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PROFESSOR ANDRZEJ LASOTA'S THEORIES IN ENGINEERING

Abstract. Professor Lasota's achievements in the field of engineering are based on two principal aspects: profound theoretical backgrounds and perfect knowledge of phenomenological processes. That allowed him to solve many different problems of engineering.

1. FOREWORD

The deficiency of original theoretical solutions of engineering problems utilizing advanced mathematical theories might be attributed to the certain pragmatism of engineers' approach. Most professionals tend to think that theoretical works help but little to achieve practical results. Obviously, we do not have in mind the theories and analytical formulas that have been well proved in engineering practice, but the new theories from fast-changing fields of modern technology. Moreover, production engineering seems to defy research efforts. The lack of clear, practical theories is mostly due to the fact that many technological processes are accompanied by very complex, not well recognised physical and chemical phenomena. In order to obtain reliable results in the theory of production engineering, profound theoretical backgrounds have to be supported by the perfect knowledge of phenomenological processes. Apparently, Prof. Lasota's achievements in the field of engineering are based on these two aspects to guarantee full success. I was lucky to be able to be Prof. Lasota's assistant during his engineering exploits.

2. TECHNOLOGY OF ROCK CUTTING

In 1968 the research program was undertaken in the Department of Manufacturing at AGH to study rock workability using cogged bits. This was a part of a vast research project commissioned by the drilling sector in Poland. No analytical method was available to determine the constraints of process parameters. First of all, it was necessary to determine the critical rotational speed of cogged bits which caused the

failures and consequent damage to the drilling tools, because in practice, parameters of the drilling process: the rotational speed and axial loading onto the cogged bits were determined from the fieldwork only.

The primary tool used for deep holes drilling (hundreds to thousands of meters deep) is the cogged bit. A cogged bit incorporates toothed rings placed on conical surfaces rolling and pressing on the hole bottom (Fig. 1).

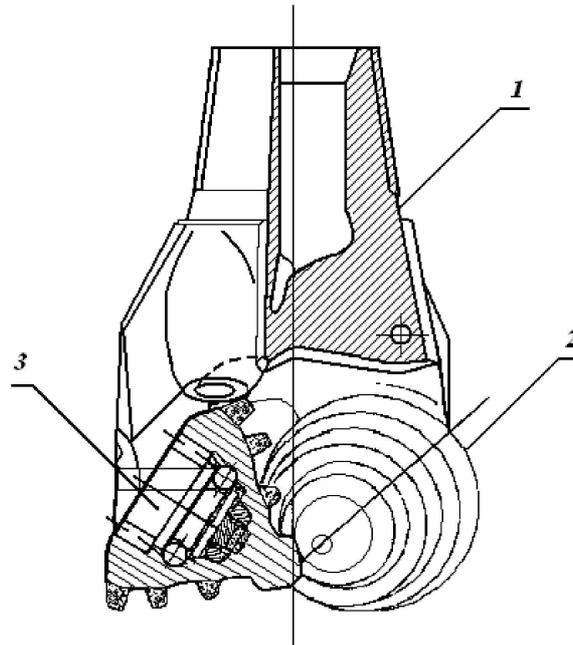


Fig. 1. Cogged bit: 1 – body; 2 – cogged roller; 3 – bearing

The bit is pressed down to the hole bottom by a long drill column. Driving torque is transmitted onto a tool by a long drill column, which connects the drilling-rig unit with the bit. It takes months to drill holes several kilometres long, and in difficult geological conditions this time is often prolonged to a year, or more. Drilling costs amount to several million zloty and each failure of the costly drilling equipment adds extra costs associated with down-time, repairs or replacement of jammed parts or elements of drill column or bits. Hence, it is necessary to predict the behaviour of the drilling tool and bit whilst in service. The model of a tool considered by A. Lasota is shown in Figure 2. This is the typical model of operation of an Egieles's cogged bit [1]. The toothed ring with the mass M and diameter $2R$ rolls with the linear velocity v . The central angle between the blades of the neighbouring teeth is 2φ . The toothed ring is pressed down to the bottom with the force F . As the cut depth is small in relation to the radius R and the tooth-to-tooth distance, the trajectory of the ring centre C is above the curve determined by arcs of circles with the radius R and

the centres located at the distance $T = 2R \sin \varphi$. This curve, called the fundamental curve, is given by the formula:

$$z = p(x)$$

$$\text{where } p(x) = \sqrt{R^2 - (x - a_k)^2} \quad \text{for } kT \leq x \leq (k+1)T, \quad (1)$$

$$a_k = (k + \frac{1}{2})T, \quad k = 0, 1, 2, \dots$$

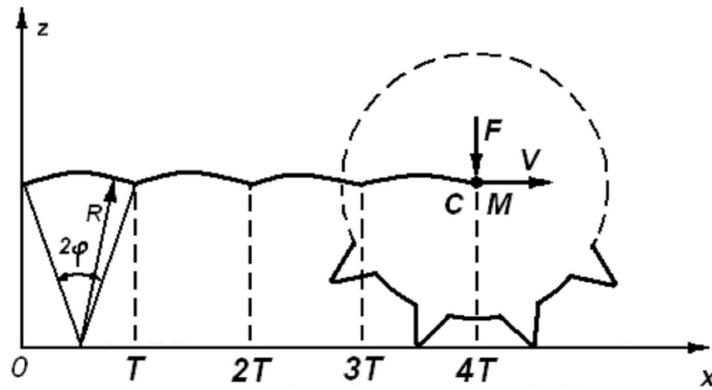


Fig. 2. Ejgieles's model of a clogged bit

As regards ring performance, the following quantity is of key importance:

$$\lambda = \frac{\nu^2 M}{FR},$$

which is Froude's number for this system.

It is worthwhile to mention that for $\lambda < 1$ the ring centre moves precisely along the fundamental curve. In order to prove this, we need to show that the component of the reaction force A_z in the direction of the z -axis should be positive

$$A_z = M \frac{d^2}{dt^2} p + F.$$

It follows from equation (1) that $d^2 p/dx^2 = -R^2/p^3$ and hence:

$$\frac{d^2}{dt^2} p = \nu^2 \frac{d^2}{dx^2} p = -\frac{R^2 \nu^2}{p^3}.$$

Finally, for the component A_z we get:

$$A_z = -\frac{MR^2 \nu^2}{p^3} + F \approx -\frac{M \nu^2}{R} + F = F(1 - \lambda) > 0.$$

For $\lambda > 1$, the component A_z in the direction of z -axis is negative, which implies that in the period between two subsequent tooth-ground contact points the ring centre C moves above the fundamental curve, in accordance with the formula:

$$M \frac{d^2 z}{dt^2} = -F; \quad \text{therefore,} \quad \frac{d^2 z}{dx^2} = -\frac{F}{M\nu^2}. \quad (2)$$

Trajectory of the point C is derived as on Figure 3.

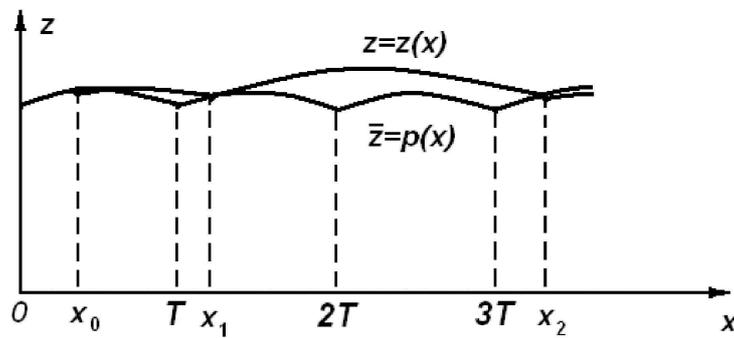


Fig. 3. Determining the sequence of node points $\{x\}$. Trajectory $z = z(x)$ in the range (x_i, x_{i+1}) becomes the solution to equation (2) and at points x_i it is a tangent to the fundamental curve $z = p(x)$

Let us assume that at the initial point $x = x_0$ ($0 \leq x_0 < T$) the solution $z(x)$ to equation (1) satisfies the following conditions:

$$z(x_0) = p(x_0), \quad z'(x_0) = p'(x_0).$$

They imply that at the initial point the trajectory begins on the fundamental curve and will be tangent to it. This solution is valid to the right of the point x_0 as long as the condition $z(x) > p(x)$ is satisfied. The first intersection point $x_1 > x_0$ is found for $z(x_1) = p(x_1)$. This procedure is then repeated, starting from the point x_1 , which implies solution of equation (1) with the initial conditions:

$$z(x_1) = p(x_1), \quad z'(x_1) = p'(x_1).$$

Let us then find the nearest point $x_2 > x_1$ where the solution intersects the fundamental curve. The next starting point is taken as $x = x_2$, and point x_3 is to be found in the same manner. Applying the recurrent procedure, we get the sequence of points:

$$0 \leq x_1 < x_2 < x_3 < \dots$$

Those points are called the node points of a trajectory. They correspond to subsequent impact points when the tool hits the bottom hole. For $\lambda < 1$ the trajectory results from kinematic excitations ($z(x) = p(x)$), so in this case node points will coincide with refraction points of the fundamental curve, it means:

$$x_n = nT, \quad n = 1, 2, \dots$$

For $\lambda = 1$ we get the first critical velocity v_I :

$$v_I^2 = HR, \quad \text{where} \quad H = \frac{F + Mg}{M}$$

at which the reaction force component A_z becomes 0. Exceeding v_I , the tool leaves the trajectory of fundamental curve, which vastly impacts on drilling performance. At lower velocities ($\lambda < 1$) the tool operation is stable and the production rate increases with the rotating speed. At higher velocities the tool partly loses contact with the hole bottom and the trajectory of tool motion becomes most complicated (see Fig. 3). It is well proved [2] that in the range $1 < \lambda < 2$ the tool motion remains stable even though the tool-ground contact is partly broken. For $\lambda = 2$ we get the second circumferential critical velocity v_{II} .

$$v_{II}^2 = 2HR.$$

Above the velocity of v_{II} , the motion is no longer stable. It appears that the transition from stable to unstable motion is associated with the critical value of Froude's number. At higher values of Froude's number the system does not display any stable periodic trajectories. It is an analogon to turbulent flows of fluids at high Reynolds numbers. For small values of Froude's number in stable motion, simple methods of analysis suffice to determine the operating parameters of the tool, particularly the mean energy transmitted to the ground by the tool. For higher values of Froude's numbers ($\lambda > 2$), the periodic trajectory is not stable (Fig. 4) and hence the methods of ergodic theory are recommended.



Fig. 4. Typical trajectory of a centre point of a single-ringed tool in unstable motion ($\lambda > 2$)

To analyse this function, let us consider the existance of invariant measure for the dynamic system formerly described. Assuming that a physical quantity \bar{f} might be expressed as a function of nodal points and can be written as $f(\hat{s}_n)$, its average value along the trajectory becomes:

$$\bar{f} = \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n f(\hat{s}_k) = \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n f(\tau_\lambda^k(s_0)).$$

In accordance with Birkhoff-Chinczyn's ergodic formula, the average value of f along the trajectory is equal to the average value of f with respect to an ergodic invariant measure (for almost each s_0). Accordingly, we get:

$$\bar{f} = \int_0^1 f(s) \mu_\lambda(ds),$$

where μ_λ stands for an invariant measure. How to determine this measure was one of the major theoretical issues associated with the solution given by A. Lasota and J. Yorke [3] in relation to Ulam's hypothesis concerning the existence of invariant measures for expanding transformations. In accordance with Ulam's hypothesis, such measure exists [4] as long as the function τ_λ satisfies the condition $|d\tau_\lambda/ds| > 1$ at fixed points. That happens when $\lambda > 2$. As it was shown in [3], the condition:

$$\inf_s \left| \frac{d\tau_\lambda^2(s)}{ds} \right| > 1$$

is sufficient for ensuring the existence of an invariant measure. The given invariant measure being known, one might proceed to obtain [5] some of the process parameters, utilising the Birkhoff-Chinczyn's individual ergodic theorem.

Remark 1. *In order to find the measure μ_λ , the specific form of the individual ergodic theorem is applicable:*

$$\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n \chi_D(\tau_\lambda^k(s)) = \mu_\lambda(D),$$

where χ_D denotes the characteristics function of the set D .

This formula might be used to analytically determine $\mu_\lambda(D)$. The calculations were performed at the Institute of Fluid Dynamics of the Maryland University (USA). It turned out that μ_λ distributions obtained numerically resemble the gamma distribution with the density $g_{bp}(s)$. Thus obtained numerical data were further utilised to derive the basic operating parameters of the clogged bit: the average impact energy and mined rock volume per one tool revolution.

In order to calculate an energy of the impact:

$$E_n = \frac{1}{2} M \left(\frac{dz}{dt} \right)_{t=t_n}^2$$

at the moment t_n which corresponds to the nodal point of a trajectory, we are going to use non-dimensional variables. The following results were obtained:

$$\begin{cases} \text{If } 0 < \lambda < 1 & \text{then } \bar{E} = \lambda K, \\ \text{If } 1 < \lambda < 2 & \text{then } \bar{E} = \lambda^{-1} K, \\ \text{If } \lambda > 2 & \text{then } \bar{E} = K \int_0^1 (1 - 2s - 2\lambda^{-1} r_\lambda(s))^2 g_{bp}(s) ds, \end{cases} \quad (3)$$

for $K = \frac{1}{2} FR \sin^2 \varphi$.

Another indicator of particular interest is expressed as the average volume of mined rock per one tool revolution, designated as h_0 . Let us assume after N.N Dawidenko [6] that the volume of rock mined during one tool impact is proportional to impact energy \bar{E} , and can be written as:

$$h_0 = \rho \bar{v} \bar{E},$$

where: \bar{v} , ρ - the average number of impacts per revolution and the proportionality factor, respectively.

Next, the formula for h_0 is obtained:

$$h_0 = \begin{cases} \lambda K_1 & \text{for } 0 < \lambda < 1, \\ \lambda^{-1} K_1 & \text{for } 1 < \lambda < 2, \\ K_1 \frac{\int_0^1 (1-2s-2\lambda^{-1}r_\lambda(s))^2 g_{bp}(s) ds}{\int_0^1 \lambda^{-1} r_\lambda(s) g_{bp}(s) ds} & \text{for } \lambda > 2, \end{cases} \quad (4)$$

where $K_1 = \frac{1}{4} \pi d \rho F \sin \varphi$.

The analysis of h_0 for large λ , yields an asymptotic value which h_0 approaches when λ tends to infinity. Namely, we get:

$$\lim_{\lambda \rightarrow \infty} h_0 \approx 0,65 K_1.$$

It appears that the theoretically predicted relationship between mined rock volume per one tool revolution and the rotating speed agree very well with the well-known experimental data [7]. It is worthwhile to compare the experimental and theoretically predicted relationship between h_0 and n - the clogged bit's rotating speed (Fig. 5 and Fig. 6).

Remark 2. For easy reference, there is velocity n of the rotational drilling in rpm on the abscissa, instead of λ . The relation between these two quantities is straightforward:

$$\lambda = \frac{Mv^2}{FR} = \frac{\pi^2 d^2 M}{FR} \left(\frac{n}{60} \right)^2,$$

where d is the clogged bit diameter (hole).

Good correspondence between the theoretical results and industrial data allowed the reliable engineering computations, prompting further research leading to the design of a new tool known as an isoangular bit.

The analysis of the clogged bit performance reveals that its elements tend to wear and tear quickly, as a result of impact. The tool itself is described by three functions:

$$N = f_1(\sigma), \quad \sigma = f_2(E), \quad v = f_3(E),$$

where:

- N - number of cycles required to damage the tool (fatigue damage),
- σ - maximal stress observed during one cycle in the worn part,
- E - energy of a single impact,
- v - rock volume mined during a single impact (cycle).

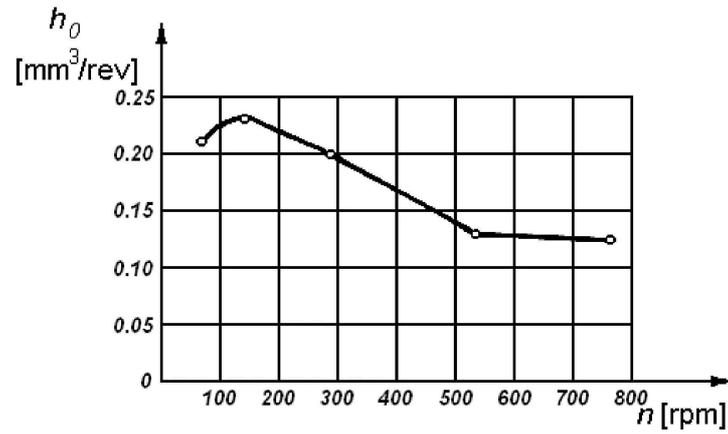


Fig. 5. Experimental relation: mined rock volume – h_0 per one tool revolution for the cogged bit $\Lambda\Delta 10-132$

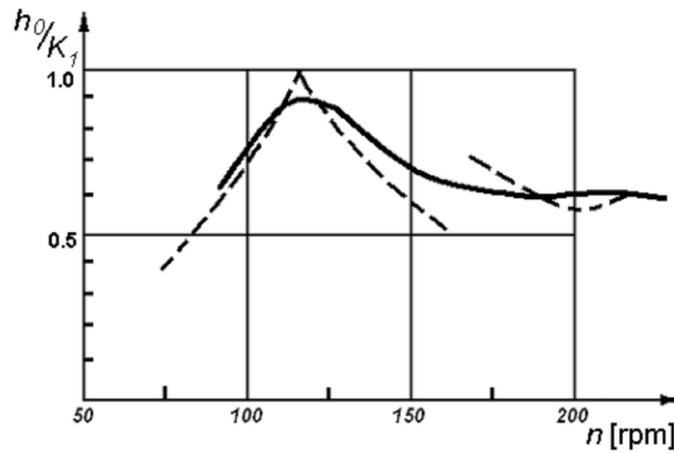


Fig. 6. Theoretical relationship between rock volume h_0 per one revolution and rotational speed n . The broken profile is derived from theoretical data; the continuous profile is the averaged one corresponding to λ variations by 20% of its nominal value

Functions f_1, f_2, f_3 are sufficient to determine the tool's fatigue endurance and the total amount of mined material V , given the predetermined constant energy. Thus:

$$N = f_1(f_2(E)) \quad \text{and} \quad V = N \cdot v = f_1(f_2(E))f_3(E) \quad (\sigma \text{ is reduced}).$$

The Palmgren hypothesis is applied to determine how effects of wear should cumulate under the variable energy of a single cycle:

$$\sum_{i=1}^k \frac{n_i}{N_i} = 1, \quad N = \sum_{i=1}^k N_i,$$

where:

- N_i – number of cycles (under stress σ_i required to damage the tool,
- n_i – number of cycles operated under the stress σ_i .

Whilst the tool is in service, positive effects (mined material) add to negative ones (loss of tool life measured by a number of cycles). After the boundary transition, the Palmgren hypothesis can be expressed with integrals:

$$N = \int_0^\infty n(E)dE, \quad \int_0^\infty \frac{n(E)dE}{f_1(f_2(E))} = 1, \quad V = \int_0^\infty f_3(E)n(E)dE.$$

Bringing up a normalised distribution function $v(E) = N^{-1}n(E)$, we get:

$$\int_0^\infty v(E)dE = 1, \quad v(E) \geq 0, \tag{5}$$

$$\frac{1}{N} = \int_0^\infty \frac{v(E)}{f_1(f_2(E))} dE, \tag{6}$$

$$V = \frac{\int_0^\infty f_3(E)v(E)dE}{\int_0^\infty \frac{v(E)dE}{f_1(f_2(E))}}. \tag{7}$$

The numerator in expression (7) has a simple physical interpretation; this is the average productivity \hat{U} per one cycle

$$\hat{U} = \frac{V}{N} = \int_0^\infty f_3(E)v(E) dE. \tag{8}$$

Formulas (5), (6), (7) highlight the following optimisation problem.

Problem I

Given the functions f_1, f_2, f_3 and the average cycle productivity \hat{U} , find the distribution v to ensure the maximal fatigue life of a tool. In terms of mathematics, the minimum of integral (6) is sought, integrals (5) and (8) being given. On account of condition (7), the solution to Problem I automatically becomes the solution to Problem II.

Problem II

Given the functions f_1 , f_2 , f_3 and the average cycle productivity \widehat{U} , find the distribution ν to ensure the maximal total performance V . (The solution to this problem suggested by Andrzej Lasota gave rise to many practical applications in the design and operation of impact tools, including cogged bits).

The solution proposed by Andrzej Lasota in his work [8] shall briefly be outlined. For simplicity, let us write $1/f_1(f_2(E)) = f_0(E)$. It is assumed that functions f_0 and f_1 belong to the class C^2 for $E \geq 0$. It is readily apparent that in considered functionals (5)–(8), the function ν is linear and hence those functionals are independent of its derivatives. That is why conventional methods of calculus of variations shall not apply and from the standpoint of Euler's equations, it becomes a very specific case. Besides, even for very regular functions f_0 , f_1 , the optimal distribution density ν cannot be chosen from the set of functions but from the set of distributions, in accordance with the following theorem:

Theorem 1. *Let $f_3(E)$ and $f_0(E_0)$ be C^2 class functions for $E \geq 0$ and let ν and E_0 be stationary positive numbers such that the following condition is satisfied:*

$$\int_0^\infty f_3(E)\delta(E_0 - E) = f_3(E_0) = \bar{U}. \quad (9)$$

If

$$\frac{f_3''(E)}{f_3'(E)} < \frac{f_0''(E)}{f_0'(E)}, \quad f_3'(E) > 0, \quad f_0'(E) > 0 \quad (10)$$

then the distribution $\nu(E) = \delta(E_0 - E)$ shall implement the maximum of the integral

$$\int_0^\infty f_0(E)\nu(E)dE$$

on the set of all distributions satisfying condition (5) and the condition:

$$\int_0^\infty f_3(E)dE = \bar{U}.$$

(The proof is provided in the Appendix to the paper [8]).

The theory of impact tool operation was applied to the design of an isoangular cogged bit, under Prof. Lasota's instructions (Fig. 7).

Analysis of performance parameters of conventional cogged bits reveals typical distributions of impact energy as the tool hits the hole bottom. Relevant indicators were obtained to account for productivity loss due to non-uniform distribution of impact energy, different from the optimal one-point distribution. Prof. Lasota's work provided the theoretical backgrounds and practical principles to further development of the new family of impact tools and machines satisfying the requirement of optimal performance in terms of maximal efficiency and improved tool life.

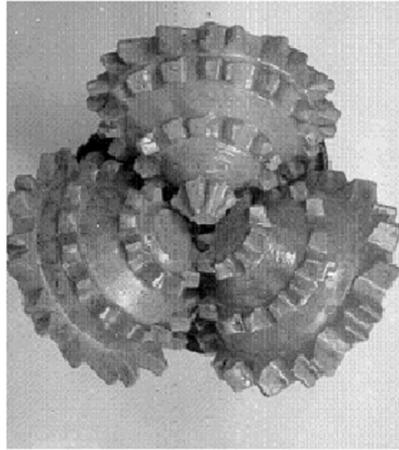


Fig. 7. Tooth design in an isoangular bit to ensure uniform impact energy whilst in service

3. MACHINING OF METALS

Metal cutting processes are of key importance in mechanical engineering. High standard surface qualities are often required. The achievements of modern engineering in a large degree depend on this surface quality. Vibrations occurring during the surface treatment negatively impact on surface quality. Numerous theoretical and practical studies highlight the evident relationships between vibrations and properties of the machined material, revealing the variable friction characteristics on the tool-machined surface contact, regenerative effects of vibrations occurring when the tool moves along the corrugated machined surface and dynamic properties of the machining tool. The most common source of vibrations accompanying nearly every machining process is the wear of the cutting tool (Fig. 8).

No earlier theoretical works are available that would highlight this particular cause of stability loss during the machining. Andrzej Lasota solved this problem basing on the theory of differential equations with a delayed parameter [9]. The simplest equation governing the tool vibrations is the following a linear equation:

$$m \frac{d^2 y}{dt^2} + b \frac{dy}{dt} + cy = F(t), \quad (11)$$

where:

- m – reduced mass involved in vibrations,
- y – cutting edge displacement normal to the axis of rotation of a machined object and to velocity vector of the cutting process,
- cy – elasticity force associated with machine tool rigidity,
- $F(t)$ – instantaneous thrust component of the cutting resistance force, actually, it is the instantaneous deviation of the thrust force from the mean value.

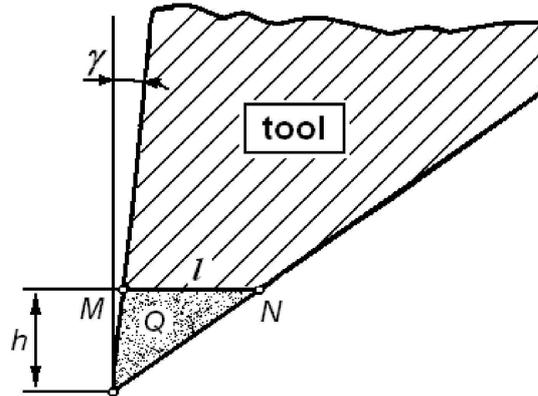


Fig. 8. Wear of a cutting tool OMN nearly triangular in shape

The force F depends on the profile of the machined surface near the cutting edge. Hence the natural transition to spatial coordinates (s, y) where $s = vt$. Assuming that $x(s) = y(s/v)$, where $s = vt$ (v -cutting speed) and $F_0(s) = F(s/v)$ equation (11) becomes:

$$mv^2 \frac{d^2x}{ds^2} + bv \frac{dx}{ds} + cx = F_0(s). \quad (12)$$

The force F depends chiefly on the profile of the machined surface along the section $[s - l, s]$ (Fig. 9).

For simplicity, let us assume that $F_0(s)$ is a linear functional and hence might be written as:

$$F_0(s) = -k \int_0^l y(s - z)p(z)dz, \quad (13)$$

where:

- l – length of the section where the cutting edge contacts the machined surface,
- k – coefficient expressing the cutting resistance,
- $p(z)$ – normalized distribution of the thrust component of the cutting force (Fig. 9).

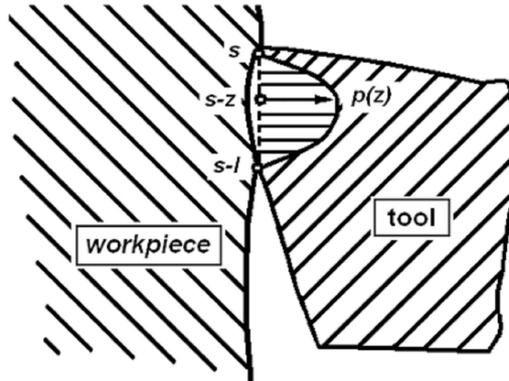


Fig. 9. Distribution of the thrust component of the cutting force on the flank surface, where l – length of the section where the cutting edge contacts the machined surface

Normalisation of the distribution p implies that:

$$\int_0^l p(z)dz = 1, \quad p(z) \geq 0.$$

Equation (12) can be rewritten as:

$$mv^2 \frac{d^2y}{ds^2} + bv \frac{dy}{ds} + cy + k \int_0^l y(s-z)p(z)dz = 0. \quad (14)$$

Substituting $x = e^{\zeta s}$ to equation (14), where $\zeta = \beta + i\omega$ is a complex number, we get two characteristic equations, which are represented separately for the real and imaginary parts:

$$\begin{aligned} mv^2(\beta^2 - \omega^2) + bv\beta + c + k \int_0^l e^{-\beta z} \cos(\omega z)p(z)dz &= 0, \\ 2\beta\omega mv^2 + bv\omega - k \int_0^l e^{-\beta z} \sin(\omega z)p(z)dz &= 0. \end{aligned} \quad (15)$$

For small values of l , the integrated function is expanded into Taylor series. Taking the linear terms only, we get a simplified system of equations:

$$\begin{aligned} mv^2(\beta^2 - \omega^2) + (bv - kq)\beta + c + k &= 0, \\ 2\beta\omega mv^2 + bv\omega + k\omega q &= 0, \end{aligned} \quad (16)$$

where:

$$q = \int_0^l zp(z)dz.$$

The parameter p is the mean value of distribution of the thrust force acting upon the cutting edge; it chiefly depends on the edge profile and can be written as follows:

$$q = \varepsilon l, \quad 1/3 < \varepsilon < 1/2$$

(coefficient ε was determined on the basis of experimental data).

Solving equation (16) with respect to b yields:

$$\beta = 1/2m^{-1}v^{-2}(k\varepsilon\ell - bv).$$

The condition for the system to be stable is that $\beta < 0$. In accordance with the presented theory, self-excited vibrations shall be generated when:

$$k\ell\varepsilon > bv. \quad (17)$$

When the machine tool system becomes unstable, the surface quality of the machined object will deteriorate. The common measure of surface quality after machining treatment is the surface roughness. In normal, stable conditions, surface roughness R_a is given by a fairly straightforward relationship:

$$R_a = \frac{1}{L} \int_0^L |z(x) - \langle z \rangle| dx, \quad (18)$$

where: R_a – basic parameter of surface roughness, expressed as average deviation of a profile given by a function $z(x)$ from the mean-value line $\langle z \rangle$, over the line section of length L .

For a widely applied profile shaped by a tool with the cutting edge in the form of a circular sector, we get:

$$R_a = \frac{p^2}{18\sqrt{3}r_\varepsilon} \quad (19)$$

where:

- p – feed rate of the cutting tool,
- r_ε – radius of the cutting edge producing the surface roughness.

Figure 10 shows typical experimental data revealing the roughness – advance rate relationship.

Major discrepancies between the experimental results, workshop practice and predicted data are observed for small roughness which determine the quality of surface finish. These discrepancies might be attributable to vibrations. However, there were no theoretical works explaining how vibrations should impact on surface quality.

The model proposed by Prof. Lasota [10] for defining surface roughness in the conditions of vibrations occurrence was developed and proposed as it is shown in Figure 11 and finally written as equation (20).

$$z_i(x) = \xi_i - \frac{1}{2r_\varepsilon} [(x - pi) - \eta_i]^2, \quad (20)$$

where: ξ and η – instantaneous displacements of the edge corner in the z and x direction, respectively.

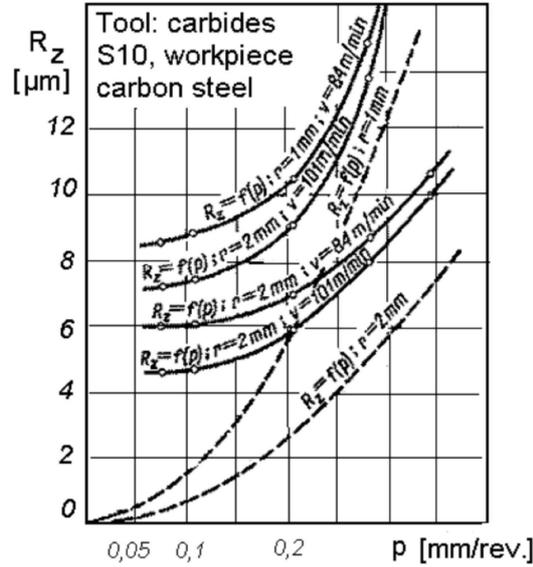


Fig. 10. Surface roughness R_z ($R_z = 1.4 - 1.6R_a$) vs feed rate when turning cylindrical bars made from carbon steel. Predicted relationships are indicated by broken line

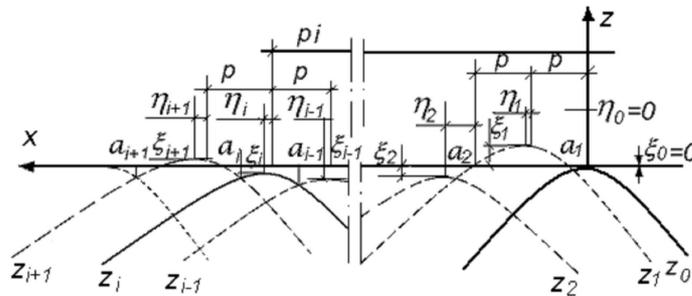


Fig. 11. Roughness profile during the rolling in the conditions of transverse (x) and longitudinal (y) vibrations

Assuming that ξ_i and η_i for $i = 1, 2, 3 \dots$ are random variables, $z(x)$ is treated as a stochastic process. Quantities $\langle z \rangle$ and R_a might be computed when the distributions ξ_i and η_i are given. It is worthwhile to mention that only variances of ξ_i and η_i are of key importance. The calculation procedure whereby all terms in order higher than the second as well as powers and products of moments of the order higher than the first are omitted as negligibly small in relation to σ_ξ^2 , σ_η^2 , yields the surface roughness R_a .

For prevalent transverse vibrations: $R_a = \left(\frac{p^4}{972r_\epsilon^2} + \frac{50}{81}\sigma_\xi^2 \right)^{\frac{1}{2}}$.

For prevalent longitudinal vibrations: $R_a = \left(\frac{p^4}{972r_\epsilon^2} + \frac{5p^2}{162r_\epsilon^2}\sigma_\eta^2 \right)^{\frac{1}{2}}$

(very few occurrences).

To find the values of $\sigma_\xi^2, \sigma_\eta^2$, it is required that vibration amplitude be determined, given the edge wearing and the stability conditions (17).

Thus obtained relationships reveal a good correspondence with workshop practice data and can be widely applied to design the surface finish operations and to determine the suitability of machine tools for use in operations to ensure the required surface finish standard.

4. ABRASIVE MACHINING

Andrzej Lasota's interests in machining technologies prompted his research in abrasive cutting processes, particularly grinding - a basic process involved in finishing treatment. A chaotic distribution of cutting edges in the grinding process, a complicated mechanism of cutting involving cutting as well as friction and plastic deformations, rendered the theoretical research a formidable task. The first reliable theoretical results were obtained thanks to the fundamental observation made by Prof. Lasota, who noticed an analogy between the Weierstrass function in the form given by B.R. Hunt [11] and the ground surface formation.

Remark 3. (*Description of the Weierstrass function by B.R.Hunt*)

$$z(x) = \sum_{n=0}^{\infty} a^n h(b^n x + \theta_n),$$

where:

- (θ_n) - is a sequence of independent random variables with a uniform distribution over the interval $[0, T]$ and T is an arbitrarily chosen positive number;
- $h(x)$ - is a periodic function (with the period T) that should not be constant and must satisfy the Lipschitz condition.

Roughness profile generated after each passage of a cutting disc changes and so does the Weierstrass function $z_n(x)$ becoming $z_{n+1}(x)$. Relating the constants a and b from the Weierstrass formula (Remark 3) to conditions of the grinding process, we obtain the fractal dimension D_z , and are able to generate (simulate) the corresponding surface and to determine how its profile (and hence properties) depend on the process parameters. The work on applications of the fractal theory [12] to the theory of engineering processes was Andrzej Lasota's final work dealing with the applications

of mathematics. Applications of fractals to the description of machining cases are now extensively studied by Prof. Lasota's fellow researchers.

Finally, a few words about Prof. Andrzej Lasota from his student and later a fellow researcher. Mathematics that he dealt with would support engineering, was utilised to highlight major phenomena and processes and the rules of their occurrence. In his work he was a brilliant discoverer, not just a gifted performer. In his research work, he never took orders from anybody. He never worked for money, just by adjusting mathematical models to fit the existing laboratory data. He was always interested in problems that would show new horizons, both for theoretical research and workshop practice. His theories were always elegant and their beauty was evident even to us, engineers. He would never slight and diminish the role of workshop practice, which he always treated as a major reference point. Simplicity of final results allowed their easy implementation. His works are also most valuable in the teaching aspect, highlighting the links between theoretical results and commonly used engineering practice, hydrodynamics, theoretical mechanics, strength of materials. His unfulfilled dream was a coherent theory of friction, he engaged in numerous experimental programs of great practical value though failing to provide a better insights into the physical aspects of friction. He was busy making plans, leaving notes and observations. Until the very end of his life he never ceased to be a most creative person, always encouraging and inspiring his fellow workers.

REFERENCES

- [1] R.M. Ejszes, *K metodike raschiota riezhimow bureniya sharoshechnymi dolotami*, Trudy MJJNH i GP im. J. Gubkina, Niedra 1967.
- [2] A. Lasota, P. Rusek, *Problemy stabilności pracy narzędzia w procesie wiercenia obrotowego świdrami gryzowymi*, Archiwum Górnictwa 15, 1970.
- [3] A. Lasota, J. Yorke, *On the existence of invariant measures for piecewise monotonic transformation*, Transactions of the American Mathematical Society **186** (1973), 481–488.
- [4] S. Ulam, *A collection of mathematical problems*, Interscience Publish., Inc., 1960.
- [5] A. Lasota, P. Rusek, *Zastosowanie teorii ergodycznej do wyznaczenia wydajności narzędzi gryzowych*, Archiwum Górnictwa 19, 1974.
- [6] L.E. Graf, D.J. Kogan, O.W. Smirnow, G.H. Fiedosiejew, *Eksperymentalnoje issledowanije pieriedachi energii udara primienitelno k gidroudarnym mashinam konstrukcji CKB Ministierstwa Gieologii SSSR*, Rabota kolektiwnaya: Tieoriya i praktika udarno wrashchatielnogo burieniya, Niedra, 1967.
- [7] J.E. Wladyslawliw, *Issledowanije wlijaniya osiej sharoshok na pokazatieli raboty triexsharoshiechnyx dolot*, Rabota kolektiwnaya: Tieoriya i tiekhnika burieniya, Niedra 1967.
- [8] A. Lasota, P. Rusek, *O problemie optymalizacji pracy narzędzi udarowych*, Archiwum Górnictwa **18** (1972).

- [9] A. Lasota, P. Rusek, *Stability of self-induced vibrations in metal cutting*, Proceedings of the Fifth World Congress on the Theory of Machines and Mechanisms, Montreal, 1979.
- [10] A. Lasota, P. Rusek, *Influence of random vibrations on the roughness of turned surfaces*, Journal of Mechanical Working Technology, Elsevier Scientific Publishing Co. **7** (1982/1983), 277–284.
- [11] B.R. Hunt, *The Hausdorff dimension of graphs of Weierstrass functions*, Proc. Amer. Math. Soc. **126** (1998) 3, 791–800.
- [12] W. Brostow, A. Lasota, P. Rusek, *Description of surfaces for tribology*, International Conference Baltrib 2007 Kaunas, Lithuania: Proceedings (Lithuan University of Agriculture) (2007), 130–133.

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