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MULTIPLICATIVE ZAGREB INDICES AND COINDICES OF SOME DERIVED GRAPHS

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Abstract. In this note, we obtain the expressions for multiplicative Zagreb indices and coindices of derived graphs such as a line graph, subdivision graph, vertex-semitotal graph, edge-semitotal graph, total graph and paraline graph.

Keywords: multiplicative Zagreb indices and coindices, derived graphs.

Mathematics Subject Classification: 05C07.

1. INTRODUCTION

In this paper, we are concerned with simple graphs without isolated vertices. Let G be such a graph with vertex set $V(G)$, $|V(G)| = n$, and edge set $E(G)$, $|E(G)| = m$. As usual, n is the order and m the size of G. The degree of a vertex $w \in V(G)$ is the number of vertices adjacent to w and is denoted by $d_G(w)$. A vertex $w \in V(G)$ is said to be pendant if $d_G(w) = 1$. The degree of an edge $e = uv$ in G, denoted by $d_G(e)$, is defined by $d_G(e) = d_G(u) + d_G(v) - 2$. We refer to [9] for unexplained terminology and notation.

A graphical invariant is a number related to a graph, in other words, it is a fixed number under graph automorphisms. In chemical graph theory, these invariants are also called the topological indices. In 1984, Narumi and Katayama [11] considered the product index as

$$NK(G) = \prod_{u \in V(G)} d_G(u)$$

for representing the carbon skeleton of a saturated hydrocarbon, and named it as a simple topological index. Tomović and Gutman renamed this molecular structure descriptor as the Narumi-Katayama index [15]. In 2010, Todeshine *et al.* [13, 14] proposed the multiplicative variants of ordinary Zagreb indices, which are defined as follows:

$$\prod\nolimits_1(G) = \prod_{u \in V(G)} d_G(u)^2 = [NK(G)]^2 \quad \text{and} \quad \prod\nolimits_2(G) = \prod_{uv \in E(G)} d_G(u) d_G(v).$$

These two graph invariants are called first and second multiplicative Zagreb indices by Gutman [6]. And recently, Eliasi *et al.* [5] introduced a further multiplicative version of the first Zagreb index as

$$\prod\nolimits_1^*(G) = \prod_{uv \in E(G)} [d_G(u) + d_G(v)].$$

In [18] and [7] the authors called it a multiplicative sum Zagreb index and modified first multiplicative Zagreb index respectively. The second multiplicative Zagreb index for any graph G can also be written as [6]

$$\prod\nolimits_2(G) = \prod_{u \in V(G)} d_G(u)^{d_G(u)}.$$

Xu *et al.* [19] defined the first and second multiplicative Zagreb coindices, respectively, as

$$\overline{\prod}_1(G) = \prod_{uv \notin E(G)} [d_G(u) + d_G(v)] \quad \text{and} \quad \overline{\prod}_2(G) = \prod_{uv \notin E(G)} d_G(u) d_G(v).$$

The main properties of multiplicative Zagreb indices are summarized in [1, 4, 10, 12, 17, 18, 20].

We introduce the modified second multiplicative Zagreb index as

$$\prod\nolimits_2^*(G) = \prod_{uv \in E(G)} [d_G(u) + d_G(v)]^{[d_G(u) + d_G(v)]}.$$

2. DERIVED GRAPHS

In recent papers [2, 3, 8], the authors obtained the expressions for Zagreb indices and coindices of derived graphs. This motivates us to find expressions for $\prod_1$, $\prod_2$, $\prod_1^*$ and $\overline{\prod}_2$ of derived graphs.

Let G be a graph with vertex set $V(G)$ and edge set $E(G)$. We are concerned with the following graphs derived from G ([8]):

- *line graph* $L = L(G)$; $V(L) = E(G)$ and the two vertices of L are adjacent if the corresponding edges of G are incident with a common vertex;
- *subdivision graph* $S = S(G)$; $V(S) = V(G) \cup E(G)$ and the vertex of S corresponding to the edge uv of G is inserted in the edge uv of G;

– *vertex-semitotal graph* $T_2 = T_2(G)$; $V(T_2) = V(G) \cup E(G)$ and $E(T_2) = E(S) \cup E(G)$;
– *edge-semitotal graph* $T_1 = T_1(G)$; $V(T_1) = V(G) \cup E(G)$ and $E(T_1) = E(S) \cup E(L)$;
– *total graph* $T = T(G)$; $V(T) = V(G) \cup E(G)$ and $E(T) = E(S) \cup E(G) \cup E(L)$;
– *paraline graph* $PL = PL(G)$ is the line graph of the subdivision graph.

In Figure 1 self-explanatory examples of these derived graphs are depicted.

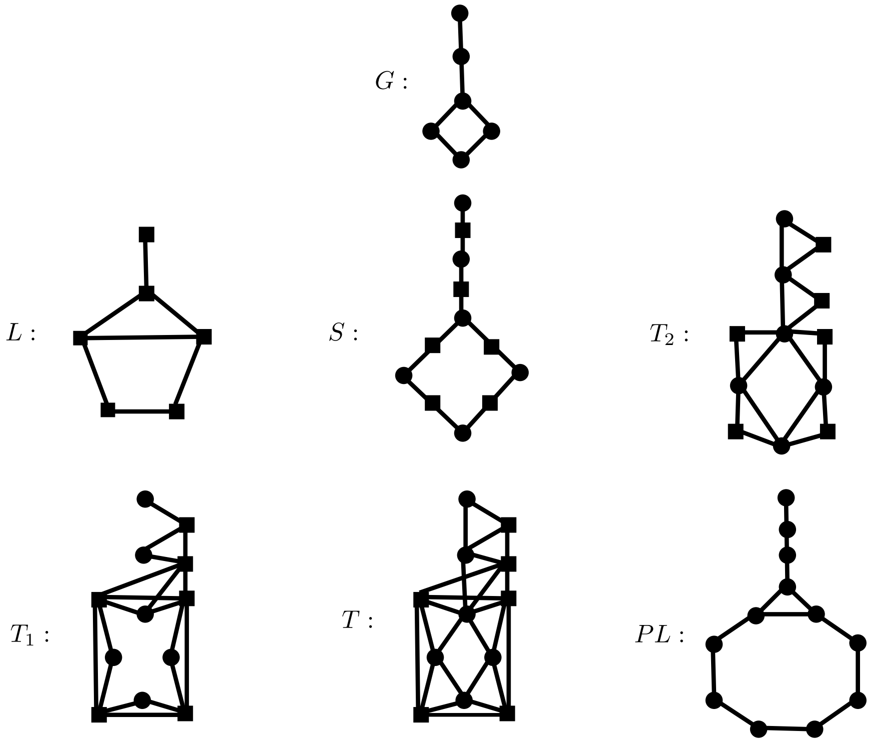

Fig. 1. Various graphs derived from the graph G. The vertices of these derived graphs (except the paraline graph PL), corresponding to the vertices of the parent graph G, are indicated by circles. The vertices of these graphs corresponding to the edges of the parent graph G are indicated by squares

In [19], Kexiang Xu *et al.* obtained the expressions for $\overline{\prod}_2(G)$ of any connected graph G as

$$\overline{\prod}_2(G) = \prod_{u \in V(G)} d_G(u)^{n-1-d_G(u)} \quad \text{and} \quad \prod_2(G)\overline{\prod}_2(G) = \left(\prod_1(G)\right)^{\frac{n-1}{2}}$$

which are not satisfied for a complete graph. The following lemmas give the correct expressions for $\overline{\prod}_2(G)$.

Lemma 2.1. *For a connected graph $G \neq K_n$, we have*

$$\overline{\prod}_2(G) = \prod_{u \in V(G)} d_G(u)^{n-1-d_G(u)}.$$

Lemma 2.2. *For a connected graph $G \neq K_n$, we have*

$$\prod_2(G)\overline{\prod}_2(G) = \left(\prod_1(G)\right)^{\frac{n-1}{2}}.$$

Next we present the values of multiplicative Zagreb indices and coindices for several classes of graphs.

Example 2.3. Let P_n be the path with n vertices. The pendant vertices have degree 1 and other vertices have degree 2. Hence,

(i) $\prod_1(P_n) = 4^{(n-2)}$,
(ii) $\prod_2(P_n) = 4^{(n-2)}$,
(iii) $\prod_1^*(P_n) = 9 \cdot 4^{(n-3)}$, $n \geq 3$,
(iv) $\overline{\prod}_1(P_n) = 2 \cdot 9^{(n-3)} \cdot 4^{(n-4)!}$, $n \geq 4$,
(v) $\overline{\prod}_2(P_n) = 2^{(n-2)(n-3)}$, $n \geq 3$,
(vi) $\prod_2^*(P_n) = 3^6 \cdot 4^{4(n-3)}$, $n \geq 3$.

Example 2.4. Consider the cycle C_n with n vertices. Since its every vertex is of degree 2, then

(i) $\prod_1(C_n) = 4^n$,
(ii) $\prod_2(C_n) = 4^n$,
(iii) $\prod_1^*(C_n) = 4^n$,
(iv) $\overline{\prod}_1(C_n) = 4^{\frac{n(n-3)}{2}}$, $n \geq 4$,
(v) $\overline{\prod}_2(C_n) = 4^{\frac{n(n-3)}{2}}$, $n \geq 4$,
(vi) $\prod_2^*(C_n) = 4^{4n}$.

Example 2.5. Let K_n be the complete graph on n vertices. All vertices of K_n have degree $n-1$ and so

(i) $\prod_1(K_n) = (n-1)^{2n}$, $n \geq 2$,
(ii) $\prod_2(K_n) = (n-1)^{n(n-1)}$, $n \geq 2$,
(iii) $\prod_1^*(K_n) = [2(n-1)]^{\frac{n(n-1)}{2}}$, $n \geq 2$,
(iv) $\overline{\prod}_1(K_n) = 0$,
(v) $\overline{\prod}_2(K_n) = 0$,
(vi) $\prod_2^*(K_n) = [2(n-1)]^{n(n-1)^2}$, $n \geq 2$.

Example 2.6. Let $K_{r,s}$ be the complete bipartite graph. Then $K_{r,s}$ has $r+s$ vertices and rs edges. Hence,

(i) $\prod_1(K_{r,s}) = r^{2s} \cdot s^{2r}$,
(ii) $\prod_2(K_{r,s}) = [rs]^{rs}$,
(iii) $\prod_1^*(K_{r,s}) = [r+s]^{rs}$,
(iv) $\overline{\prod}_1(K_{r,s}) = [2r]^{\frac{s(s-1)}{2}} \cdot [2s]^{\frac{r(r-1)}{2}}$, $r \neq 1$ and $s \neq 1$,
(v) $\overline{\prod}_2(K_{r,s}) = r^{s(s-1)} \cdot s^{r(r-1)}$, $r \neq 1$ and $s \neq 1$,
(vi) $\prod_2^*(K_{r,s}) = [r+s]^{rs(r+s)}$.

Example 2.7. Let W_n be the wheel on n vertices. Its central vertex has degree $n-1$ and its other vertices have degree 3. This implies

(i) $\prod_1(W_n) = (n-1)^2 \cdot 3^{2(n-1)}$,
(ii) $\prod_2(W_n) = [3(n-1)]^{(n-1)} \cdot 3^{2(n-1)}$,
(iii) $\prod_1^*(W_n) = [n+2]^{(n-1)} \cdot 6^{(n-1)}$,
(iv) $\overline{\prod}_1(W_n) = 6^{\frac{(n-1)(n-4)}{2}}$, $n \geq 5$,
(v) $\overline{\prod}_2(W_n) = 9^{\frac{(n-1)(n-4)}{2}}$, $n \geq 5$,
(vi) $\prod_2^*(W_n) = 6^{6(n-1)} \cdot [n+2]^{(n-1)(n+2)}$.

3. RESULTS

Theorem 3.1. *Let G be a graph of order n and size m. Then*

$$\prod_1(S) = 4^m \prod_1(G).$$

Proof. Note that S has $n+m$ vertices.

$$\prod_1(S) = \prod_{u \in V(S)} d_S(u)^2 = \prod_{u \in V(S) \cap V(G)} d_S(u)^2 \prod_{e \in V(S) \cap E(G)} d_S(e)^2.$$

Note that for $u \in V(S) \cap V(G)$, $d_S(u) = d_G(u)$ and for $e \in V(S) \cap E(G)$, $d_S(e) = 2$.

$$\prod_1(S) = \prod_{u \in V(G)} d_G(u)^2 \prod_{e \in E(G)} 2^2 = 4^m \prod_1(G). \qquad \square$$

Theorem 3.2. *Let G be a graph of order n and size m. Then*

$$\prod_2(S) = 4^m \prod_2(G).$$

Proof. Since S has $n+m$ vertices, then

$$\prod_2(S) = \prod_{u \in V(S)} d_S(u)^{d_S(u)} = \prod_{u \in V(S) \cap V(G)} d_S(u)^{d_S(u)} \prod_{e \in V(S) \cap E(G)} d_S(e)^{d_S(e)}.$$

Since for $u \in V(S) \cap V(G)$, $d_S(u) = d_G(u)$ and for $e \in V(S) \cap E(G)$, $d_S(e) = 2$.

$$\prod_2(S) = \prod_{u \in V(G)} d_G(u)^{d_G(u)} \prod_{e \in E(G)} 2^2 = 4^m \prod_2(G). \qquad \square$$

Theorem 3.3. *Let G be a graph of order n and size m. Then*

$$\prod_1^*(S) = \prod_{u \in V(G)} [2 + d_G(u)]^{d_G(u)}.$$

Proof. Since S has $n+m$ vertices, then we have

$$\prod_1^*(S) = \prod_{uv \in E(S)} [d_S(u) + d_S(e)].$$

Since for $u \in V(S) \cap V(G)$, $d_S(u) = d_G(u)$ and for $e \in V(S) \cap E(G)$, $d_S(e) = 2$.

$$\prod_1^*(S) = \prod_{u \in V(G)} [2 + d_G(u)]^{d_G(u)}.$$ □

Corollary 3.4. *Let G be a connected graph of order n and size m. Then*

$$\overline{\prod}_2(S) = \frac{2^{m^2+nm-3m}[\prod_1(G)]^{\frac{n+m-1}{2}}}{\prod_2(G)}.$$

Proof. From Lemma 2.2 we have

$$\overline{\prod}_2(S) = \frac{[\prod_1(S)]^{\frac{n+m-1}{2}}}{\prod_2(S)}.$$

From Theorems 3.1 and 3.2 we get the result. □

Theorem 3.5. *Let G be a graph of order n and size m. Then*

$$\prod_1(T_2) = 4^{n+m} \prod_1(G).$$

Proof. Note that T_2 has $n+m$ vertices.

$$\prod_1(T_2) = \prod_{u \in V(T_2)} d_{T_2}(u)^2 = \prod_{u \in V(T_2) \cap V(G)} d_{T_2}(u)^2 \prod_{e \in V(T_2) \cap E(G)} d_{T_2}(e)^2.$$

Note that for $u \in V(T_2) \cap V(G)$, $d_{T_2}(u) = 2d_G(u)$ and for $e \in V(T_2) \cap E(G)$, $d_{T_2}(e) = 2$.

$$\prod_1(T_2) = \prod_{u \in V(G)} [2d_G(u)]^2 \prod_{e \in E(G)} 2^2 = 4^{n+m} \prod_1(G).$$ □

Theorem 3.6. *Let G be a graph of order n and size m. Then*

$$\prod_2(T_2) = 64^m \prod_1(G) \prod_2(G).$$

Proof. Since T_2 has $n+m$ vertices and $3m$ edges, then we have

$$\begin{aligned}\prod\nolimits_2(T_2) &= \prod_{uv\in E(T_2)} d_{T_2}(u)d_{T_2}(v)\\ &= \prod_{uv\in E(T_2)\cap E(G)} d_{T_2}(u)d_{T_2}(v) \prod_{ue\in E(T_2)\setminus E(G)} d_{T_2}(u)d_{T_2}(e).\end{aligned}$$

Since for $u\in V(T_2)\cap V(G)$, $d_{T_2}(u)=2d_G(u)$ and for $e\in V(T_2)\cap E(G)$, $d_{T_2}(e)=2$.

$$\begin{aligned}\prod\nolimits_2(T_2) &= \prod_{uv\in E(G)} 2d_G(u)2d_G(v) \prod_{ue\in E(T_2)\setminus E(G)} (2)2d_G(u)\\ &= 4^m \prod_{uv\in E(G)} d_G(u)d_G(v)4^{2m} \prod_{u\in V(G)} d_G(u)^2\\ &= 64^m \prod\nolimits_1(G)\prod\nolimits_2(G).\end{aligned}$$

□

Theorem 3.7. *Let G be a graph of order n and size m. Then*

$$\prod\nolimits_1^*(T_2) = 8^m \prod\nolimits_1^*(G) \prod_{u\in V(G)} [1+d_G(u)]^{d_G(u)}.$$

Proof. Since T_2 has $n+m$ vertices and $3m$ edges, then we have

$$\begin{aligned}\prod\nolimits_1^*(T_2) &= \prod_{uv\in E(T_2)} [d_{T_2}(u)+d_{T_2}(v)]\\ &= \prod_{uv\in E(T_2)\cap E(G)} [d_{T_2}(u)+d_{T_2}(v)] \prod_{ue\in E(T_2)\setminus E(G)} [d_{T_2}(u)+d_{T_2}(e)].\end{aligned}$$

Since for $u\in V(T_2)\cap V(G)$, $d_{T_2}(u)=2d_G(u)$ and for $e\in V(T_2)\cap E(G)$, $d_{T_2}(e)=2$.

$$\begin{aligned}\prod\nolimits_1^*(T_2) &= \prod_{uv\in E(G)} [2d_G(u)+2d_G(v)] \prod_{ue\in E(T_2)\setminus E(G)} [2+2d_G(u)]\\ &= 2^m \prod_{uv\in E(G)} [d_G(u)+d_G(v)]2^{2m} \prod_{u\in V(G)} [1+d_G(u)]^{d_G(u)}\\ &= 8^m \prod\nolimits_1^*(G) \prod_{u\in V(G)} [1+d_G(u)]^{d_G(u)}.\end{aligned}$$

□

Corollary 3.8. *Let G be a connected graph of order n and size m. Then*

$$\overline{\prod}_2(T_2) = \frac{2^{(n+m)^2-n-7m}[\prod_1(G)]^{\frac{n+m-3}{2}}}{\prod_2(G)}.$$

Proof. From Lemma 2.2 we have

$$\overline{\prod}_2(T_2) = \frac{[\prod_1(T_2)]^{\frac{n+m-1}{2}}}{\prod_2(T_2)}.$$

From Theorems 3.5 and 3.6 we get the result. □

Theorem 3.9. *Let G be a graph of order n and size m. Then*

$$\prod\nolimits_1(T_1) = \prod\nolimits_1(G)\Big[\prod\nolimits_1^*(G)\Big]^2.$$

Proof. Note that T_1 has $n+m$ vertices.

$$\begin{aligned}\prod\nolimits_1(T_1) &= \prod_{u\in V(T_1)} d_{T_1}(u)^2\\ &= \prod_{u\in V(T_1)\cap V(G)} d_{T_1}(u)^2 \prod_{e_i\in V(T_1)\cap E(G)} d_{T_1}(e_i)^2.\end{aligned}$$

Note that for $u \in V(T_1)\cap V(G)$, $d_{T_1}(u) = d_G(u)$ and for $e_i \in V(T_1)\cap E(G)$, $d_{T_1}(e_i) = d_G(u_i) + d_G(v_i)$.

$$\begin{aligned}\prod\nolimits_1(T_1) &= \prod_{u\in V(G)} d_G(u)^2 \prod_{u_iv_i\in E(G)} [d_G(u_i) + d_G(v_i)]^2\\ &= \prod\nolimits_1(G)\Big[\prod\nolimits_1^*(G)\Big]^2.\end{aligned}$$ □

Theorem 3.10. *Let G be a graph of order n and size m. Then*

$$\prod\nolimits_2(T_1) = \prod\nolimits_2(G)\prod\nolimits_2^*(G).$$

Proof. Since T_1 has $n+m$ vertices, then we have

$$\begin{aligned}\prod\nolimits_2(T_1) &= \prod_{u\in V(T_1)} d_{T_1}(u)^{d_{T_1}(u)}\\ &= \prod_{u\in V(T_1)\cap V(G)} d_{T_1}(u)^{d_{T_1}(u)} \prod_{e_i\in V(T_1)\cap E(G)} [d_{T_1}(e_i)]^{[d_{T_1}(e_i)]}.\end{aligned}$$

Since for $u \in V(T_1)\cap V(G)$, $d_{T_1}(u) = d_G(u)$ and for $e_i \in V(T_1)\cap E(G)$, $d_{T_1}(e_i) = d_G(u_i) + d_G(v_i)$.

$$\begin{aligned}\prod\nolimits_2(T_1) &= \prod_{u\in V(G)} d_G(u)^{d_G(u)} \prod_{u_iv_i\in E(G)} [d_G(u_i) + d_G(v_i)]^{[d_G(u_i)+d_G(v_i)]}\\ &= \prod\nolimits_2(G)\prod\nolimits_2^*(G).\end{aligned}$$ □

Corollary 3.11. *Let G be a connected graph of order n and size m. Then*

$$\overline{\prod}_2(T_1) = \frac{[\prod_1(G)]^{\frac{n+m-1}{2}}\,[\prod_1^*(G)]^{n+m-1}}{\prod_2(G)\prod_2^*(G)}.$$

Proof. From Lemma 2.2 we have

$$\overline{\prod}_2(T_1) = \frac{[\prod_1(T_1)]^{\frac{n+m-1}{2}}}{\prod_2(T_1)}.$$

From Theorems 3.9 and 3.10 we get the result. □

In [16], an incorrect expression for $\prod_1(T)$ was established. The following theorem gives the correct expression for $\prod_1(T)$.

Theorem 3.12. *Let G be a graph of order n and size m. Then*

$$\prod\nolimits_1(T) = 4^n \prod\nolimits_1(G)\Big[\prod\nolimits_1^*(G)\Big]^2.$$

Proof. Note that T has $n+m$ vertices.

$$\prod\nolimits_1(T) = \prod_{u\in V(T)} d_T(u)^2 = \prod_{u\in V(T)\cap V(G)} d_T(u)^2 \prod_{e_i\in V(T)\cap E(G)} d_T(e_i)^2.$$

Note that for $u \in V(T)\cap V(G)$, $d_T(u) = 2d_G(u)$ and for $e_i \in V(T)\cap E(G)$, $d_T(e_i) = d_G(u_i) + d_G(v_i)$.

$$\prod\nolimits_1(T) = \prod_{u\in V(G)} [2d_G(u)]^2 \prod_{u_iv_i\in E(G)} [d_G(u_i)+d_G(v_i)]^2 = 4^n \prod\nolimits_1(G)\Big[\prod\nolimits_1^*(G)\Big]^2. \quad \square$$

Theorem 3.13. *Let G be a graph of order n and size m. Then*

$$\prod\nolimits_2(T) = 16^m \prod\nolimits_2^*(G)\Big[\prod\nolimits_2(G)\Big]^2.$$

Proof. Since T has $n+m$ vertices, then we have

$$\prod\nolimits_2(T) = \prod_{u\in V(T)} d_T(u)^{d_T(u)} = \prod_{u\in V(T)\cap V(G)} d_T(u)^{d_T(u)} \prod_{e_i\in V(T)\cap E(G)} d_T(e_i)^{d_T(e_i)}.$$

Note that for $u \in V(T)\cap V(G)$, $d_T(u) = 2d_G(u)$ and for $e_i \in V(T)\cap E(G)$, $d_T(e_i) = d_G(u_i) + d_G(v_i)$.

$$\begin{aligned}\prod\nolimits_2(T) &= \prod_{u\in V(G)} [2d_G(u)]^{[2d_G(u)]} \prod_{u_iv_i\in E(G)} [d_G(u_i)+d_G(v_i)]^{[d_G(u_i)+d_G(v_i)]}\\ &= \prod_{u\in V(G)} 2^{[2d_G(u)]}[d_G(u)]^{2d_G(u)} \prod\nolimits_2^*(G)\\ &= 16^m \prod\nolimits_2^*(G)\Big[\prod\nolimits_2(G)\Big]^2.\end{aligned}$$

□

Corollary 3.14. *Let G be a connected graph of order n and size m. Then*

$$\overline{\prod}_2(T) = \frac{2^{n(n+m-1)-4m}[\prod_1(G)]^{\frac{n+m-1}{2}}[\prod_1^*(G)]^{n+m-1}}{\prod_2^*(G)[\prod_2(G)]^2}.$$

Proof. From Lemma 2.2 we have

$$\overline{\prod}_2(T) = \frac{[\prod_1(T)]^{\frac{n+m-1}{2}}}{\prod_2(T)}.$$

From Theorems 3.12 and 3.13 we get the result. □

Theorem 3.15. *Let G be a graph of order n and size m. Then*

$$\prod_1(PL) = \Big[\prod_2(G)\Big]^2.$$

Proof. Note that paraline graph PL has $2m$ vertices, and $d_G(u)$ of its vertices have the same degree as the vertex u of the graph G.

$$\prod_1(PL) = \prod_{u\in V(PL)} d_{PL}(u)^2 = \prod_{u\in V(G)} d_G(u)^{[2d_G(u)]} = \Big[\prod_2(G)\Big]^2. \qquad \square$$

Theorem 3.16. *Let G be a graph of order n and size m. Then*

$$\prod_2(PL) = \prod_{u\in V(G)} [d_G(u)]^{[d_G(u)]^2}.$$

Proof. Since PL has $2m$ vertices, then we have

$$\begin{aligned}\prod_2(PL) &= \prod_{uv\in E(PL)} d_{PL}(u)d_{PL}(v)\\ &= \prod_2(G)\prod[d_G(u)]^{[d_G(u)(d_G(u)-1)]} = \prod_{u\in V(G)} [d_G(u)]^{[d_G(u)]^2}. \qquad \square\end{aligned}$$

Theorem 3.17. *Let G be a graph of order n and size m. Then*

$$\prod_1^*(PL) = \prod_1^*(G) \prod_{u\in V(G)} [2d_G(u)]^{\frac{d_G(u)(d_G(u)-1)}{2}}.$$

Proof. Since PL has $2m$ vertices, then we have

$$\prod_1^*(PL) = \prod_{uv\in E(PL)} [d_{PL}(u)+d_{PL}(v)] = \prod_1^*(G) \prod_{u\in V(G)} [2d_G(u)]^{\frac{d_G(u)(d_G(u)-1)}{2}}. \qquad \square$$

Corollary 3.18. *Let G be a connected graph of order n and size m. Then*

$$\overline{\prod}_2(PL) = \frac{[\prod_2(G)]^{2m-1}}{\prod\limits_{u\in V(G)} [d_G(u)]^{[d_G(u)]^2}}$$

Proof. From Lemma 2.2 we have

$$\overline{\prod}_2(PL) = \frac{[\prod_1(PL)]^{\frac{2m-1}{2}}}{\prod_2(PL)}.$$

From Theorems 3.15 and 3.16 we get the result. $\square$

One can easily obtain the expressions for multiplicative Zagreb indices and coindices of line graph L of graph G by considering edge degrees of G.

Theorem 3.19. *Let G be a graph of order n and size m. Then*

(i) $\prod_1(L) = \prod\limits_{e \in E(G)} d_G(e)^2$,

(ii) $\prod_2(L) = \prod\limits_{e_i \sim e_j} d_G(e_i) d_G(e_j)$,

(iii) $\prod_1^*(L) = \prod\limits_{e_i \sim e_j} [d_G(e_i) + d_G(e_j)]$,

(iv) $\overline{\prod}_1(L) = \prod\limits_{e_i \nsim e_j} [d_G(e_i) + d_G(e_j)]$,

(v) $\overline{\prod}_2(L) = \prod\limits_{e_i \nsim e_j} d_G(e_i) d_G(e_j)$,

where $e_i \sim e_j$ (resp. $e_i \nsim e_j$) means that the edges e_i and e_j are adjacent (resp. not adjacent) in G.

It remains a task for the future to find the expressions for $\prod_1^*(T_1)$ and $\prod_1^*(T)$.

In [19], the total multiplicative sum Zagreb index $\overset{T}{\prod}(G)$ of a graph G is defined as

$$\overset{T}{\prod}(G) = \prod_{u,v \in V(G)} [d_G(u) + d_G(v)].$$

Lemma 3.20 ([19])**.** *For a connected graph G, we have $\prod_1^*(G)\overline{\prod}_1(G) = \overset{T}{\prod}(G)$.*

By Lemma 3.20, one can find the expression for $\overline{\prod}_1$ of derived graphs. But obtaining the expression for $\overset{T}{\prod}$ of derived graphs is a difficult task.

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