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CORRIGENDUM TO "ACYCLIC SUM-LIST-COLOURING OF GRIDS AND OTHER CLASSES OF GRAPHS" [OPUSCULA MATH. 37, NO. 4 (2017), 535–556]

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Abstract. This note provides some minor corrections to the article [*Acyclic sum-list-colouring of grids and other classes of graphs*, Opuscula Math. 37, no. 4 (2017), 535–556].

Keywords: sum-list colouring, acyclic colouring, grids, generalized Petersen graphs.

Mathematics Subject Classification: 05C30, 05C15.

1. CORRECTION OF LEMMA 5.18 OF [1]

We present the following corrected version of Lemma 5.18.

Lemma 1.1 (Correction of Lemma 5.18). *Let G be a graph and $\mathbf{L}' = \{L'(v)\}_{v\in V(G)}$ be a list assignment for G. Next, let $a \in \mathbb{N}$ and v_1, v_2 be two nonadjacent vertices of G that do not belong to the same 4-vertex cycle and for which $L'(v_1) = L'(v_2) = \{a\}$. Let H be a graph obtained from G by the identification of v_1 with v_2 into w and $\mathbf{L} = \{L(v)\}_{v\in V(H)}$ be the list assignment for H defined by $L(v) = L'(v)$ for $v \in V(H) \setminus \{w\}$ and $L(w) = \{a\}$. If H is $(\mathbf{L}, D_1)$-colourable, then G is $(\mathbf{L}', D_1)$-colourable or equivalently if G is not $(\mathbf{L}', D_1)$-colourable, then H is not $(\mathbf{L}, D_1)$-colourable.*

The correction of Lemma 5.18 consists in adding the assumption: there is no 4-vertex cycle that contains both v_1, v_2 vertices.

The change in the form of Lemma 5.18 has no consequences in the paper. The proof of the lemma and the application of the lemma are given under this assumption.

Next we give the corrected version of the whole section "Concluding remarks". We substitute "$n \equiv$" with "$m \equiv$" in two equations describing $G_{n,m}$ and $\chi_{sc}^{D_1}(G_{n,m})$, respectively. Moreover, the exact value $G_{n,m}$ given as a function of n, m, and consequently the lower bound on $\chi_{sc}^{D_1}(P_n \square P_m)$ given in Corollary 6.1 are corrected (it was a calculating mistake).

2. CONCLUDING REMARKS

Several supporting results presented in the paper can be used as tools for other research in this field. Some of them could help us to establish the exact values of $\chi_{sc}^{D_1}(P_n \Box P_m)$, when both $n, m \geq 5$. We are able to calculate some special numbers of this type but, in general, Problem 5.3 is still open. The method that is used in this work when $\min\{n, m\} \leq 4$ fails in the possible analogue of Lemma 5.14.

On the other hand, Theorem 5.21 implies the improvement of Corollary 5.1. Actually, graphs $P_n \Box P_m$ with $n \geq 1$, $m \geq 4$ contain subgraphs $G_{n,m}$ of the forms

$$G_{n,m} = \begin{cases} \lfloor \frac{m}{4} \rfloor (P_n \Box P_4), & \text{if } m \equiv 0 \pmod 4, \\ \lfloor \frac{m}{4} \rfloor (P_n \Box P_4) \cup (P_n \Box P_1), & \text{if } m \equiv 1 \pmod 4, \\ \lfloor \frac{m}{4} \rfloor (P_n \Box P_4) \cup (P_n \Box P_2), & \text{if } m \equiv 2 \pmod 4, \\ \lfloor \frac{m}{4} \rfloor (P_n \Box P_4) \cup (P_n \Box P_3) & \text{if } m \equiv 3 \pmod 4. \end{cases}$$

As we mentioned previously, $\chi_{sc}^{D_1}(G_1 \cup G_2) = \chi_{sc}^{D_1}(G_1) + \chi_{sc}^{D_1}(G_2)$, which gives

$$\chi_{sc}^{D_1}(G_{n,m}) = \begin{cases} \lfloor \frac{m}{4} \rfloor \lceil \frac{11n-3}{2} \rceil, & \text{if } m \equiv 0 \pmod 4, \\ \lfloor \frac{m}{4} \rfloor \lceil \frac{11n-3}{2} \rceil + n, & \text{if } m \equiv 1 \pmod 4, \\ \lfloor \frac{m}{4} \rfloor \lceil \frac{11n-3}{2} \rceil + (2n + \lfloor \frac{n}{2} \rfloor), & \text{if } m \equiv 2 \pmod 4, \\ \lfloor \frac{m}{4} \rfloor \lceil \frac{11n-3}{2} \rceil + (4n - 1), & \text{if } m \equiv 3 \pmod 4. \end{cases}$$

Careful calculation of these numbers yields

$$\chi_{sc}^{D_1}(G_{n,m}) = nm + \left\lceil \frac{(n-1)(m-1)}{2} \right\rceil - \left\lfloor \frac{m-1}{4} \right\rfloor \left\lfloor \frac{(n-1)}{2} \right\rfloor, \text{ for } n \geq 1, m \geq 4.$$

Observe that $V(G_{n,m}) = V(P_n \Box P_m)$. Thus Remark 2.1(i) implies

$$\chi_{sc}^{D_1}(G_{n,m}) \leq \chi_{sc}^{D_1}(P_n \Box P_m) \text{ for } n \geq 1, m \geq 4.$$

Note that $\chi_{sc}^{D_1}(G_{n,m})$ is an asymptotically better lower bound on $\chi_{sc}^{D_1}(P_n \Box P_m)$ than that one which was obtained in Corollary 5.1.

We turn out Lemma 5.2 to write its two alternative statements by one inequality.

$$\chi_{sc}^{D_1}(P_n \Box P_m) \leq nm + \left\lceil \frac{(n-1)(m-1)}{2} \right\rceil \text{ for } m, n \in \mathbb{N}.$$

Now we combine all the above observations and known values of $\chi_{sc}^{D_1}(P_n \Box P_m)$ that were discussed in the previous section in the following result.

Corollary 2.1 (Correction of Corollary 6.1)**.** *If* $n, m \in \mathbb{N}$*, then*

$$nm + \left\lceil \frac{(n-1)(m-1)}{2} \right\rceil - \left\lfloor \frac{m-1}{4} \right\rfloor \left\lfloor \frac{(n-1)}{2} \right\rfloor \leq \chi_{sc}^{D_1}(P_n \Box P_m) \leq nm + \left\lceil \frac{(n-1)(m-1)}{2} \right\rceil.$$

REFERENCES

[1] E. Drgas-Burchardt, A. Drzystek, *Acyclic sum-list-colouring of grids and other classes of graphs*, Opuscula Math. **37** (2017) 4, 535–556.

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