Opuscula Mathematica • Vol. 28 • No. 2 • 2008

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APPLICATION OF CHEBYSHEV AND TRIGONOMETRIC POLYNOMIALS TO THE APPROXIMATION OF A SOLUTION OF A SINGULAR INTEGRAL EQUATION WITH A MULTIPLICATIVE CAUCHY KERNEL IN THE HALF-PLANE

Abstract. In this article Chebyshev and trigonometric polynomials are used to construct an approximate solution of a singular integral equation with a multiplicative Cauchy kernel in the half-plane.

Keywords: singular integral equation, Cauchy kernel, multiplicative kernel, approximate solution, Chebyshev polynomials, trigonometric polynomials.

Mathematics Subject Classification: 45E05, 45L05, 65R20.

1. INTRODUCTION

Let us consider a singular integral equation of the form

$$\frac{1}{(\pi i)^2}\iint\limits_D \frac{\varphi(\sigma_1,\sigma_2)}{(\sigma_1-x)(\sigma_2-y)}d\sigma_1 d\sigma_2 = f(x,y), \tag{1}$$

where $(x,y)\in D=\{(x,y): 0<\operatorname{Re} z<\infty,\ -\infty<\operatorname{Im} z<\infty,\ z=x+iy\}$, $f(x,y)$ is a given function and $\varphi(x,y)$ is an unknown function. Let us notice that the surface of integration is the complex half-plane. In the case of surface of integration being the quarter-plane or the whole complex plane, the exact solutions are presented in [7]. Theory of (1) is presented in [4]. Let us shortly recall the explicit solution of (1).

Definition 1.1. *We write* $\varphi(x,y)\in h(0,\infty)\times h(\infty)$, $0\leqslant x<\infty$, $-\infty<y<\infty$, *if the function*

$$\varphi^*(t_1,t_2)=\varphi\left(\frac{1+t_1}{1-t_1}, i\frac{1+t_2}{1-t_2}\right),\quad (t_1,t_2)\in[-1,1)\times L,\quad L=\{t_2:|t_2|=1\},$$

satisfies the conditions

$$|\varphi^*(t_1', t_2') - \varphi^*(t_1'', t_2'')| \leqslant K_1 |t_1' - t_1''|^{\mu_1} + K_2 |t_2' - t_2''|^{\mu_2}, \tag{2}$$

$K_1,\ K_2 > 0,\ 0 < \mu_1,\ \mu_2 \leqslant 1$, *in each closed domain contained in* $(-1,1) \times L$ *and*

$$\lim_{t_1 \to 1-0} \varphi^*(t_1, t_2) = \lim_{x \to \infty} \varphi(x, y) = 0, \quad \forall t_2 \in L \quad (\forall y \in (-\infty, \infty)). \tag{3}$$

Theorem 1.1. *Let* $f(x,y) \in h(0,\infty) \times h(\infty)$ *and let*

$$\lim_{|y| \to \infty} f(x,y) = 0, \quad x \in [0,\infty).$$

Then the solution $\varphi(x,y)$ *of* (1) *in the function class* $h(0,\infty) \times h(\infty)$, *satisfying the relations*

$$\varphi(x, \infty) = 0, \quad x \in [0,\infty), \tag{4}$$

and

$$\frac{1}{\pi i} \int_0^\infty \frac{\varphi(\sigma_1, y)}{\sigma_1 + 1} d\sigma_1 = \frac{i}{(\pi i)^2} \int_0^\infty \int_{-\infty}^\infty \frac{f(\sigma_1, \sigma_2)\, d\sigma_1 d\sigma_2}{\sqrt{\sigma_1}(\sigma_1 + 1)(\sigma_2 - y)}, \tag{5}$$

is given by the following formula

$$\varphi(x,y) = \frac{\sqrt{x}}{(\pi i)^2} \int_0^\infty \int_{-\infty}^\infty \frac{f(\sigma_1, \sigma_2)\, d\sigma_1 d\sigma_2}{\sqrt{\sigma_1}(\sigma_1 - x)(\sigma_2 - y)}. \tag{6}$$

In this paper we present the method of finding on approximate solution of (1) in the function class $h(0,\infty) \times h(\infty)$ based on Chebyshev and trigonometric polynomials. Let us notice that the surface of integration is unbounded. In the literature [2,3,5,8], the methods of approximating a solution of equation (1) are well-known in the case of D bounded only. We have not found, in the literature, any study of the equation in which the surface of integration is a half-plane.

2. APPROXIMATE SOLUTION

Using the following identities

$$\frac{1}{\sigma_1 - x} = \frac{x+1}{\sigma_1 + 1} \frac{1}{\sigma_1 - x} + \frac{1}{\sigma_1 + 1}, \quad \frac{1}{\sigma_2 - y} = \frac{y+i}{\sigma_2 + i} \frac{1}{\sigma_2 - y} + \frac{1}{\sigma_2 + i},$$

and substitutions

$$\sigma_1 = \frac{1+\tau_1}{1-\tau_1}, \quad x = \frac{1+t_1}{1-t_1}, \quad \sigma_2 = i\frac{1+\tau_2}{1-\tau_2}, \quad y = i\frac{1+t_2}{1-t_2},$$

$\tau_1,\, t_1 \in (-1,1)\,,\, \tau_2,\, t_2 \in L$, we can rewrite equation (1) in the form

$$\frac{1}{(\pi i)^2}\int_{-1}^{1}\int_{L}\frac{\varphi^*(\tau_1,\tau_2)}{(\tau_1-t_1)(\tau_2-t_2)}d\tau_1 d\tau_2-\frac{1}{(\pi i)^2}\int_{-1}^{1}\int_{L}\frac{\varphi^*(\tau_1,\tau_2)}{(\tau_1-t_1)(\tau_2-1)}d\tau_1 d\tau_2-$$
$$-\frac{1}{(\pi i)^2}\int_{-1}^{1}\int_{L}\frac{\varphi^*(\tau_1,\tau_2)}{(\tau_1-1)(\tau_2-t_2)}d\tau_1 d\tau_2+\frac{1}{(\pi i)^2}\int_{-1}^{1}\int_{L}\frac{\varphi^*(\tau_1,\tau_2)}{(\tau_1-1)(\tau_2-1)}d\tau_1 d\tau_2= \tag{7}$$
$$=f^*(t_1,t_2)\,,$$

where

$$\varphi^*(\tau_1,\tau_2)=\varphi\left(\frac{1+\tau_1}{1-\tau_1},i\frac{1+\tau_2}{1-\tau_2}\right),\quad f^*(t_1,t_2)=f\left(\frac{1+t_1}{1-t_1},i\frac{1+t_2}{1-t_2}\right).$$

Let us introduce a new unknown function $u(t_1,t_2)$ using the relation

$$\varphi^*(t_1,t_2)=\sqrt{1-t_1^2}\,u(t_1,t_2)\,. \tag{8}$$

Substituting (8) into (7), (4) and (5), we get

$$\frac{1}{(\pi i)^2}\int_{-1}^{1}\int_{L}\frac{\sqrt{1-\tau_1^2}\,u(\tau_1,\tau_2)}{(\tau_1-t_1)(\tau_2-t_2)}d\tau_1 d\tau_2-\frac{1}{(\pi i)^2}\int_{-1}^{1}\int_{L}\frac{\sqrt{1-\tau_1^2}\,u(\tau_1,\tau_2)}{(\tau_1-t_1)(\tau_2-1)}d\tau_1 d\tau_2-$$
$$-\frac{1}{(\pi i)^2}\int_{-1}^{1}\int_{L}\frac{\sqrt{1-\tau_1^2}\,u(\tau_1,\tau_2)}{(\tau_1-1)(\tau_2-t_2)}d\tau_1 d\tau_2+\frac{1}{(\pi i)^2}\int_{-1}^{1}\int_{L}\frac{\sqrt{1-\tau_1^2}\,u(\tau_1,\tau_2)}{(\tau_1-1)(\tau_2-1)}d\tau_1 d\tau_2= \tag{9}$$
$$=f^*(t_1,t_2)\,,$$

$$u(t_1,1)=0, \tag{10}$$

$$\frac{1}{\pi i}\int_{-1}^{1}\frac{\sqrt{1-\tau_1^2}\,u(\tau_1,t_2)}{1-\tau_1}d\tau_1=\frac{i(1-t_2)}{(\pi i)^2}\int_{-1}^{1}\int_{L}\sqrt{\frac{1-\tau_1}{1+\tau_1}}\frac{f^*(\tau_1,\tau_2)}{(1-\tau_1)(1-\tau_2)(\tau_2-t_2)}d\tau_1 d\tau_2. \tag{11}$$

Now we approximate the function $f^*(t_1,t_2)$ with an interpolating polynomial $f^*_{n,n}(t_1,t_2)$ of the form (cf. [6])

$$f^*(t_1,t_2)\approx f^*_{n,n}(t_1,t_2)=\sum_{k=0}^{n}{}'\sum_{j=-n}^{n}F_{kj}T_k(t_1)\,t_2^j, \tag{12}$$

where

$$F_{kj}=\frac{2}{n+1}\sum_{r=0}^{n}f_{rj}T_k(t_{1,r})\,,\qquad f_{r,j}=\frac{1}{2n+1}\sum_{p=0}^{2n}t_{2,p}^{-j}f(t_{1,r},t_{2,p})\,, \tag{13}$$

$T_k(t_1) = \cos(n \arccos t_1)$, $k = 0, 1, \ldots, n$, are Chebyshev polynomials of the first kind, the points $t_{1,k} = \cos\frac{(2k+1)\pi}{2(n+1)}$, $k = 0, 1, \ldots, n$, are Chebyshev nodes, and $t_{2,j} = e^{is_j}$, where $s_j = \frac{2\pi j}{2n+1}$, $j = 0, 1, \ldots, 2n$. Here we approximate $f^*(t_1, t_2)$ with Chebyshev polynomials of the first kind with respect to the first variable and by trigonometric polynomials with respect to the second variable.

We are going to get the error estimations for an approximate solution of (9)–(11) using the following lemma.

Lemma 2.1. ([8]) *If the function $f^*(t_1, t_2)$ satisfies Hölder condition with respect to both variables on $[-1, 1] \times L$, $L : |t_2| = 1$, i.e., $f^*(t_1, t_2) \in H(\mu_1, \mu_2)$, and $f^*_{n,n}(t_1, t_2)$ is a polynomial of form* (12), *then the following inequality holds*

$$\left\| f^*(t_1, t_2) - f^*_{n,n}(t_1, t_2) \right\|_\infty \leqslant C \frac{\ln^2 n}{n^\mu},$$

where $\mu = \min\{\mu_1, \mu_2\}$ and C is an arbitrary constant.

We approximate the unknown function $u(t_1, t_2)$ with a polynomial $u_{n-1,n}(t_1, t_2)$ of the form

$$u(t_1, t_2) \approx u_{n-1,n}(t_1, t_2) = \sum_{k=0}^{n-1} \sum_{j=-n}^{n} c_{kj} U_k(t_1) t_2^j, \tag{14}$$

where $U_k(t_1) = \frac{\sin((k+1)\arccos t_1)}{\sin(\arccos t_1)}$, $k = 0, 1, \ldots, n-1$, are Chebyshev polynomials of the second kind, and c_{kj} are unknown coefficients.

An approximate solution $u_{n-1,n}(t_1, t_2)$ of (9)–(11) is defined as a solution of the following problem

$$\begin{gathered}
\frac{1}{(\pi i)^2} \int_{-1}^{1} \int_L \frac{\sqrt{1-\tau_1^2}\, u_{n-1,n}(\tau_1, \tau_2)}{(\tau_1 - t_1)(\tau_2 - t_2)} d\tau_1 d\tau_2 - \frac{1}{(\pi i)^2} \int_{-1}^{1} \int_L \frac{\sqrt{1-\tau_1^2}\, u_{n-1,n}(\tau_1, \tau_2)}{(\tau_1 - t_1)(\tau_2 - 1)} d\tau_1 d\tau_2 - \\
- \frac{1}{(\pi i)^2} \int_{-1}^{1} \int_L \frac{\sqrt{1-\tau_1^2}\, u_{n-1,n}(\tau_1, \tau_2)}{(\tau_1 - 1)(\tau_2 - t_2)} d\tau_1 d\tau_2 + \frac{1}{(\pi i)^2} \int_{-1}^{1} \int_L \frac{\sqrt{1-\tau_1^2}\, u_{n-1,n}(\tau_1, \tau_2)}{(\tau_1 - 1)(\tau_2 - 1)} d\tau_1 d\tau_2 = \\
= G^*_{n,n}(t_1, t_2) - G^*_{n,n}(t_1, 1) - G^*_{n,n}(1, t_2) + G^*_{n,n}(1, 1),
\end{gathered} \tag{15}$$

where $G^*_{n,n}(t_1, t_2) = f^*_{n,n}(t_1, t_2) + Q^*_n(t_2)$,

$$u_{n-1,n}(t_1, 1) = 0, \tag{16}$$

$$\frac{1}{\pi i} \int_{-1}^{1} \frac{\sqrt{1-\tau_1^2}\, u_{n-1,n}(\tau_1, t_2)}{1 - \tau_1} d\tau_1 = \frac{i(1 - t_2)}{(\pi i)^2} \int_{-1}^{1} \int_L \sqrt{\frac{1-\tau_1}{1+\tau_1}} \frac{f^*_{n,n}(\tau_1, \tau_2) + Q^*_n(\tau_2)}{(1-\tau_1)(1-\tau_2)(\tau_2 - t_2)} d\tau_1 d\tau_2. \tag{17}$$

Here we need the polynomial $Q^*_n(t_2) = \sum\limits_{j=-n}^{n} q^*_j t_2^j$ to satisfy (5). However, the unknown coefficients c_{kj} are independent of $Q^*_n(t_2)$.

Substituting (13) and (14) into (15), (16) and (17), we obtain

$$\sum_{k=0}^{n-1}\sum_{j=-n}^{n} c_{kj}\left\{\frac{1}{\pi i}\int_{-1}^{1}\sqrt{1-\tau_1^2}\frac{U_k(\tau_1)}{\tau_1-t_1}d\tau_1\frac{1}{\pi i}\int_L\frac{\tau_2^j}{\tau_2-t_2}d\tau_2-\right.$$
$$-\frac{1}{\pi i}\int_{-1}^{1}\sqrt{1-\tau_1^2}\frac{U_k(\tau_1)}{\tau_1-t_1}d\tau_1\frac{1}{\pi i}\int_L\frac{\tau_2^j}{\tau_2-1}d\tau_2-$$
$$-\frac{1}{\pi i}\int_{-1}^{1}\sqrt{1-\tau_1^2}\frac{U_k(\tau_1)}{\tau_1-1}d\tau_1\frac{1}{\pi i}\int_L\frac{\tau_2^j}{\tau_2-t_2}d\tau_2+ \tag{18}$$
$$\left.+\frac{1}{\pi i}\int_{-1}^{1}\sqrt{1-\tau_1^2}\frac{U_k(\tau_1)}{\tau_1-1}d\tau_1\frac{1}{\pi i}\int_L\frac{\tau_2^j}{\tau_2-1}d\tau_2\right\}=$$
$$=\sum_{k=0}^{n}{}'\sum_{j=-n}^{n}F_{kj}\left\{T_k(t_1)\,t_2^j-T_k(t_1)-t_2^j+1\right\},$$

$$\sum_{k=0}^{n-1}\sum_{j=-n}^{n}c_{kj}U_k(t_1)=0, \tag{19}$$

$$\sum_{k=0}^{n-1}\sum_{j=-n}^{n}\frac{c_{kj}}{\pi i}\int_{-1}^{1}\frac{\sqrt{1-\tau_1^2}U_k(\tau_1)\,t_2^j}{1-\tau_1}d\tau_1=$$
$$=\sum_{k=0}^{n}\sum_{j=-n}^{n}\frac{i\,(1-t_2)}{(\pi i)^2}\int_{-1}^{1}\int_L\sqrt{\frac{1-\tau_1}{1+\tau_1}}\frac{F_{kj}T_k(\tau_1)\,\tau_2^j}{(1-\tau_1)(1-\tau_2)(\tau_2-t_2)}d\tau_1d\tau_2+ \tag{20}$$
$$+\sum_{j=-n}^{n}\frac{i\,(1-t_2)}{(\pi i)^2}\int_{-1}^{1}\int_L\sqrt{\frac{1-\tau_1}{1+\tau_1}}\frac{q_j^*\tau_2^j}{(1-\tau_1)(1-\tau_2)(\tau_2-t_2)}d\tau_1d\tau_2.$$

Since (cf. [1])

$$\frac{1}{\pi}\int_{-1}^{1}\frac{\sqrt{1-\tau_1^1}U_k(\tau_1)\,d\tau_1}{t_1-\tau_1}=T_{k+1}(t_1),\quad t_1\in(-1,1),\quad k=0,1,\dots,$$
$$\frac{1}{\pi i}\int_L\frac{\tau_2^j}{\tau_2-t_2}d\tau_2=\begin{cases} t_2^j, & j\geq 0,\\ -t_2^j, & j<0,\end{cases}$$

it follows that (18) takes the form

$$\sum_{k=0}^{n-1}\sum_{j=-n}^{n} c_{kj} i \left(T_{k+1}(t_1)\operatorname{sgn}(j)\, t_2^j + T_{k+1}(t_1)\operatorname{sgn}(j) + \operatorname{sgn}(j)\, t_2^j + \operatorname{sgn}(j)\right) = \\ = {\sum_{k=0}^{n}}' \sum_{j=-n}^{n} F_{kj} \left\{T_k(t_1)\, t_2^j - T_k(t_1) - t_2^j + 1\right\}. \tag{21}$$

From (21) we get

$$c_{kj} = -i \operatorname{sgn}(j)\, F_{k+1,j}, \quad k = 0, 1, \ldots, n-1, \quad j \neq 0. \tag{22}$$

Next, from (19) we derive

$$c_{k0} = -\sum_{\substack{j=-n \\ j\neq 0}}^{n} c_{kj}, \quad k = 0, \ldots, n-1. \tag{23}$$

Now we find the coefficients of $Q_n^*(t_2)$. Using the equality

$$\frac{1}{\pi i}\int_{-1}^{1}\sqrt{\frac{1-\tau_1}{1+\tau_1}}\frac{\tau_1^k}{\tau_1 - t_1} d\tau_1 = i\left(t_1^k + p_1 t_1^{k-1} + \ldots + p_k\right), \quad k = 0, 1, \ldots,$$

where $p_1, p_2, \ldots, p_k$ are the coefficients of the expansion

$$\sqrt{\frac{z-1}{z+1}} z^k = z^k\left(1 + \frac{p_1}{z} + \frac{p_2}{z^2} + \ldots\right),$$

from (20) we obtain

$$q_j^* = -{\sum_{k=0}^{n}}' F_{kj} \sum_{r=0}^{k} d_r^{(k)} \sum_{l=0}^{r} p_l - i \operatorname{sgn}(j) \sum_{k=0}^{n-1} c_{kj}, \tag{24}$$

where $d_r^{(k)}$, $k = 0, \ldots, n$, $r = 0, \ldots, k$ are the coefficients of Chebyshev polynomials of the first kind $T_k(t_1)$, i.e., $T_k(t_1) = \sum_{r=0}^{k} d_r^{(k)} t_1^r$.

Let us summarize our considerations in the following theorem.

Theorem 2.1. *Let $f^*(t_1, t_2)$, being the right side of (9) satisfy conditions (2) and (3) and let $f^*(t_1, t_2)$ be approximated by the polynomial $f_{n,n}^*(t_1, t_2)$ of form (12). Moreover, let $u(t_1, t_2)$ be an unknown function in (9)–(11). An approximate solution $u_{n-1,n}(t_1, t_2)$ of (15)–(17), corresponding to an exact solution $u(t_1, t_2)$ of (9)–(11) is given by polynomial (14), with coefficients c_{kj}, $k = 0, \ldots, n-1$, $j = -n, \ldots, n$ given by (22) and (23).*

Moreover, the coefficients of the polynomial $Q_n^*(t_2)$, which coefficients necessarily satisfy (5), are given by (24).

Example. Let

$$f(x,y)=\frac{1}{(x+2)(y+1+i)}\qquad\left(f^{*}(t_1,t_2)=\frac{1-t_1}{3-t_1}\frac{1-t_2}{1+2i-t_2}\right),$$

Then the solution $\varphi(x,y)\in h(0,\infty)\times h(\infty)$ of (1) satisfying (4) and (5) is given by the formula

$$\varphi(x,y)=\frac{i\sqrt{2}}{2}\frac{\sqrt{x}}{x+2}\frac{1}{y+i+1},\qquad\left(u(t_1,t_2)=\frac{i\sqrt{2}}{2}\frac{1}{3-t_1}\frac{1-t_2}{1+2i-t_2}\right).$$

The values of $u(t_1,t_2)$, $u_{n-1,n}(t_1,t_2)$ for $n=20$ are shown in Table 1. The values of the exact and the approximate solutions of (1) are tabulated in Table 2.

Table 1

Comparison of the values of $u(t_1,t_2)$, $u_{n-1,n}(t_1,t_2)$

t_1	t_2	$u(t_1,t_2)$ / $u_{n-1,n}(t_1,t_2)$
0.99720379718118	0.988280423780349 + 0.152649284218874i	0.00242478904699766 − 0.0291584615303826i
		0.00242485214964709 − 0.0291584381822308i
0.930873748644204	0.896165556961056 + 0.44371983786696i	0.0291719127489146 − 0.0954895773730349i
		0.0291719754738018 − 0.0954895448496493i
0.680172737770919	0.606225410966638 + 0.795292871273427i	0.149437423868678 − 0.152376154798167i
		0.149437429952641 − 0.152376185255323i
−0.433883739117558	−0.40906863717134 + 0.91250361647655i	0.183172195569413 + 0.0645510819588097i
		0.183172200692985 + 0.0645511046137608i
−0.78183148246803	−0.99706580118374 − 0.0765492528364957i	0.0899725258754509 + 0.0934212503128208i
		0.0899725314804843 + 0.0934212607760827i

Table 2

Comparison of the values of exact and approximate solutions of (1)

x	y	$\varphi(x,y)$ / $\varphi_{n-1,n}(x,y)$
714.255698384602	−13.0251539268895	0.000181204712409472 − 0.00217901455900165i
		0.00018120942807637 − 0.00217901281419209i
27.9325684638375	−4.27333960563102	0.0106576964864156 − 0.0348862600137787i
		0.0106577194023901 − 0.0348862481316517i
5.25337560675948	−2.01966529436476	0.109545383288362 − 0.111699625497027i
		0.109545387748222 − 0.111699647823703i
0.394813223302777	−0.647593447476322	0.165032445673374 + 0.0581585152343051i
		0.165032450289554 + 0.0581585356457107i
0.122440600965128	0.0383308616026182	0.0560969523308147 + 0.0582471968469358i
		0.0560969558254958 + 0.0582472033706729i

3. CONCLUSIONS

In this paper a numerical solution of equation (1) in the function class $h\,(0,\infty)\times h\,(\infty)$ is presented. Numerical experiments show that the method gives very accurate results and may be useful in practice. However, estimation of the error of the approximate solution remains a research problem.

REFERENCES

[1] H. Bateman, A. Erdélyi, *Higher Transcendental Functions,* vol. 2, McGraw-Hill Book Company, New York, 1953.

[2] S.M. Belocerkovskii, I.K. Lifanov, *Numerical Methods for Singular Integral Equations and its Applications in Aerodynamics, Theory of Elasticity and Electrodynamics,* Nauka, Moscow, 1985 [in Russian].

[3] B.G. Gabdulkhaev, *Finite-dimensional approximations of singular integrals and direct methods of solution of singular integral and integrodifferential equations,* J. Math. Sci. (N.Y.), **18** (1982) 4, 593–627.

[4] P. Karczmarek, *Singular integral equation with a multiplicative Cauchy kernel in the half-plane,* Opuscula Math., **28** (2008), 63–72.

[5] I.K. Lifanov, *Singular Integral Equations and Discrete Vortices,* VSP, Amsterdam, 1996.

[6] S. Paszkowski, *Numerical applications of Chebyshev polynomials and series,* PWN, Warsaw, 1975 [in Polish].

[7] D. Pylak, M.A. Sheshko, *Inversion of singular integrals with Cauchy kernels in the case of an infinite integration domain*, Differ. Equ., **41** (2005), 1297–1310.

[8] M. Sheshko, *Singular Integral Equations with Cauchy and Hilbert Kernels and Theirs Approximated Solutions*, The Learned Society of the Catholic University of Lublin, Lublin, 2003.

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Received: July 24, 2007.
Revised: October 30, 2007.
Accepted: November 11, 2007.