

DECOMPOSING COMPLETE 3-UNIFORM HYPERGRAPH $K_n^{(3)}$ INTO 7-CYCLES

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Abstract. We use the Katona–Kierstead definition of a Hamiltonian cycle in a uniform hypergraph. A decomposition of complete k -uniform hypergraph $K_n^{(k)}$ into Hamiltonian cycles was studied by Bailey–Stevens and Meszka–Rosa. For $n \equiv 2, 4, 5 \pmod{6}$, we design an algorithm for decomposing the complete 3-uniform hypergraphs into Hamiltonian cycles by using the method of edge-partition. A decomposition of $K_n^{(3)}$ into 5-cycles has been presented for all admissible $n \leq 17$, and for all $n = 4^m + 1$ when m is a positive integer. In general, the existence of a decomposition into 5-cycles remains open. In this paper, we show if $42 \mid (n-1)(n-2)$ and if there exist $\lambda = \frac{(n-1)(n-2)}{42}$ sequences $(k_{i_0}, k_{i_1}, \dots, k_{i_6})$ on $D_{all}(n)$, then $K_n^{(3)}$ can be decomposed into 7-cycles. We use the method of edge-partition and cycle sequence. We find a decomposition of $K_{37}^{(3)}$ and $K_{43}^{(3)}$ into 7-cycles.

Keywords: uniform hypergraph, 7-cycle, cycle decomposition.

Mathematics Subject Classification: 05C65, 05C85.

1. INTRODUCTION

The notion of hamiltonicity in hypergraphs was introduced by Berge in 1970. A problem on decomposing complete 3-uniform hypergraphs into Hamilton cycles has been completely solved by Verall in 1994 [12]. It is also worth to mention a definition of a cycle by Berge. Recently, in [5] there were given necessary and sufficient conditions for a decomposition of the complete 3-uniform hypergraph of order n into 4-cycles. Meanwhile, many papers studied the two different definitions of a Hamiltonian cycle in [6, 13], which are due to Katona and Kierstead, Wang and Lee, respectively. In fact, the two different definitions of a Hamiltonian cycle are the same. A decomposition of complete k -uniform hypergraphs into Hamiltonian cycles has been considered in [1, 3, 4, 6, 10, 13–15]. In the paper [10], Hamiltonian decompositions of $K_n^{(3)}$ for all admissible $n \leq 32$ has been resolved. Recently, by programming, using the method of edge-partition and cycle

sequence, we have obtained some results for all admissible $32 < n \leq 46$ and $n \neq 43$ in [2]. Meszka and Rosa have introduced a necessary condition for the existence of 5-cycles such a decomposition is that $n \equiv 1, 2, 5, 7, 10$ or $11 \pmod{15}$. In [10], the problem of decomposing the complete 3-uniform hypergraph into 5-cycles and $\ell (\geq 5)$ -cycles are open. A decomposition of $K_n^{(3)}$ into 5-cycles exists for all admissible $n \leq 17$, for all $n = 4^m + 1$, m a positive integer. In [7, 8], the author proposed to find a decomposition of $K_n^{(3)}$ into 5-cycles for $n \in \{5, 7, 10, 11, 16, 17, 20, 22, 26\}$ and shown that if $K_n^{(3)}$ can be decomposed into 5-cycles, then $K_{5n}^{(3)}$ can also be decomposed into 5-cycles. In [9], we found a decomposition of $K_n^{(3)}$ into 7-cycles for $n \in \{7, 8, 14, 16, 22, 23\}$ and have shown if $K_n^{(3)}$ can be decomposed into 7-cycles, then $K_{7n}^{(3)}$ can also be decomposed into 7-cycles. However, the problem of decomposing the complete 3-uniform hypergraph into 7-cycles are difficult. In this paper, under some conditions, we show if $42 \mid (n - 1)(n - 2)$ and if there exist $\lambda = \frac{(n-1)(n-2)}{42}$ sequences $(k_{i_0}, k_{i_1}, \dots, k_{i_6})$ on $D_{all}(n)$, then $K_n^{(3)}$ can be decomposed into 7-cycles.

2. PRELIMINARIES

A hypergraph $H = (V, E)$ consists of a finite set V of vertices with a family E of subsets of V , called hyperedges (or simply edges). If each (hyper)edge has a size k , we say that H is a k -uniform hypergraph. In particular, the complete k -uniform hypergraph on n vertices has all k -subsets of $\mathbb{Z}_n = \{0, 1, \dots, n - 1\}$ as edges, denote it by $K_n^{(k)}$. The set of (hyper)edges of $K_n^{(3)}$ is denoted by $\varepsilon(K_n^{(3)})$.

Definition 2.1. Let $H = (V, E)$ be a k -uniform hypergraph. An ℓ -cycle in H is a cyclic ordering $(v_0, v_1, \dots, v_{\ell-1})$ of the elements of V such that each mod k consecutive k -tuple of vertices is an edge of H , where $3 \leq k < \ell - 1$.

Definition 2.2. An ℓ -cycle decomposition of H is a partition of the set of (hyper)edges of H into mutually-edge-disjoint ℓ -cycles.

We introduce the method of edge-partition and the cycle-model from [3].

Definition 2.3 ([1]). Let $T = \{a, b, c\}$ be a triple of distinct elements of \mathbb{Z}_n . Then its *difference pattern* $\pi(T)$ is the equivalence class of ordered triples containing cyclic rotations of $(b - a, c - b, a - c)$ and $(c - a, b - c, a - b)$ (where the differences are taken modulo n).

Clearly, the three differences sum to zero. Therefore, if we know that the first two differences are x and y , then the third is $n - x - y$. Omitting the third number, we obtain a difference pair. Using edge-partition of $K_n^{(3)}$ as in paper [3], all difference pairs of the hypergraph $K_n^{(3)}$ may be obtained. Let \mathbb{Z} be the set of integers, n be a fixed positive integer, and $\mathbb{Z}_n = \{0, 1, \dots, n - 1\}$. Let

$$D_{all}(n) = \{(k_1, k_2) \mid 1 \leq k_1, k_2 \leq n - 1, \text{ and } k_1 + k_2 \neq n\},$$

$$D(n) = D_e \cup D_t \cup D_m,$$

where

$$\begin{aligned} D_e &= \left\{ (k_1, k_2) \in D_{all}(n) \mid k_1 = k_2 = k, 1 \leq k < \frac{n}{2} \right\}, \\ D_l &= \left\{ (k_1, k_2) \in D_{all}(n) \mid 1 \leq k_1 < k_2 < \frac{n - k_1}{2} \right\}, \\ D_m &= \left\{ (k_2, k_1) \in D_{all}(n) \mid (k_1, k_2) \in D_l \right\}. \end{aligned}$$

Given a difference pair $(k_1, k_2) \in D_{all}(n)$ and an integer $m \in \mathbb{Z}_n$, define a subhypergraph of $K_n^{(3)}$ generated by (k_1, k_2) as follows:

$$E(m; k_1, k_2) = \{m, m + k_1, m + k_1 + k_2\} \pmod{n},$$

We introduce the notation

$$H(k_1, k_2) = \{E(m; k_1, k_2) \mid m \in \mathbb{Z}_n\},$$

where the addition is performed modulo n .

Now we repeat some of the results from [3] to make the paper self-contained.

Lemma 2.4 ([3]). *Let (k_1, k_2) and (k'_1, k'_2) be arbitrary two distinct difference pairs in $D_{all}(n)$ we have either*

$$H(k_1, k_2) \cap H(k'_1, k'_2) = \emptyset$$

or

$$H(k_1, k_2) = H(k'_1, k'_2)$$

and a necessary and sufficient condition for the second equation is

$$(k_1, k_2) \equiv \begin{cases} (k'_1, k'_2) & \text{or} \\ (k'_1 + k'_2, -k'_2) & \text{or} \\ (-k'_1, k'_1 + k'_2) & \text{or} \\ (k'_2, -k'_1 - k'_2) & \text{or} \\ (-k'_1 - k'_2, k'_1) & \text{or} \\ (-k'_2, -k'_1) \end{cases} \pmod{n}.$$

Definition 2.5 ([3]). Let (k_1, k_2) and (k'_1, k'_2) be arbitrary two distinct difference pairs in $D_{all}(n)$. We say (k_1, k_2) and (k'_1, k'_2) are equivalent if $H(k_1, k_2) = H(k'_1, k'_2)$. This is denoted by $(k'_1, k'_2) \sim (k_1, k_2)$.

Lemma 2.6 (Edge-partition of $K_n^{(3)}$, [3]). *For any $K_n^{(3)}$,*

$$\varepsilon(K_n^{(3)}) = \bigcup_{(k_1, k_2) \in D(n)} H(k_1, k_2),$$

where $(k_1, k_2) \in D(n)$. If $k_1 \neq k_2$, for convenience, we use (k_1, k_2) to denote (k_1, k_2) and (k_2, k_1) .

Definition 2.7. Let n be a positive integer. For any $0 \leq i, j \leq \ell - 1$, $(k_i, k_{i+1}) \in D_{all}(n)$, and $(k_i, k_{i+1}) \not\sim (k_j, k_{j+1})$ ($i \neq j$), let $(k_0, k_1, \dots, k_{\ell-1})$ be a sequence on $D_{all}(n)$. The sequence $(k_0, k_1, \dots, k_{\ell-1})$ induces the cycle sequence

$$(r_0, r_1, \dots, r_{\ell-1}) \tag{2.1}$$

Sequence (2.1) satisfies the following two conditions:

- (a) $r_0 = 0, \sum_{i=0}^j k_i \equiv r_j \pmod{n}, r_\ell = 0.$
- (b) For any i, j ($i \neq j$), $r_i \neq r_j.$

Then $(r_0, r_1, \dots, r_{\ell-1})$ is an ℓ -cycle, denoted by $C_\ell = (r_0, r_1, \dots, r_{\ell-1})$, called *base cycle*. According to the definition of difference pattern $\pi(T)$, we obtain the set of ℓ -cycles $\{C_\ell + i \mid i \in \mathbb{Z}_n\}$, where $C_\ell + i = (r_0 + i, r_1 + i, \dots, r_{\ell-1} + i) \pmod{n}$. In particular, if $\ell = n$, then $(r_0, r_1, \dots, r_{n-1})$ is a base Hamiltonian cycle, denoted by $C_n = (r_0, r_1, \dots, r_{n-1})$.

Definition 2.8. Let n be a positive integer, for any $0 \leq i, j \leq \ell - 1$, $(k_i, k_{i+1}) \in D_{all}(n)$, and $(k_i, k_{i+1}) \not\sim (k_j, k_{j+1})$ ($i \neq j$). Given a sequence $(k_0, k_1, \dots, k_{\ell-1})$ in $D_{all}(n)$ and an integer $m \in \mathbb{Z}_n$, define a subhypergraph of $K_n^{(3)}$ generated by $(k_0, k_1, \dots, k_{\ell-1})$ as follows:

$$\begin{aligned}
 (m; k_0, k_1, \dots, k_{\ell-1}) &= \left\{ m, m + k_0, m + k_0 + k_1, \dots, \sum_{j=0}^{\ell-1} k_j \right\} \pmod{n}, \\
 E(m; k_0, k_1, \dots, k_{\ell-1}) &= \left\{ m + \sum_{j=0}^{i-1} k_j, m + \sum_{j=0}^i k_j, m + \sum_{j=0}^{i+1} k_j \right\} \pmod{n},
 \end{aligned}$$

where $k_\ell = k_0$.

We introduce the notation

$$H(k_0, k_1, \dots, k_{\ell-1}) = \{E(m; k_0, k_1, \dots, k_{\ell-1}) \mid m \in \mathbb{Z}_n\},$$

where the addition is performed modulo n .

Lemma 2.9. Let n be a positive integer, for any $0 \leq i, j \leq \ell - 1$, $(k_i, k_{i+1}) \in D_{all}(n)$, and $(k_i, k_{i+1}) \not\sim (k_j, k_{j+1})$ ($i \neq j$). If $(k_0, k_1, \dots, k_{\ell-1})$ is a sequence on $D_{all}(n)$, then

$$H(k_0, k_1, \dots, k_{\ell-1}) = \bigcup_{i=0}^{\ell-1} H(k_i, k_{i+1}),$$

where $k_\ell = k_0$.

Proof. For any $0 \leq i \leq \ell - 1$ and an integer $m \in \mathbb{Z}_n$, there is the edge of the $H(k_0, k_1, \dots, k_{\ell-1})$ as follows:

$$\left\{ m + \sum_{j=0}^{i-1} k_j, m + \sum_{j=0}^i k_j, m + \sum_{j=0}^{i+1} k_j \right\} \pmod{n}.$$

Obviously, the edge $\left\{ m + \sum_{j=0}^{i-1} k_j, m + \sum_{j=0}^i k_j, m + \sum_{j=0}^{i+1} k_j \right\} \pmod n$ induces the difference pair (k_i, k_{i+1}) , that is

$$H(k_i, k_{i+1}) = \left\{ m + \sum_{j=0}^{i-1} k_j, m + \sum_{j=0}^i k_j, m + \sum_{j=0}^{i+1} k_j \right\} \pmod n.$$

Hence, we have

$$H(k_0, k_1, \dots, k_{\ell-1}) = \bigcup_{i=0}^{\ell-1} H(k_i, k_{i+1}). \quad \square$$

3. DECOMPOSING $K_n^{(3)}$ INTO 7-CYCLES

Theorem 3.1. *A necessary condition for the decomposition of complete 3-uniform hypergraph $K_n^{(3)}$ into 7-cycles is $42 \mid n(n-1)(n-2)$.*

Theorem 3.2. *If there exist $\lambda = \frac{(n-1)(n-2)}{42}$ sequences $(k_{i_0}, k_{i_1}, \dots, k_{i_6})$ on $D_{all}(n)$ by Definition 2.5, then the sequences $(k_{i_0}, k_{i_1}, \dots, k_{i_6})$ satisfy the following condition: if $(k_{i_\alpha}, k_{i_{(\alpha+1)}}) \not\sim (k_{j_\beta}, k_{j_{(\beta+1)}})$ ($i \neq j, \alpha \neq \beta, i, j \in [1, \lambda], \alpha, \beta \in [0, 5]$), then $K_n^{(3)}$ can be decomposed into 7-cycles.*

Proof. Let there exist $\lambda = \frac{(n-1)(n-2)}{42}$ sequences $(k_{i_0}, k_{i_1}, \dots, k_{i_6})$ on $D_{all}(n)$ by Definition 2.5. The sequence $(k_{i_0}, k_{i_1}, \dots, k_{i_6})$ induces the cycle sequence

$$(r_{i_0}, r_{i_1}, \dots, r_{i_6}), \quad 1 \leq i \leq \lambda. \tag{3.1}$$

Sequence (3.1) satisfies the following two conditions:

- (a) $r_{i_0} = 0, \sum_{\alpha=0}^j k_{i_\alpha} \equiv r_{i_j} \pmod n, r_{i_7} = 0.$
- (b) For any $\alpha, \beta \in [0, 6]$ ($\alpha \neq \beta$), $r_{i_\alpha} \neq r_{i_\beta}.$

Obviously, we obtain the set of base cycle λ 7 cycles. We can decompose the edges of $K_n^{(3)}$ into $n\lambda$ 7-cycles produced by λ -base 7-cycles. By the structure of base cycle, we obtain the set of 7-cycles $\{C_{7_\alpha} + i \mid i \in \mathbb{Z}_n, \alpha \in [1, \lambda]\}$. By the method of edge-partition, we obtain a decomposition of $K_n^{(3)}$ into $n\lambda$ 7-cycles, that is,

$$\begin{aligned} \varepsilon(K_n^{(3)}) &= \bigcup_{(k_1, k_2) \in (D(n))} H(k_1, k_2) = \bigcup_{(k_{i_0}, k_{i_1}, \dots, k_{i_6}) \in D_{all}(n)} H(k_{i_0}, k_{i_1}, \dots, k_{i_6}) \\ &= \bigcup_{i=1}^{\lambda} \{C_{7_i} + j, j \in \mathbb{Z}_n\}. \end{aligned}$$

Hence, we obtain a decomposition of $K_n^{(3)}$ into $n\lambda$ 7- cycles. □

Example 3.3. $K_{37}^{(3)}$ can be decomposed into 7-cycles.

Proof. We can decompose the edges of $K_{37}^{(3)}$ into 1110 7-cycles produced by 30 base 7-cycles as follows. According to our method, we have

$$|\varepsilon(K_{37}^{(3)})| = \binom{37}{3} = 7770$$

edges and $7 \mid 7770$. We have

$$D(37) = \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6), (7, 7), (8, 8), (9, 9), (10, 10), (11, 11), (12, 12), (13, 13), (14, 14), (15, 15), (16, 16), (17, 17), (18, 18), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (1, 8), (1, 9), (1, 10), (1, 11), (1, 12), (1, 13), (1, 14), (1, 15), (1, 16), (1, 17), (2, 3), (2, 4), (2, 5), (2, 6), (2, 7), (2, 8), (2, 9), (2, 10), (2, 11), (2, 12), (2, 13), (2, 14), (2, 15), (2, 16), (2, 17), (3, 4), (3, 5), (3, 6), (3, 7), (3, 8), (3, 9), (3, 10), (3, 11), (3, 12), (3, 13), (3, 14), (3, 15), (3, 16), (4, 5), (4, 6), (4, 7), (4, 8), (4, 9), (4, 10), (4, 11), (4, 12), (4, 13), (4, 14), (4, 15), (4, 16), (5, 6), (5, 7), (5, 8), (5, 9), (5, 10), (5, 11), (5, 12), (5, 13), (5, 14), (5, 15), (6, 7), (6, 8), (6, 9), (6, 10), (6, 11), (6, 12), (6, 13), (6, 14), (6, 15), (7, 8), (7, 9), (7, 10), (7, 11), (7, 12), (7, 13), (7, 14), (8, 9), (8, 9), (8, 10), (8, 11), (8, 12), (8, 13), (8, 14), (9, 10), (9, 11), (9, 12), (9, 13), (10, 11), (10, 12), (10, 13), (11, 12)\}.$$

Now, we find the decomposition of $K_{37}^{(3)}$. On $D(37)$, according to Definition 2.5, we obtain $\lambda = 30$ sequences as follows:

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|-----------------------------------|-----------------------------------|-----------------------------------|
| (1) (1, 1, 2, 1, 3, 1, 28), | (2) (1, 4, 35, 5, 31, 5, 30), | (3) (1, 7, 33, 6, 8, 2, 25), |
| (4) (1, 9, 30, 4, 1, 10, 27), | (5) (1, 12, 27, 2, 9, 1, 22), | (6) (1, 13, 26, 3, 2, 12, 17), |
| (7) (1, 15, 24, 3, 3, 12, 16), | (8) (1, 16, 23, 3, 5, 8, 18), | (9) (1, 21, 18, 2, 15, 4, 13), |
| (10) (1, 24, 15, 2, 16, 5, 11), | (11) (1, 26, 14, 4, 29, 31, 7), | (12) (2, 18, 22, 16, 6, 16, 14), |
| (13) (2, 23, 17, 3, 6, 14, 9), | (14) (2, 28, 12, 28, 30, 5, 6), | (15) (3, 7, 3, 26, 4, 10, 21), |
| (16) (3, 14, 28, 11, 9, 10, 16), | (17) (3, 16, 25, 31, 10, 11, 15), | (18) (3, 21, 20, 18, 11, 13, 12), |
| (19) (3, 23, 18, 32, 8, 17, 20), | (20) (4, 5, 4, 25, 4, 21, 11), | (21) (4, 16, 6, 22, 4, 23, 9), |
| (22) (4, 9, 7, 18, 5, 24, 7), | (23) (4, 14, 30, 27, 24, 32, 5), | (24) (5, 5, 7, 22, 5, 15, 15), |
| (25) (5, 18, 26, 28, 17, 7, 10), | (26) (5, 20, 28, 27, 18, 6, 7), | (27) (6, 11, 12, 9, 15, 8, 13), |
| (28) (6, 19, 26, 17, 13, 14, 16), | (29) (6, 21, 8, 9, 9, 13, 8), | (30) (7, 19, 25, 27, 25, 23, 14), |

Let D be a collection of 30 sequences above. Thus, they correspond to 30-base 7-cycles:

$$\begin{aligned}
 C_{7(1)} &= (0, 1, 2, 4, 5, 8, 9), & C_{7(2)} &= (0, 1, 5, 3, 8, 2, 7), \\
 C_{7(3)} &= (0, 1, 8, 4, 2, 10, 12), & C_{7(4)} &= (0, 1, 10, 3, 7, 8, 18), \\
 C_{7(5)} &= (0, 1, 13, 3, 5, 14, 15), & C_{7(6)} &= (0, 1, 14, 3, 6, 8, 20), \\
 C_{7(7)} &= (0, 1, 16, 3, 6, 9, 21), & C_{7(8)} &= (0, 1, 17, 3, 6, 11, 19), \\
 C_{7(9)} &= (0, 1, 22, 3, 5, 20, 24), & C_{7(10)} &= (0, 1, 25, 3, 5, 21, 26), \\
 C_{7(11)} &= (0, 1, 27, 3, 7, 36, 30), & C_{7(12)} &= (0, 2, 20, 5, 1, 7, 23), \\
 C_{7(13)} &= (0, 2, 25, 5, 814, 28), & C_{7(14)} &= (0, 2, 30, 5, 33, 26, 31), \\
 C_{7(15)} &= (0, 3, 10, 13, 2, 6, 16), & C_{7(16)} &= (0, 3, 17, 8, 2, 11, 21), \\
 C_{7(17)} &= (0, 3, 19, 7, 1, 11, 22), & C_{7(18)} &= (0, 3, 24, 7, 1, 12, 25), \\
 C_{7(19)} &= (0, 3, 26, 7, 2, 10, 27), & C_{7(20)} &= (0, 4, 9, 13, 1, 5, 26), \\
 C_{7(21)} &= (0, 4, 10, 16, 1, 5, 28), & C_{7(22)} &= (0, 4, 13, 20, 1, 6, 30), \\
 C_{7(23)} &= (0, 4, 18, 11, 1, 25, 20), & C_{7(24)} &= (0, 5, 10, 17, 2, 7, 22), \\
 C_{7(25)} &= (0, 5, 23, 12, 3, 20, 27), & C_{7(26)} &= (0, 5, 25, 16, 6, 24, 30), \\
 C_{7(27)} &= (0, 6, 17, 29, 1, 16, 24), & C_{7(28)} &= (0, 6, 25, 14, 31, 7, 21), \\
 C_{7(29)} &= (0, 6, 27, 35, 7, 16, 29), & C_{7(30)} &= (0, 7, 26, 14, 4, 29, 15),
 \end{aligned}$$

By the method of edge-partition, we obtain the decomposition of $K_{37}^{(3)}$ into 1110 7-cycles, that is

$$\begin{aligned}
 \varepsilon(K_{37}^{(3)}) &= \bigcup_{(k_1, k_2) \in D(37)} H(k_1, k_2) \\
 &= \bigcup_{(k_0, k_1, \dots, k_6) \in D_{all}(37)} H(k_0, k_1, \dots, k_6) \\
 &= \bigcup_{i=1}^{30} \{C_{7(i)} + j, j \in \mathbb{Z}_{37}\}.
 \end{aligned}$$

Hence, we obtain the decomposition of $K_{37}^{(3)}$ into 1110 7-cycles. □

Example 3.4. $K_{43}^{(3)}$ can be decomposed into 7-cycles.

Proof. We can decompose the edges of $K_{43}^{(3)}$ into 1763 7-cycles produced by 41 base 7-cycles as follows. According to our method, we have

$$|\varepsilon(K_{43}^{(3)})| = \binom{43}{3} = 12341$$

edges and $7 \mid 12341$.

$$D(43) = \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6), (7, 7), (8, 8), (9, 9), (10, 10), (11, 11), (12, 12), (13, 13), (14, 14), (15, 15), (16, 16), (17, 17), (18, 18), (19, 19), (20, 20), (21, 21), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (1, 8), (1, 9), (1, 10), (1, 11), (1, 12), (1, 13), (1, 14), (1, 6), (1, 7), (1, 8), (1, 9), (1, 10), (1, 11), (1, 12), (1, 13), (1, 14), (1, 15), (1, 16), (1, 15), (1, 16), (1, 17), (1, 18), (1, 19), (1, 20), (1, 21), (2, 3), (2, 4), (2, 5), (2, 6), (2, 7), (2, 8), (2, 9), (2, 10), (2, 11), (2, 12) (2, 13), (2, 14), (2, 15), (2, 16), (2, 17), (2, 18), (2, 19), (2, 20), (3, 4), (3, 5), (3, 6), (3, 7), (3, 8), (3, 9), (3, 10), (3, 11), (3, 12), (3, 13), (3, 14), (3, 15), (3, 16), (3, 17), (3, 18), (3, 19), (4, 5), (4, 6), (4, 7), (4, 8), (4, 9), (4, 10)(4, 11), (4, 12), (4, 13), (4, 14), (4, 15), (4, 16), (4, 17), (4, 18), (4, 19), (5, 6), (5, 7), (5, 8), (5, 9), (5, 10), (5, 11), (5, 12), (5, 13), (5, 14), (5, 15), (5, 16), (5, 17), (5, 18), (6, 7), (6, 8), (6, 9), (6, 10), (6, 11), (6, 12), (6, 13), (6, 14), (6, 15), (6, 16), (6, 17), (6, 18), (7, 8), (7, 9), (7, 10), (7, 11), (7, 12), (7, 13), (7, 14), (7, 16), (7, 17), (8, 9), (8, 10), (8, 11)(8, 12), (8, 13), (8, 14)(8, 15), (8, 16), (8, 17), (9, 10), (9, 11), (9, 12), (9, 13)(9, 14), (9, 15), (9, 16), (10, 11), (10, 12), (10, 13), (10, 14), (10, 15), (10, 16), (11, 12), (11, 13), (11, 14), (11, 15)}\}.$$

Now, we need to find the decomposition of $K_{43}^{(3)}$. On $D(43)$, according to Definition 2.5, we obtain 41 sequences as follows:

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| (1) (1, 1, 2, 1, 3, 1, 34), | (2) (1, 4, 41, 5, 37, 5, 36), |
| (3) (1, 7, 39, 41, 8, 2, 31), | (4) (1, 9, 36, 4, 1, 10, 25), |
| (5) (1, 12, 33, 2, 9, 1, 28), | (6) (1, 13, 32, 3, 2, 12, 23), |
| (7) (1, 15, 30, 3, 3, 12, 22), | (8) (1, 16, 29, 3, 5, 8, 24), |
| (9) (1, 21, 24, 2, 15, 3, 20), | (10) (1, 23, 22, 3, 6, 13, 18), |
| (11) (1, 25, 20, 4, 2, 18, 16), | (12) (1, 27, 18, 3, 7, 17, 13), |
| (13) (1, 30, 15, 2, 16, 11, 11), | (14) (1, 32, 13, 4, 35, 37, 7), |
| (15) (2, 20, 26, 39, 9, 14, 19), | (16) (2, 24, 22, 39, 10, 18, 14), |
| (17) (22, 29, 17, 3, 9, 20, 6), | (18) (2, 32, 14, 4, 32, 38, 7), |
| (19) (3, 13, 24, 37, 9, 7, 26), | (20) (3, 15, 32, 37, 10, 19, 13), |
| (21) (3, 16, 31, 7, 11, 10, 21), | (22) (3, 24, 23, 37, 12, 20, 10), |
| (23) (3, 28, 19, 25, 13, 21, 8), | (24) (3, 31, 16, 38, 8, 26, 7), |
| (25) (4, 7, 5, 28, 5, 16, 23), | (26) (4, 8, 4, 28, 7, 6, 29), |
| (27) (4, 13, 38, 8, 5, 18, 17), | (28) (4, 14, 34, 35, 15, 6, 21), |
| (29) (4, 15, 37, 7, 17, 9, 16), | (30) (4, 29, 19, 36, 20, 12, 9), |
| (31) (5, 5, 6, 10, 23, 15, 22), | (32) (5, 7, 7, 25, 5, 19, 18), |
| (33) (5, 12, 8, 19, 6, 23, 13), | (34) (5, 15, 7, 18, 8, 16, 17), |
| (35) (5, 17, 7, 16, 35, 33, 16), | (36) (6, 16, 15, 8, 22, 8, 11), |
| (37) (7, 15, 20, 13, 16, 34, 24), | (38) (7, 20, 34, 17, 27, 29, 22), |
| (39) (7, 22, 38, 29, 28, 30, 25), | (40) (9, 9, 11, 15, 18, 11, 13), |
| (41) (9, 10, 11, 17, 12, 12, 15). | |

Let D be a collection of the 41 sequences above. Thus they correspond to 41 base 7-cycles:

$$\begin{aligned}
 C_{7(1)} &= (0, 1, 2, 4, 5, 8, 9), & C_{7(2)} &= (0, 1, 5, 3, 8, 2, 7), \\
 C_{7(3)} &= (0, 1, 8, 4, 2, 10, 12), & C_{7(4)} &= (0, 1, 10, 3, 7, 8, 18), \\
 C_{7(5)} &= (0, 1, 13, 3, 5, 14, 15), & C_{7(6)} &= (0, 1, 14, 3, 6, 8, 20), \\
 C_{7(7)} &= (0, 1, 16, 3, 6, 9, 21), & C_{7(8)} &= (0, 1, 17, 3, 6, 11, 19), \\
 C_{7(9)} &= (0, 1, 22, 3, 5, 20, 23), & C_{7(10)} &= (0, 1, 24, 3, 6, 12, 25), \\
 C_{7(11)} &= (0, 1, 26, 3, 7, 9, 2), & C_{7(12)} &= (0, 1, 28, 3, 6, 13, 30), \\
 C_{7(13)} &= (0, 1, 31, 3, 5, 21, 32), & C_{7(14)} &= (0, 1, 33, 3, 7, 42, 36), \\
 C_{7(15)} &= (0, 2, 22, 5, 1, 10, 24), & C_{7(16)} &= (0, 2, 26, 5, 1, 11, 29), \\
 C_{7(17)} &= (0, 2, 31, 5, 8, 17, 37), & C_{7(18)} &= (0, 2, 34, 5, 9, 41, 36), \\
 C_{7(19)} &= (0, 3, 16, 7, 1, 10, 17), & C_{7(20)} &= (0, 3, 18, 7, 1, 11, 30), \\
 C_{7(21)} &= (0, 3, 19, 7, 1, 12, 22), & C_{7(22)} &= (0, 3, 27, 7, 1, 13, 33), \\
 C_{7(23)} &= (0, 3, 31, 7, 1, 14, 35), & C_{7(24)} &= (0, 3, 34, 7, 2, 10, 36), \\
 C_{7(25)} &= (0, 4, 11, 16, 1, 6, 20), & C_{7(26)} &= (0, 4, 12, 16, 1, 8, 14), \\
 C_{7(27)} &= (0, 4, 17, 12, 3, 8, 26), & C_{7(28)} &= (0, 4, 18, 9, 1, 16, 22), \\
 C_{7(29)} &= (0, 4, 19, 13, 1, 18, 27), & C_{7(30)} &= (0, 4, 33, 9, 2, 22, 34), \\
 C_{7(31)} &= (0, 5, 10, 16, 26, 6, 21), & C_{7(32)} &= (0, 5, 12, 19, 1, 6, 25), \\
 C_{7(33)} &= (0, 5, 17, 25, 1, 7, 30), & C_{7(34)} &= (0, 5, 20, 27, 2, 10, 26), \\
 C_{7(35)} &= (0, 5, 22, 29, 2, 37, 27), & C_{7(36)} &= (0, 6, 22, 37, 2, 24, 32), \\
 C_{7(37)} &= (0, 7, 22, 42, 12, 28, 19), & C_{7(38)} &= (0, 7, 27, 18, 8, 35, 21), \\
 C_{7(39)} &= (0, 7, 29, 17, 3, 31, 18), & C_{7(40)} &= (0, 9, 18, 29, 1, 19, 30), \\
 C_{7(41)} &= (0, 9, 19, 30, 4, 16, 28).
 \end{aligned}$$

By the method of edge-partition, we obtain the decomposition of $K_{43}^{(3)}$ into 1763 7-cycles, that is,

$$\begin{aligned}
 \varepsilon(K_{43}^{(3)}) &= \bigcup_{(k_1, k_2) \in D(43)} H(k_1, k_2) = \bigcup_{(k_0, k_1, \dots, k_6) \in D_{all}(43)} H(k_0, k_1, \dots, k_6) \\
 &= \bigcup_{i=1}^{41} \{C_{7(i)} + j, j \in \mathbb{Z}_{43}\}.
 \end{aligned}$$

Hence we obtain the decomposition of $K_{43}^{(3)}$ into 1763 7-cycles. \square

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REFERENCES

- [1] R. Bailey, B. Stevens, *Hamiltonian decompositions of complete k -uniform hypergraphs*, Discrete Math. **310** (2010), 3088–3095.
- [2] H. Huo, L.Q. Zhao, W. Feng, Y.S. Yang, Jirimutu, *Decomposing the complete 3-uniform hypergraph $K_n^{(3)}$ into Hamiltonian cycles*, Journal of Mathematics **58** (2015), 965–976.
- [3] Jirimutu, J. Wang, *Study on graph labelings and hypergraph decomposition*, Doctoral Dissertation, Dalian University of Technology, 2006.
- [4] Jirimutu, J.F. Wang, *Hamiltonian decompositions of complete bipartite hypergraphs*, Acta Math. Appl. Sin. **4** (2001), 563–566.
- [5] H. Jordon, G. Newkirk, *4-cycle decompositions of complete 3-uniform hypergraphs*, Australas. J. Combin. **71** (2017), 312–323.
- [6] G.Y. Katona, H.A. Kierstead, *Hamiltonian chains in hypergraphs*, J. Graph Theory **30** (1999), 205–212.
- [7] G.R. Li, Y.M. Lei, L.Q. Zhao, Y.S. Yang, Jirimutu, *Decomposing the complete 3-uniform hypergraph $K_n^{(3)}$ into 5-cycles*, Applied Mechanics and Materials **672–674** (2014), 1935–1939.
- [8] G.R. Li, Y.M. Lei, L.Q. Zhao, Y.S. Yang, Jirimutu, *Decomposing the complete 3-uniform hypergraph $K_n^{(3)}$ into 5-cycles*, J. Math. Res. Appl. **36** (2016), 9–14.
- [9] G.R. Li, Y.M. Lei, Y.S. Yang, Jirimutu, *Decomposing the complete 3-uniform hypergraph $K_n^{(3)}$ into 7-cycles* (2014), submitted.
- [10] M. Meszka, A. Rosa, *Decomposing complete 3-uniform hypergraph into Hamiltonian cycles*, Australas. J. Combin. **45** (2009), 291–302.
- [11] M. Meszka, A. Rosa, *A possible analogue of ρ -labellings for 3-uniform hypergraphs*, Electron. Notes Discrete Math. **60** (2017), 33–37.
- [12] H. Verrall, *Hamiltonian decompositions of complete 3-uniform hypergraphs*, Discrete Math. **132** (1994), 333–348.
- [13] J.F. Wang, Tony T. Lee, *Paths and Cycles of Hypergraphs*, Science in China (A) **42** (1999), 1–12.
- [14] J.F. Wang, G.Y. Yan, *On cycle structure of hypergraph*, Chinese Science Bulletin **19** (2001), 1585–1589.
- [15] B.G. Xu, J.F. Wang, *On the Hamiltonian decompositions of complete 3-uniform hypergraphs*, Electron. Notes Discrete Math. **11** (2002), 722–733.

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