, we observe that $\alpha_1 = 4$, $\beta_1 = 6$, $\alpha_2 = 6$ and $\beta_2 = 4$. Therefore, by Corollary 2.4 (1), $K_4^{(n)}$ is TMC if $n \not\equiv 1 \pmod 4$. □

Theorem 4.5. *$K_5^{(n)}$ is TMC for all $n \ge 1$.*

Proof. Define $f : V(K_5^{(n)}) \cup E(K_5^{(n)}) \to \{0,1\}$ as follows:

$$f(v_i) = \begin{cases} 0 & \text{if } i \neq 5, \\ 1 & \text{if } i = 5 \end{cases}$$

and

$$f(v_iv_j) = \begin{cases} 1 & \text{if } 1 \le i, j \le 4, \\ 0 & \text{if } i = 5 \text{ or } j = 5. \end{cases}$$

Clearly, $\alpha = \beta + 1 = 8$. Therefore, by Corollary 2.2, $K_5^{(n)}$ is TMC for all $n \ge 1$. □

Theorem 4.6. *$K_6^{(n)}$ is TMC for all $n \ge 1$.*

Proof. Let f_1 and f_2 be the labelings from $V(K_6^{(n)}) \cup E(K_6^{(n)})$ into $\{0,1\}$. Then, under the labelings f_1 and f_2 the graph K_6 can be decomposed as $K_1 \cup K_5 \cup K_{1,5}$ and $K_2 \cup K_4 \cup K_{2,4}$ respectively. Clearly, $\alpha_1 = 10$, $\beta_1 = 11$, $\alpha_2 = 12$ and $\beta_2 = 9$. Hence, by Corollary 2.4 (2a), $K_6^{(n)}$ is TMC for all $n \ge 1$. □

Theorem 4.7. *$K_7^{(n)}$ is TMC for all $n \ge 1$.*

Proof. Let f_1, f_2 and f_3 be the labelings from $V(K_7^{(n)}) \cup E(K_7^{(n)})$ into $\{0,1\}$. Then under the labelings f_1, f_2 and f_3 the graph K_7 can be decomposed as $K_3 \cup K_4 \cup K_{3,4}$, $K_4 \cup K_3 \cup K_{4,3}$ and $K_5 \cup K_2 \cup K_{5,2}$ respectively. We observe that $\alpha_1 = 16$, $\beta_1 = 12$, $\alpha_2 = 15$, $\beta_2 = 13$, $\alpha_3 = 12$ and $\beta_3 = 16$. Hence, by Corollary 2.4 (2b), $K_7^{(n)}$ is TMC for all $n \ge 1$. □

Theorem 4.8. *$K_8^{(n)}$ is TMC if and only if $n \not\equiv 3 \pmod 4$.*

Proof. Necessity follows from Theorem 4.3 and for sufficiency we define the labelings f_1 and f_2 as follows: under the labelings f_1 and f_2 the graph K_8 can be decomposed as $K_2 \cup K_6 \cup K_{2,6}$ and $K_3 \cup K_5 \cup K_{3,5}$ respectively. Clearly, $\alpha_1 = 18$, $\beta_1 = 18$, $\alpha_2 = 20$ and $\beta_2 = 16$. Hence, by Corollary 2.4 (3), $K_8^{(n)}$ is TMC if $n \not\equiv 3 \pmod 4$. □

Theorem 4.9. *$K_9^{(n)}$ is TMC for all $n \ge 1$.*

Proof. Under the labelings f_1, f_2 and f_3 the graph K_9 can be decomposed as $K_2 \cup K_7 \cup K_{2,7}$, $K_3 \cup K_6 \cup K_{3,6}$ and $K_4 \cup K_5 \cup K_{4,5}$ respectively. We observe that $\alpha_1 = 21$, $\beta_1 = 24$, $\alpha_2 = 24$, $\beta_2 = 21$, $\alpha_3 = 25$ and $\beta_3 = 20$. Therefore, by Corollary 2.4 (2c), the graph $K_9^{(n)}$ is TMC for all $n \geq 1$. □

5. ONE-POINT UNION OF WHEELS

A wheel W_m is obtained by joining the vertices $v_1, v_2, \ldots, v_m$ of a cycle C_m to an extra vertex v called the centre. We consider W_m as a rooted graph with v as its root.

Theorem 5.1. *Let $W_m^{(n)}$ be the one-point union of n copies of a wheel W_m.*

(i) *If $m \equiv 0(mod\ 4)$, then $W_m^{(n)}$ is TMC for all $n \geq 1$.*
(ii) *If $m \equiv 1(mod\ 4)$, then $W_m^{(n)}$ is TMC for $n \not\equiv 3(mod\ 4)$.*
(iii) *If $m \equiv 2(mod\ 4)$, then $W_m^{(n)}$ is TMC for all $n \geq 1$.*
(iv) *If $m \equiv 3(mod\ 4)$, then $W_m^{(n)}$ is TMC for $n \not\equiv 1(mod\ 4)$.*

Proof. Define the labelings f_1,f_2,f_3,f_4 and f_5 as follows: $f_j(v) = 0$ for $j = 1, 2, 3, 4, 5$. $f_1(v_m v_1) = 0$,

$$f_1(v_i) = \begin{cases} 1 & \text{if} \quad i \equiv 0(\text{mod } 4), \\ 0 & \text{if} \quad i \not\equiv 0(\text{mod } 4), \end{cases} \qquad f_1(v_i v_{i+1}) = \begin{cases} 1 & \text{if} \quad i \equiv 1, 2(\text{mod } 4), \\ 0 & \text{if} \quad i \equiv 0, 3(\text{mod } 4) \end{cases}$$

and

$$f_1(vv_i) = \begin{cases} 1 & \text{if} \quad i \not\equiv 0(\text{mod } 4), \\ 0 & \text{if} \quad i \equiv 0(\text{mod } 4). \end{cases}$$

$f_2(v_i) = f_2(v_i v_{i+1}) = 1$, $f_2(vv_i) = 0$ for $i = 1, 2, \ldots, m$ and $f_2(v_m v_1) = 1$. $f_3(v_i) = f_1(v_i)$, $f_3(v_i v_{i+1}) = f_1(v_i v_{i+1})$, $f_3(vv_i) = f_1(vv_i)$ for $i = 1, 2, \ldots, m$ and $f_3(v_m v_1) = 1$. $f_4(v_1) = 1$, $f_4(v_1 v_2) = f_4(v_m v_1) = 0$, $f_4(v_i) = f_3(v_i)$, $f_4(v_i v_{i+1}) = f_3(v_i v_{i+1})$, $f_4(vv_i) = f_3(vv_i)$ for $i = 2, 3, \ldots, m$ and $f_4(vv_1) = 0$.

$$f_5(v_i) = \begin{cases} 1 & \text{if} \quad i \equiv 1(\text{mod } 2), \\ 0 & \text{if} \quad i \equiv 0(\text{mod } 2), \end{cases} \qquad f_5(vv_i) = \begin{cases} 0 & \text{if} \quad i \equiv 1(\text{mod } 2), \\ 1 & \text{if} \quad i \equiv 0(\text{mod } 2), \end{cases}$$

$f_5(v_i v_{i+1}) = f_5(v_m v_1) = 0$.

Case 1. $m \equiv 0$ (mod 4).
If we consider the labeling f_1 we have, $n_{f_1}(0) = n_{f_1}(1) + 1$. Then, by Corollary 2.2, $W_m^{(n)}$ is TMC for all $n \geq 1$.

Case 2. $m \equiv 1$ (mod 4).
If we consider the labelings f_2, f_3 and f_4. We have $\alpha_2 = \frac{3m+1}{2}$, $\beta_2 = \frac{3m+1}{2}$, $\alpha_3 = \frac{3m+5}{2}$, $\beta_3 = \frac{3m-3}{2}$, $\alpha_4 = m + 1$, $\beta_4 = 2m$. Then, system (2.1) in Theorem 2.1 becomes $|-x_2 + 3x_3 - (m+1)x_4 + 1| \leq 1$, $x_2 + x_3 + x_4 = n$. When $n = 4t$, $x_2 = 3t$, $x_3 = t$, $x_4 = 0$ is a solution. When $n = 4t+1$, $x_2 = 3t+1$, $x_3 = t$, $x_4 = 0$ is a solution. When $n = 4t+2$, $x_2 = 3t+2$, $x_3 = t$, $x_4 = 0$ is a solution. When $n = 4t+3$, the system has no solution. Hence, by Theorem 2.1, $W_m^{(n)}$ is TMC if $n \not\equiv 3(\text{mod } 4)$.

Case 3. $m \equiv 2 \pmod 4$.
If we consider the labelings f_2, f_3, f_4 and f_5, we have $\alpha_2 = m+1$, $\beta_2 = 2m$, $\alpha_3 = \frac{3m}{2}$, $\beta_3 = \frac{3m+2}{2}$, $\alpha_4 = \frac{3m+4}{2}$, $\beta_4 = \frac{3m-2}{2}$, $\alpha_5 = 2m+1$, $\beta_5 = m$. Thus, system (2.1) in Theorem 2.1 becomes $|-mx_2 - 2x_3 + 2x_4 + mx_5 + 1| \leq 1$, $x_2 + x_3 + x_4 + x_5 = n$. When $n = 4t$, $x_2 = x_3 = x_4 = x_5 = t$ is a solution. When $n = 4t+1$, $x_2 = t$, $x_3 = t+1$, $x_4 = t$, $x_5 = t$ is a solution. When $n = 4t+2$, $x_2 = t+1$, $x_2 = t$, $x_4 = t$, $x_5 = t+1$ is a solution. When $n = 4t+3$, $x_2 = t+1$, $x_3 = t+1$, $x_4 = t$, $x_5 = t+1$ is a solution. Hence, by Theorem 2.1, $W_m^{(n)}$ is TMC for all $n \geq 1$.

Case 4. $m \equiv 3 \pmod 4$.
If we consider the labelings f_3 and f_4. We have $\alpha_3 = \frac{3m-1}{2}$, $\beta_3 = \frac{3m+3}{2}$, $\alpha_4 = \frac{3m+3}{2}$ and $\beta_4 = \frac{3m-1}{2}$. Therefore, system (2.1) in Theorem 2.1 becomes, $|-3x_3 + x_4 + 1| \leq 1$, $x_3 + x_4 = n$. When $n = 4t$, $x_3 = t$, $x_4 = 3t$ is a solution. When $n = 4t+1$,the system has no solution. When $n = 4t+2$, $x_3 = t+1$, $x_4 = 3t+1$ is a solution. When $n = 4t+3$, $x_3 = t+1$, $x_4 = 3t+2$ is a solution. Hence, by Theorem 2.1, $W_m^{(n)}$ is TMC if $n \not\equiv 1 (\mathrm{mod}\ 4)$. □

Acknowledgments
The authors sincerely thank the referee for the valuable suggestions which were used in this paper.

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Received: June 10, 2013.
Revised: September 14, 2013.
Accepted: September 21, 2013.