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ESTIMATION OF THE DISTORTION RISK PREMIUM FOR HEAVY-TAILED LOSSES UNDER SERIAL DEPENDENCE

Hakim Ouadjed

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Abstract. In the actuarial literature, many authors have studied estimation of the reinsurance premium for heavy tailed i.i.d. sequences, especially for the Proportional Hazard (PH) due to Wang. The main aim of this paper is to extend this estimation for heavy tailed dependent sequences satisfying some mixing dependence structure. In this study we prove that the new estimator is asymptotically normal. The behavior of the estimator is examined using simulation for MA(1) process.

Keywords: extreme value theory, mixing processes, tail index estimation.

Mathematics Subject Classification: 60G70, 62G32.

1. INTRODUCTION

Let χ denote the set of nonnegative random variables on the probability space $(\Omega, \mathcal{A}, P)$.

In this study, we use the following distortion risk measure

$$\mathcal{M}_g : \chi \longrightarrow \mathbf{R}^+,$$
$$X \longrightarrow \mathcal{M}_g(X) = \int_0^\infty g(S_X(x))dx,$$

where $S_X(x) = 1 - F_X(x)$ is the survival function of X and g is a concave increasing function such that $g(0) = 0$, $g(1) = 1$. One remarks that this distortion measure is sub-additive. It was introduced by Denneberg [6] and Wang [19]. It verifies also axioms of coherent risk measure proposed by Artzner *et al.* [1]. In the field of insurance the risk measure $\mathcal{M}_g$ is called the risk premium.

A standard product of reinsurance is excess-of-loss reinsurance, which means the reinsurer only offset the loss of cedent exceeding a certain amount of retention $R > 0$.

By purchasing excess-of-loss reinsurance, the cedants limit their risk at a certain level. The premium of excess-of-loss reinsurance follows as:

$$\mathcal{M}_{g,R}(X) = \int_R^\infty g(S_X(x))dx.$$

If $g(x) = x^{1/\rho}, \rho \geq 1$ (called the distortion parameter), we obtain the PH-premium of excess-of-loss reinsurance (Wang [19])

$$\pi_{\rho,R}(X) = \int_R^\infty (S_X(x))^{1/\rho}dx. \tag{1.1}$$

The parameter ρ is called also the risk-aversion index. It controls the amount of risk loading in the premium.

The heavy-tailed nature of insurance claims requires that special attention be put into the analysis of the tail of a loss distribution. The reinsurance companies need to calculate the premium $\pi_{\rho,R}$ for covering such excess claims, which is usually very large. Extreme value theory has become one of the main theories in developing statistical models for extreme insurance losses. Many authors have studied estimation of the reinsurance premium when sequences are i.i.d. for different distributions, particularly for heavy tailed ones (see Vandewalle and Beirlant [18] and Necir *et al.* [13], Rassoul [16]). It is interesting to extend this estimation for dependent sequences with heavy tailed marginals since in economics real data sets are most often dependent.

The rest of this paper is organized as follows. In Section 2, we discus about the behavior of the tail empirical process under dependence. In Section 3, we construct a reinsurance premium estimation for positive stationary β-mixing sequence with heavy-tailed marginals which is the main result. In Section 4, we compute confidence bounds for $\pi_{\rho,R}$ by some simulations. Section 5 is devoted to the proofs.

2. TAIL EMPIRICAL PROCESS UNDER DEPENDENCE

Let $\{X_i\}$ a stationary sequence with common distribution function F of an insured risk $X > 0$ satisfy the following condition of β-mixing dependence structure

$$\beta(l) := \sup_{m\in\mathbf{N}} \mathbf{E}\left(\sup_{A\in\mathcal{B}_{m+l+1}^\infty} |P(A|\mathcal{B}_1^m) - P(A)|\right) \to 0,$$

as $l \to \infty$, where $\mathcal{B}_1^m$ and $\mathcal{B}_{m+l+1}^\infty$ denote the σ-fields generated by $(X_i)_{1\leq i\leq m}$ and $(X_i)_{m+l+1\leq i}$, respectively.

We assume that $S_X(x) = 1 - F_X(x)$ has regular variation function near infinity with index $-\alpha$, that is,

$$\lim_{v\to\infty} \frac{S_X(vx)}{S_X(v)} = x^{-\alpha}, \text{ for any } x > 0 \text{ and } 1 < \alpha < 2. \tag{2.1}$$

(see, e.g., de Haan and Ferreria [9]). Such d.f. constitute a major subclass of the family of heavy-tailed distributions. It includes distributions such those Pareto's, Burr's, Student's and log-gamma, which are known to be appropriate models of fitting large insurance claims, large fluctuations of prices, log-returns, and so on (see Beirlant *et al.* [2], Reiss and Thomas [17] for more details). A high quantile x_p situated in the border or even beyond the range of the available data is denoted

$$x_p := Q(1-p) = F^{-1}(1-p),\ p = p_n \to 0,\ \text{as } n \to \infty,\ np_n \to \omega \geq 0,$$

where $Q(s) = \inf\{x \in \mathbf{R} : F(x) \geq s,\ 0 < s < 1\}$ is the quantile function associated to the d.f. F. Note that the condition (2.1) is equivalent to

$$\lim_{t\to\infty} \frac{U(tx)}{U(t)} = x^{1/\alpha},\ \text{for any } x > 0, \tag{2.2}$$

where $U(t) = Q(1 - 1/t), t \geq 1$.

To get asymptotic normality of estimators of parameters of extreme events, it is necessary to quantify the speed of convergence in (2.2), then is usual to assume the following extra second regular variation condition, that involves a second order parameter $\eta < 0$:

$$\lim_{t\to\infty} (A(t))^{-1} \left(\frac{U(tx)}{U(t)} - x^{1/\alpha} \right) = x^{1/\alpha} \frac{x^{\eta} - 1}{\eta},\ \text{for any } x > 0, \tag{2.3}$$

where A is a suitably chosen function of constant sign near infinity. The most popular estimator of α, is the Hill estimator [11], with the form

$$\widehat{\alpha} = \left(\frac{1}{k} \sum_{i=1}^{k} \log X_{n-i,n} - \log X_{n-k+1,n} \right)^{-1}, \tag{2.4}$$

where $X_{1,n} \leq X_{2,n} \leq \ldots \leq X_{n,n}$ are the order statistics and $k = k_n$ is an intermediate sequence such that

$$k \to \infty\ ,\ k/n \to 0,\ n \to \infty. \tag{2.5}$$

For the high quantile estimation, we recall the classical semi-parametric Weissman-type estimator of x_p (Weissman [20])

$$\widehat{x}_p = X_{n-k,n} \left(\frac{np}{k} \right)^{-1/\widehat{\alpha}}. \tag{2.6}$$

Drees [8] established, for stationary β-mixing time series, the asymptotic behavior of the tail empirical quantile function (q.f.), $Q_n(t) := X_{n-[k_n t],n}$ where $k_n = k$ is an intermediate sequence. This result is reached considering a weighted approximation for $Q_n(t)$ and the following conditions:

(C1) Assumed that there exists a sequence l_n, $n \in \mathbf{N}$, such that

$$\lim_{n\to\infty} \frac{\beta(l_n)}{l_n} n + l_n k^{-1/2} \log^2 k = 0.$$

(C2) A regularity condition for the joint tail of (X_1, X_{1+m}):

$$c_m(x,y) = \lim_{x\to\infty} \frac{n}{k} P\left[X_1 > F_X^{-1}\left(1 - \frac{k}{n}x\right), X_{1+m} > F_X^{-1}\left(1 - \frac{k}{n}y\right)\right],$$

for all $m \in \mathbf{N},\ x > 0,\ y \le 1+\varepsilon,\ \varepsilon > 0$ and F^{-1} denoting the inverse function of F.

(C3) A uniform bound on the probability that both X_1 and X_{1+m} belong to an extreme interval:

$$\frac{n}{k} P\left(X_1 \in I_n(x,y), X_{1+m} \in I_n(x,y)\right) \le (y-x)\left(\widetilde{\rho}(m) + D_1 \frac{k}{n}\right),$$

for all $m \in \mathbf{N}$, $0 < x,\ y \le 1+\varepsilon$, where $D_1 \ge 0$ is a constant, $\widetilde{\rho}(m)$, is a sequence satisfying $\sum_{m=1}^{\infty} \widetilde{\rho}(m) < \infty$ and $I_n(x,y) =]F^{-1}(1-yk/n), F^{-1}(1-xk/n)]$.

(C4) The q.f. admits the following representation:

$$F^{-1}(1-t) = dt^{-\alpha^{-1}}(1 - r(t)),$$

with $|r(t)| \le \Phi(t)$, for some constant $d > 0$ and a function Φ which is τ-varying at 0 for $\tau > 0$, or $\tau = 0$ and Φ is non decreasing with $\lim_{t\downarrow 0} \Phi(t) = 0$.

(C5) A limiting behavior for k

$$\lim_{n\to\infty} \sqrt{(}k)\Phi(k/n) \to 0.$$

Under the conditions (C1)-(C5) with $l_n k/n \to 0$ as $n \to \infty$, Drees [8] proved that there exist versions of the tail empirical q.f. Q_n and a centered Gaussian process e with covariance function c given by

$$c(x,y) = x \wedge y + \sum_{m=1}^{\infty} [c_m(x,y) + c_m(y,x)], \tag{2.7}$$

such that

$$\sup_{t\in(0,1]} t^{1/\alpha+1/2}(1 + |\log t|)^{-1/2} \left| k^{1/2}\left(\frac{Q_n(t)}{F^{-1}(1-k/n)} - t^{-1/\alpha}\right) - \alpha^{-1} t^{-(1/\alpha+1)} e(t)\right| \xrightarrow{P} 0,$$

with $\xrightarrow{P}$ stands for convergence in probability.

Drees [8] observe that almost every estimator $\widehat{\alpha}$ of the tail index parameter α that are based only on the $k_n + 1$ largest order statistics, can be represented as a smooth functional T (verifying some regularity conditions) applied to the tail empirical q.f.. Hill estimator [11], Pickands estimator (Pickands [15]) and the moment estimator proposed by Dekkers *et al.* [5] are some examples. Drees [7] established the asymptotic normality of these estimators.

More precisely,

$$\sqrt{k}\left(\widehat{\alpha}-\alpha\right) \xrightarrow{D} \mathcal{N}\left(0, \sigma_{T,\alpha}^{2}\right), \tag{2.8}$$

where $\xrightarrow{D}$ stands for convergence in distribution and

$$\sigma_{T,\alpha}^{2} = \alpha^{2} \int\limits_{(0,1]} \int\limits_{(0,1]} (st)^{-(1+1/\alpha)} c(s,t) \nu_{T,\alpha}(ds) \nu_{T,\alpha}(dt), \tag{2.9}$$

with c is the function defined in (2.7) and $\nu_{T,\alpha}$ is a signed measure. For the Hill estimator (2.4) it can be proved that, it has signed measure given by

$$\nu_{H,\alpha}(dt) = t^{\alpha^{-1}} dt - \delta_{1}(dt), \tag{2.10}$$

with δ_1 the Dirac measure with mass 1 at 1.

3. DEFINING THE ESTIMATOR AND MAIN RESULTS

To estimate the risk measure $\pi_{\rho,R}(X)$, given in (1.1), when X is a positive stationary β-mixing process. Notice that for a suitable economic interest, the threshold R must be so large and depends on the sample size n of claims income. For this reason we will suppose that $R = Q(1-k/n)$, where $k = k_n$ is a sequence of integers defining in (2.5). This leads to rewrite $\pi_{\rho,R}(X)$ into

$$\pi_{\rho,R}(X) = \int\limits_{Q(1-k/n)}^{\infty} \left(S_X(x)\right)^{1/\rho} dx. \tag{3.1}$$

We present now our risk measure $\pi_{\rho,R}(X)$ as

$$\pi_{\rho,R}(X) = -\int\limits_{0}^{k/n} s^{1/\rho} dQ(1-s). \tag{3.2}$$

We derive the estimator for $Q(1-s)$ in (2.6), and after an integration, we obtain the following estimator for $\pi_{\rho,R}(X)$

$$\widehat{\pi}_{\rho,R}(X) = \frac{\rho\ (k/n)^{1/\rho}}{\widehat{\alpha}-\rho} X_{n-k,n}. \tag{3.3}$$

The asymptotic normality of $\widehat{\pi}_{\rho,R}(X)$ is established in the following theorem.

Theorem 3.1. *Suppose that X_i is a positive stationary β-mixing sequence with continuous common marginal distribution function F satisfying the conditions* (C1)–(C5). *Assume that* (2.3) *holds with $t^{-1/\rho}Q(1-1/t) \to 0$ as $t \to \infty$, and $k = k_n$ be such that*

$k \to \infty$, $k/n \to 0$, $l_n k/n \to 0$ and $\sqrt{n}A(n/k) \to 0$ as $n \to \infty$. Then, for $1 \le \rho < \alpha$, we have

$$\frac{(k/n)^{-1/\rho} k^{1/2}}{X_{n-k,n}} [\widehat{\pi}_{\rho,R}(X) - \pi_{\rho,R}(X)] \xrightarrow{D} \mathcal{N}(0, \sigma^2), \quad \text{as } n \to \infty,$$

where

$$\sigma^2 = (b-a)^2 c(1,1) + 2ab \int_0^1 t^{-1} c(1,t) dt, \tag{3.4}$$

with $a = \frac{\rho\alpha}{(\alpha-\rho)^2}$ and $b = \frac{\rho}{\alpha(\alpha-\rho)}$.

4. SIMULATION STUDY

Several approaches to the automated determination of an optimal sample fraction k for the Hill estimator have been studied (see e.g. Cheng and Peng [4], Neves and Fraga Alves [14]). Recently Caeiro and Gomes [3] proposed an algorithm essentially based on sample path stability (PS), using this algorithm we calculate k_{opt} an optimal value of k. The optimal premium can therefore be estimated by

$$\widehat{\pi}_\rho(X) = \frac{\rho\ (k_{opt}/n)^{1/\rho}}{\widehat{\alpha} - \rho} X_{n-k_{opt},n}. \tag{4.1}$$

4.1. MOVING AVERAGE MODEL

Consider now the stationary solution of the MA(1) equation

$$X_t = \lambda Z_{t-1} + Z_t,\ 1 \le t \le n, \tag{4.2}$$

where $0 < \lambda < 1$ and $\{Z_t\}$ i.i.d. innovations such that

$$F_Z(x) = (1 - x^{-\alpha})\mathbf{1}_{\{x \ge 1\}},\ 1 < \alpha < 2.$$

Drees [8] shows that conditions C1-C5 hold for the MA(1) in (4.2) with

$$l_n = [C \log n], \tag{4.3}$$

for a sufficiently large constant $C > 0$ (here $[x]$ denotes the largest integer smaller than or equal to x) and k_n satisfying.

$$\log^2 n \log^4 (\log n) = o(k_n). \tag{4.4}$$

We have that $1 - F_X(x) \sim (1 + \lambda^\alpha)(1 - F_Z(x))$ as $x \to \infty$ and from de Haan *et al.* [10], the covariance function c is given by

$$c(x,y) = x \wedge y + (1 + \lambda^\alpha)^{-1}(x \wedge y\lambda^\alpha + y \wedge x\lambda^\alpha).$$

Then we have

$$\sigma_H^2 = \frac{\rho^2\alpha^4 + \rho^2\alpha^2 - 2\alpha\rho^3 + \rho^4}{\alpha^2(\alpha-\rho)^4}\left(\frac{1+3\lambda^\alpha}{1+\lambda^\alpha}\right) + \frac{2\rho^2}{(\alpha-\rho)^3}\left(\frac{-\alpha\lambda^\alpha \ln\lambda}{1+\lambda^\alpha}\right).$$

To illustrate the performance of our estimator, we fix the distortion parameter $\rho = 1.1$ and $\rho = 1.2$, then we generate 100 replications of the time series $(X_1, \ldots, X_n)$ for different sample sizes $(1000, 2000)$, where X_t is an MA(1) process satisfying (4.2), where $\lambda = 0.4$, and we use two tail indices $\alpha = 1.6$ and $\alpha = 1.7$. The simulation results are presented in the Table 1, where *lb* and *ub* stand respectively for lower bound and upper bound of the confidence interval, we calculate also the absolute bias (abias) and the root mean squared error (RMSE). We remark what follows.

1. The premium increases with ρ, since as $\log[S_X(x)] < 0$, we have

$$\frac{d\pi_{\rho,R}}{d\rho} = -\frac{1}{\rho^2}\int_R^\infty [S_X(x)]^{1/\rho}\log[S_X(x)]dx > 0, \tag{4.5}$$

   justifying the risk-aversion index interpretation of ρ.
2. The abias and RMSE of our estimator decrease when the sample size increases, which indicates that the estimator is consistent.
3. For the same ρ and different values of α the premium increase when α decrease, this is caused by the tail of the distribution that becomes heavier.

Table 1. 95% confidence intervals for the premium

α	1.6		1.7	
ρ	1.1	1.2	1.1	1.2
$n = 1000$				
π	2.24808	3.132809	1.807002	2.422459
$\widehat{\pi}$	2.303153	3.028236	1.879957	2.375734
abias	0.05507251	0.104573	0.07295575	0.04672427
RMSE	0.1301033	0.1525836	0.1192907	0.1191978
lb	1.330498	1.331333	1.179637	1.25169
ub	3.275807	4.725138	2.580277	3.499779
length	1.945309	3.393805	1.40064	2.248089
$n = 2000$				
π	2.207413	3.123468	1.745688	2.38437
$\widehat{\pi}$	2.245965	3.044279	1.777907	2.346778
abias	0.03855116	0.0791887	0.03221853	0.03759216
RMSE	0.1221333	0.1253208	0.1084543	0.1006299
lb	1.546671	1.844361	1.273527	1.541767
ub	2.945258	4.244197	2.282286	3.151788
length	1.398587	2.399837	1.008759	1.610021

5. PROOFS

5.1. PROOF OF THEOREM 3.1

Denoting

$$H_1 = \rho\ (k/n)^{1/\rho} X_{n-k,n} \left\{ \frac{1}{\widehat{\alpha}-\rho} - \frac{1}{\alpha-\rho} \right\},$$
$$H_2 = \frac{\rho\ (k/n)^{1/\rho} Q(1-k/n)}{\alpha-\rho} \left\{ \frac{X_{n-k,n}}{Q(1-k/n)} - 1 \right\},$$
$$H_3 = \frac{\rho\ (k/n)^{1/\rho} Q(1-k/n)}{\alpha-\rho} - \int_{Q(1-k/n)}^{\infty} (S_X(x))^{1/\rho} dx.$$

Then, we can easily verify that

$$\widehat{\pi}_{\rho,R}(X) - \pi_{\rho,R}(X) = H_1 + H_2 + H_3.$$

H_1 can be written also

$$H_1 = \frac{\rho\widehat{\alpha}\alpha\ (k/n)^{1/\rho} X_{n-k,n}}{(\widehat{\alpha}-\rho)(\alpha-\rho)} \left\{ \frac{1}{\widehat{\alpha}} - \frac{1}{\alpha} \right\}.$$

Since $\widehat{\alpha}$ is a consistent estimator for α (see Hsing [12]), then for all large n

$$H_1 = (1+o_P(1)) \frac{\rho\alpha^2\ (k/n)^{1/\rho} Q(1-k/n)}{(\alpha-\rho)^2} \left\{ \frac{1}{\widehat{\alpha}} - \frac{1}{\alpha} \right\}$$

and

$$H_2 = (1+o_P(1)) \frac{\rho\ (k/n)^{1/\rho} Q(1-k/n)}{\alpha-\rho} \left\{ \frac{X_{n-k,n}}{Q(1-k/n)} - 1 \right\},$$

and

$$H_3 = \frac{\rho\ (k/n)^{1/\rho} Q(1-k/n)}{\alpha-\rho} - \int_{Q(1-k/n)}^{\infty} (S_X(x))^{1/\rho} dx.$$

Dress [8] has been shown that for all large n

$$\sqrt{k} \left(\frac{1}{\widehat{\alpha}} - \frac{1}{\alpha} \right) = \alpha^{-1} \int_{(0,1]} t^{-(1+1/\alpha)} e(t) \nu_{T,\alpha}(dt) + o_P(1),$$

$$\sqrt{k} \left(\frac{X_{n-k,n-1}}{Q(1-k/n)} - 1 \right) = \alpha^{-1} e(1) + o_P(1),$$

and

$$\frac{X_{n-k,n}}{Q(1-k/n)} = 1 + o_P(1),$$

where $e(1)$ is a centered Gaussian process with covariance function c defined in (2.7).

This implies that for all large n

$$H_1 = (1+o_P(1))\frac{\rho\alpha^2(k/n)^{1/\rho}Q(1-k/n)}{\sqrt{k}(\alpha-\rho)^2}\left(\alpha^{-1}\int\limits_{(0,1]} t^{-(1+1/\alpha)}e(t)\nu_{T,\alpha}(dt)+o_P(1)\right),$$

$$H_2 = (1+o_P(1))\frac{\rho(k/n)^{1/\rho}Q(1-k/n)}{\sqrt{k}(\alpha-\rho)}\left(\alpha^{-1}e(1)+o_P(1)\right).$$

Hence

$$\frac{(k/n)^{-1/\rho}\sqrt{k}}{Q(1-k/n)}(H_1+H_2)\to\mathcal{N}(0,\sigma^2),$$

where

$$\begin{aligned}\sigma^2 &= b^2c(1,1)+2ab\int\limits_{(0,1]} t^{-(1+1/\alpha)}c(1,t)\nu_{T,\alpha}(dt)\\ &\quad + a^2\int\limits_{(0,1]}\int\limits_{(0,1]}(st)^{-(1+1/\alpha)}c(s,t)\nu_{T,\alpha}(dt)\nu_{T,\alpha}(ds),\end{aligned} \tag{5.1}$$

with $a=\frac{\rho\alpha}{(\alpha-\rho)^2}$ and $b=\frac{\rho}{\alpha(\alpha-\rho)}$.

For H_3 we have

$$\frac{(k/n)^{-1/\rho}\sqrt{k}}{Q(1-k/n)}H_3=(k/n)^{-1/\rho}\sqrt{k}\left(\frac{\rho\;(k/n)^{1/\rho}}{\alpha-\rho}-\frac{\pi_{\rho,R}}{U(n/k)}\right)+o_P(1),$$

where $\pi_{\rho,R}=\int_{U(n/k)}^{\infty}(S(x))^{1/\rho}dx$. Since $x^{-1/\rho}U(x)\to 0$ as $x\to\infty$, then an integration by parts with a change of variables yields

$$\pi_{\rho,R}=(k/n)^{1/\rho}\left[\frac{1}{\rho}\int\limits_1^{\infty}x^{-1-1/\rho}U(nx/k)-U(n/k)\right].$$

Therefore

$$\begin{aligned}\frac{(k/n)^{-1/\rho}\sqrt{k}}{Q(1-k/n)}H_3 &= \sqrt{k}\left[\frac{\alpha}{\alpha-\rho}-\frac{1}{\rho}\int\limits_1^{\infty}x^{-1-1/\rho}\frac{U(nx/k)}{U(n/k)}dx\right]+o_P(1)\\ &= -\frac{1}{\rho}\sqrt{k}\int\limits_1^{\infty}x^{-1-1/\rho}\left[\frac{U(nx/k)}{U(n/k)}-x^{1/\alpha}\right]dx+o_P(1).\end{aligned}$$

From Theorem 2.3.9 of de Haan and Ferreira [9], for $1 < \alpha < 2$ and $1/\alpha < 1/\rho$, we obtain the limit as $n \to \infty$

$$\begin{aligned}\frac{(k/n)^{-1/\rho}\sqrt{k}}{Q(1-k/n)}H_3 &= -\frac{1}{\rho}\sqrt{k}A\left(\frac{n}{k}\right)\int_1^\infty x^{1/\alpha-1/\rho-1}\frac{x^\eta-1}{\eta}dx\,(1+o(1))+o_P(1)\\ &= \sqrt{k}A\left(\frac{n}{k}\right)\frac{\alpha^2\rho}{(\alpha-\rho)(\rho-\alpha\eta\rho-\alpha)}\,(1+o(1))+o_P(1),\end{aligned}$$

and since $\sqrt{k}A\left(\frac{n}{k}\right) \to 0$ as $n \to \infty$ we get

$$\frac{(k/n)^{-1/\rho}\sqrt{k}}{Q(1-k/n)}H_3 \to 0,\ \ as\ n \to \infty.$$

Replacing the signed measure (2.10) in the asymptotic variance (5.1), then

$$\begin{aligned}\sigma^2 &= b^2c(1,1)+2ab\left[\int_0^1 t^{-1}c(1,t)dt-c(1,1)\right]+a^2\left[\int_0^1\int_0^1(st)^{-1}c(s,t)ds\ dt\right.\\ &\quad\left.-2\int_0^1 s^{-1}c(s,1)ds+c(1,1)\right]\\ &= b^2c(1,1)+2ab\left[\int_0^1 t^{-1}c(1,t)dt-c(1,1)\right]+a^2c(1,1)\\ &= (b-a)^2c(1,1)+2ab\int_0^1 t^{-1}c(1,t)dt.\end{aligned}$$

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Hakim Ouadjed
o_hakim77@yahoo.fr

Department of Gestion
Faculty of Science of Gestion, Economic and Commerce
Mustapha Stambouli University, Mascara, Algeria

Mathematics Laboratory
Djillali Liabes University of Sidi Bel Abbes, Algeria

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