

A MEIR-KEELER TYPE COMMON FIXED POINT THEOREM FOR FOUR MAPPINGS

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Abstract. In this paper, we prove a general common fixed point theorem for two pairs of weakly compatible self-mappings of a metric space satisfying a weak Meir-Keeler type contractive condition by using a class of implicit relations. In particular, our result generalizes and improves a result of K. Jha, R.P. Pant, S.L. Singh, by removing the assumption of continuity, relaxing compatibility to weakly compatibility property and replacing the completeness of the space with a set of four alternative conditions for maps satisfying an implicit relation. Also, our result improves the main result of H. Bouhadjera, A. Djoudi.

Keywords: common fixed point for four mappings, weakly compatible mappings, Meir-Keeler type contractive condition, complete metric spaces.

Mathematics Subject Classification: 54H25, 47H10.

1. INTRODUCTION

We start by recalling on some concepts of weak commutativity used in fixed point theory.

Two self-mappings A and S of a metric space (X, d) are called compatible (see Jungck [7]) if

$$\lim_{n \rightarrow \infty} d(ASx_n, SAx_n) = 0,$$

whenever $\{x_n\}$ is a sequence in X such that

$$\lim_{n \rightarrow \infty} Ax_n = \lim_{n \rightarrow \infty} Sx_n = t$$

for some t in X .

In 1993, Jungck, Murthy and Cho [9] define S and T to be compatible of type (A) if $\lim_{n \rightarrow \infty} d(TSx_n, S^2x_n) = 0$ and $\lim_{n \rightarrow \infty} d(STx_n, T^2x_n) = 0$, whenever $\{x_n\}$ is a sequence in X such that $\lim_{n \rightarrow \infty} Sx_n = \lim_{n \rightarrow \infty} Tx_n = x$ for some $x \in X$.

By [9, Ex. 2.1 and Ex. 2.2] it follows that the notions of compatible mappings and compatible mappings of type (A) are independent.

In 1995, Pathak and Khan [22] introduced a new concept of compatible mappings of type (B) as a generalization of compatible mappings of type (A). Two mappings S and T are said to be compatible of type (B) if

$$\lim_{n \rightarrow \infty} d(STx_n, T^2x_n) \leq \frac{1}{2} \left[\lim_{n \rightarrow \infty} d(STx_n, St) + \lim_{n \rightarrow \infty} d(St, S^2x_n) \right]$$

and

$$\lim_{n \rightarrow \infty} d(TSx_n, S^2x_n) \leq \frac{1}{2} \left[\lim_{n \rightarrow \infty} d(TSx_n, Tt) + \lim_{n \rightarrow \infty} d(Tt, T^2x_n) \right],$$

whenever $\{x_n\}$ is a sequence in X such that $\lim_{n \rightarrow \infty} Sx_n = \lim_{n \rightarrow \infty} Tx_n = t$ for some $t \in X$.

Clearly, compatible mappings of type (A) are compatible of type (B). By [22, Ex. 2.4] it follows that the converse is not true.

In [23], the concept of compatible mappings of type (P) was introduced and compared with compatible mappings and compatible mappings of type (A). We recall that two self-mappings S and T of a metric space (X, d) are said to be compatible of type (P) if $\lim_{n \rightarrow \infty} d(S^2x_n, T^2x_n) = 0$ whenever $\{x_n\}$ is a sequence in X such that $\lim_{n \rightarrow \infty} Sx_n = \lim_{n \rightarrow \infty} Tx_n = t$ for some $t \in X$.

In 1994, Pant [15] introduced the notion of pointwise R-weakly commuting mappings. Two self mappings A and S of a metric space (X, d) are called R -weakly commuting at a point $x \in X$ if $d(ASx, SAx) \leq Rd(Ax, Sx)$ for some $R > 0$. The mappings A and S are called pointwise R -weakly commuting if given x in X , there exists $R > 0$ such that $d(ASx, SAx) \leq Rd(Ax, Sx)$. It is proved in [16] that the notion of pointwise R -weakly commuting is equivalent to commutativity at coincidence points.

In 1996, Jungck [8] defines S and T to be weakly compatible if $Sx = Tx$ implies $STx = TSx$. Thus S and T are weakly compatible if and only if S and T are pointwise R -weakly commuting mappings.

Lemma 1.1 ([7], resp. [9, 22, 23]). *Let S and T be compatible (resp. compatible of type (A), compatible of type (B), compatible of type (P)) self mappings of a metric space (X, d) . If $Sx = Tx$ for some $x \in X$, then $STx = TSx$.*

Remark 1.2. By Lemma 1.1, it follows that every compatible (compatible of type (A), compatible of type (B), compatible of type (P)) pair of mappings is weakly compatible. In [25], V. Popa has given a pair of mappings which is weakly compatible but not compatible (compatible of type (A), compatible of type (B), compatible of type (P)).

2. PRELIMINARIES

In 1969, Meir and Keeler [12] established a fixed point theorem for self mappings of a metric space (X, d) satisfying the following condition:

For every $\epsilon > 0$, there exists a $\delta > 0$ such that

$$\epsilon \leq d(x, y) < \epsilon + \delta \implies d(fx, fy) < \epsilon. \quad (2.1)$$

In 1975, in connection to (2.1), J. Matkowski (see [11]) has proved the following fixed point result.

Theorem 2.1 (J. Matkowski [11]). *Let f be a self-mapping of a complete metric space (X, d) and let*

$$d(f(x), f(y)) < d(x, y) \quad \text{for all } x, y \in X, x \neq y. \quad (2.2)$$

If for every $\epsilon > 0$ there exists a $\delta > 0$ such that

$$\epsilon < d(x, y) < \epsilon + \delta \implies d(f(x), f(y)) \leq \epsilon, \quad (2.3)$$

then there exists exactly one fixed point of f ; moreover, its domain of attraction coincides with the whole of X .

For a self-mapping f of a metric space (X, d) , we consider the following conditions: for every $\epsilon > 0$, there exists a $\delta > 0$ such that

$$d(x, y) < \epsilon + \delta \implies d(fx, fy) \leq \epsilon, \quad (2.4)$$

and

$$x, y \in X, d(x, y) > 0 \implies d(fx, fy) < d(x, y). \quad (2.5)$$

Conditions (2.4) and (2.5) are implied by (2.1).

In [10], Maiti and Pal proved a fixed point theorem for a self-mapping f of a metric space (X, d) satisfying the following condition, which is a generalization of (2.1):

for every every $\epsilon > 0$, there exists a $\delta > 0$ such that

$$\epsilon \leq \max\{d(x, y), d(x, fx), d(y, fy)\} < \epsilon + \delta \implies d(fx, fy) < \epsilon. \quad (2.6)$$

In [21] and [26], Park-Rhodes and Rao-Rao have extended this result to the case of two self-mappings f and g of a metric space (X, d) satisfying the following condition:

for every every $\epsilon > 0$, there exists a $\delta > 0$ such that

$$\begin{aligned} \epsilon \leq \max\{d(fx, fy), d(fx, gx), d(fy, gy), \frac{1}{2}[d(fx, gy) + d(fy, gx)]\} < \epsilon + \delta \\ \implies d(fx, gy) < \epsilon. \end{aligned} \quad (2.7)$$

In 1986, Jungck [7] and Pant [13] extended these results for four mappings. It is known from Jungck [7] and Pant [14, 16–18] and other papers the fact that in the case of four mappings $A, B, S, T : (X, d) \rightarrow (X, d)$, a contractive condition of Meir-Keeler type is not sufficient to ensure the existence of a common fixed point. So some additional conditions are needed. Generally, these conditions are a weak type commutativity between the maps and some topological conditions.

To simplify notations, for all $x, y \in X$, we set

$$M(x, y) := \max \left\{ d(Sx, Ty), d(Ax, Sx), d(By, Ty), \frac{d(Sx, By) + d(Ax, Ty)}{2} \right\}$$

and

$$\sigma(x, y) := d(Sx, Ty) + d(Ax, Sx) + d(By, Ty) + d(Sx, By) + d(Ax, Ty).$$

For four self-mappings A, B, S and T of a metric space (X, d) , K. Jha, R.P. Pant and S.L. Singh (see [6]) considered the following contractive condition of Meir-Keeler type:

given $\epsilon > 0$ there exists a $\delta > 0$ such that

$$\epsilon \leq M(x, y) < \epsilon + \delta \implies d(Ax, By) < \epsilon \quad (2.8)$$

and have established the following theorem.

Theorem 2.2 ([6]). *Let (A, S) and (B, T) be two compatible pairs of self-mappings of a complete metric space (X, d) such that:*

- (i) $AX \subset TX$, $BX \subset SX$,
- (ii) given $\epsilon > 0$ there exists a $\delta > 0$ such that

$$\epsilon \leq M(x, y) < \epsilon + \delta \implies d(Ax, By) < \epsilon, \quad \text{and}$$

- (iii) $d(Ax, By) < k\sigma(x, y)$ for all $x, y \in X$, for $0 \leq k \leq \frac{1}{3}$.

If one of the mappings A, B, S and T is continuous then A, B, S and T have a unique common fixed point.

Remark 2.3. If A, B, S and T have a common fixed point, then the symbol ' $<$ ' in the condition (iii) must be replaced by the symbol ' \leq '. Otherwise, (iii) would give $0 < 0$ which is impossible. This change will suggest the new condition $0 \leq k < \frac{1}{3}$ on k instead of $0 \leq k \leq \frac{1}{3}$.

In [18] and [20] other similar results are published.

In [25], V. Popa introduced a class of implicit relations to generalize the results of [6].

In this paper, by using a combination of methods used in [4, 24] and [27], we improve the result of [6] by removing the assumption of continuity, relaxing compatibility to a weakly compatibility property and replacing the completeness of the space with a set of four alternative conditions for four functions satisfying an implicit relation.

After the introduction and preliminaries, in the third section, we introduce a new class of implicit relations (called \mathcal{P}_4) that will be used in our main result. In the fourth section, we present and prove our main result (see Theorem 4.2).

3. IMPLICIT RELATIONS

Let \mathbb{R}_+ be the set of non-negative real numbers and let \mathcal{P}_4 be the set of all functions $F(t_1, \dots, t_4) : \mathbb{R}_+^4 \rightarrow \mathbb{R}$ which are lower semi-continuous and satisfying the following conditions:

$$(P): F(u, 0, u, u) \leq 0 \implies u = 0.$$

It is easy to see that all the following functions satisfy property (P).

Example 3.1. $F(t_1, \dots, t_4) = t_1 - k[t_2 + t_3 + t_4]$, where k is such that $0 \leq k < \frac{1}{2}$.

Example 3.2. $F(t_1, \dots, t_4) = t_1 - at_2 - bt_3 - ct_4$, where $a, b, c \geq 0$ are such that $0 \leq b + c < 1$.

Example 3.3. $F(t_1, \dots, t_4) = t_1 - q \max\{t_2, t_3, t_4\}$, where $0 \leq q < 1$.

Example 3.4. $F(t_1, \dots, t_4) = t_1^2 - a[t_2^2 + t_3^2 + t_4^2]$, where $0 \leq a < \frac{1}{2}$.

Example 3.5. $F(t_1, \dots, t_4) = t_1^p - k[t_2^p + t_3^p + t_4^p]$, where $p > 0$ and $0 \leq k < \frac{1}{2}$.

Example 3.6. $F(t_1, \dots, t_4) = t_1^2 - t_2^2 - \frac{bt_3t_4}{1+t_2+t_3}$, where $0 \leq b < 1$.

Example 3.7. $F(t_1, \dots, t_4) = t_1 - \max\{t_2, \frac{t_3}{2}, \frac{kt_4}{2}\}$, where $0 \leq k \leq 1$.

Example 3.8. $F(t_1, \dots, t_4) = t_1 - \max\{k_1t_2, \frac{k_2}{2}t_3, \frac{t_4}{2}\}$, where $0 \leq k_1 \leq 1$ and $1 \leq k_2 < 2$.

4. COMMON FIXED POINT RESULT

The following lemma (see [5]) played a crucial role in the proofs of the main results of [6] and [25] and will be used to prove the main result of this paper.

Lemma 4.1 (2.2 of [5]). *Let A, B, S and T be self mappings of a metric space (X, d) such that $AX \subset TX$ and $BX \subset SX$. Assume further that given $\epsilon > 0$ there exists $\delta > 0$ such that for all x, y in X*

$$\epsilon < M(x, y) < \epsilon + \delta \implies d(Ax, By) \leq \epsilon, \quad (4.1)$$

and

$$d(Ax, By) < M(x, y), \quad \text{whenever } M(x, y) > 0. \quad (4.2)$$

Then for each x_0 in X , the sequence $\{y_n\}$ in X defined by the rule

$$y_{2n} = Ax_{2n} = Tx_{2n+1}, \quad y_{2n+1} = Bx_{2n+1} = Sx_{2n+2} \quad \forall n \in \mathbb{N}$$

is a Cauchy sequence.

The main result of this paper reads as follows.

Theorem 4.2. *Let S, T, I and J be the self-mappings of a metric space (X, d) such that:*

(H1) $SX \subseteq JX$ and $TX \subseteq IX$,

(H2) (a) given $\epsilon > 0$ there exists a $\delta > 0$ such that

$$\epsilon < M(x, y) < \epsilon + \delta \implies d(Sx, Ty) \leq \epsilon, \quad \text{and}$$

(H2) (b) $x, y \in X, M(x, y) > 0 \implies d(Sx, Ty) < M(x, y)$,
where

$$M(x, y) := \max \left\{ d(Ix, Jy), d(Ix, Sx), d(Jy, Ty), \frac{d(Ix, Ty) + d(Sx, Jy)}{2} \right\};$$

(H3) there exists $F \in \mathcal{P}_4$ such that the following inequality

$$F(d(Sx, Ty), d(Ix, Jy), d(Ix, Sx) + d(Jy, Ty), d(Ix, Ty) + d(Jy, Sx)) \leq 0 \quad (4.3)$$

holds for all x, y in X .

If one of $S(X), T(X), I(X)$ and $J(X)$ is a complete subspace of (X, d) , then:

- (i) S and I have a coincidence point,
- (ii) T and J have a coincidence point.

Moreover, if the pairs (S, I) and (T, J) are weakly compatible, then the mappings S, T, I and J have a unique common fixed point.

Proof. Let x_0 be an arbitrary point in X . Then by virtue of (H1), we can define inductively two sequences $\{x_n\}$ and $\{y_n\}$ in X by the rule:

$$y_{2n} = Sx_{2n} = Jx_{2n+1} \quad \text{and} \quad y_{2n+1} = Tx_{2n+1} = Ix_{2n+2}, \quad (4.4)$$

for each nonnegative integer n . By Lemma 4.1, it follows that the sequence $\{y_n\}$ is a Cauchy sequence.

(1) Suppose that $S(X)$ is a complete subspace of (X, d) . Then there exists a point (say) z in $S(X)$ such that

$$z = \lim_{n \rightarrow \infty} y_{2n} = \lim_{n \rightarrow \infty} Sx_{2n} = \lim_{n \rightarrow \infty} Jx_{2n+1}. \quad (4.5)$$

Since $\lim_{n \rightarrow \infty} d(y_n, y_{n+1}) = 0$, then by (4.5) it follows that we have

$$z = \lim_{n \rightarrow \infty} Sx_{2n} = \lim_{n \rightarrow \infty} Jx_{2n+1} = \lim_{n \rightarrow \infty} Ix_{2n} = \lim_{n \rightarrow \infty} Tx_{2n+1}. \quad (4.6)$$

Since $S(X) \subset J(X)$, then there exists $v \in X$ such that $z = Jv$. By (H3), we get

$$F(d(Sx_{2n}, Tv), d(Ix_{2n}, Jv), d(Ix_{2n}, Sx_{2n}) + d(Jv, Tv), d(Ix_{2n}, Tv) + d(Jv, Sx_{2n})) \leq 0.$$

Letting $n \rightarrow \infty$ and using the lower semi-continuity of F , we obtain

$$F(d(Jv, Tv), 0, d(Jv, Tv), d(Jv, Tv)) \leq 0.$$

By the property (P), it follows that $Jv = Tv$. Thus, we have $z = Jv = Tv$. Since $T(X) \subset I(X)$, and $z = Tv \in T(X)$, then there exists $w \in X$ such that $z = Tv = Iw$. Then $z = Jv = Tv = Iw$. By applying the inequality (H3), we get

$$\begin{aligned} 0 &\geq F(d(Sw, Tv), d(Iw, Jv), d(Sw, Iw) + d(Jv, Tv), d(Iw, Tv) + d(Jv, Sw)) = \\ &= F(d(Sw, Iw), 0, d(Sw, Iw), d(Sw, Iw)), \end{aligned}$$

which, by virtue of (P), implies that $Sw = Iw$. Hence, we obtain

$$z = Jv = Tv = Iw = Sw. \tag{4.7}$$

The conclusions in (4.7) will be obtained by similar arguments, if we suppose that $J(X)$, $T(X)$ or $I(X)$ is a complete subspace of X . This proves (i) and (ii).

(2) Suppose that the pairs $\{S, I\}$ and $\{T, J\}$ are weakly compatible. Then it follows

$$Sz = Iz \quad \text{and} \quad Tz = Jz. \tag{4.8}$$

Now, we show that $z = Tz$. To get a contradiction, let us suppose that $d(z, Sz) > 0$. We start by observing that by setting

$$\epsilon := \max\{d(Iw, Jz), d(Iw, Sw), d(Jz, Tz), [d(Iw, Tz) + d(Sw, Jz)]/2\} = d(z, Tz) > 0.$$

Then, by virtue of assumption (H2)(b), we get

$$d(z, Tz) = d(Sw, Tz) < \epsilon = d(z, Tz),$$

which is a contradiction. Thus we have $z = Tz = Jz$.

Now, we show that $z = Sz$. To obtain a contradiction, let us suppose the contrary. We observe that

$$\epsilon := \max\{d(Iz, Jv), d(Iz, Sz), d(Jv, Tv), [d(Iz, Tv) + d(Sz, Jv)]/2\} = d(Sz, z) > 0.$$

Then, by virtue of assumption (H2)(b), we get

$$d(Sz, z) = d(Sz, Tv) < \epsilon = d(Sz, z),$$

which is a contradiction. Thus we have $z = Sz = Iz$. Thus, we have $z = Sz = Iz = Jz = Tz$. We conclude that z is a common fixed point for S, T, I and J .

(3) Suppose that y is another common fixed point for the mappings S, T, I and J , such that $y \neq z$. Obviously we have

$$\epsilon := \max\{d(Iy, Jz), d(Iy, Sy), d(Jz, Tz), [d(Iy, Tz) + d(Sy, Jz)]/2\} = d(y, z) > 0.$$

Then, by applying condition (H2)(b), we obtain

$$d(y, z) = d(Sy, Tz) < \epsilon = d(y, z),$$

which is a contradiction. So the mappings S, T, I and J have a unique common fixed point. This completes the proof. \square

Corollary 4.3. *Let S, T, I and J be the self mappings of a complete metric spaces satisfying conditions (H1), (H2)(a), (H2)(b) and (H3) of Theorem 4.2. Then the conditions (i) and (ii) of Theorem 4.2 hold. Moreover, if the pair (S, I) and (T, J) are compatible (compatible of type (A), compatible of type (B), compatible of type (P)) then S, T, I and J have a unique common fixed point.*

Proof. It follows by Theorem 4.2 and Remark 1.2. □

Corollary 4.4. *Let (S, I) and (T, J) be two weakly compatible pairs of self-mappings of a complete metric space (X, d) such that:*

- (a) $SX \subseteq JX$ and $TX \subseteq IX$,
- (b) one of SX, JX, TX or IX is closed,
- (c) given $\epsilon > 0$ there exists a $\delta > 0$ such that

$$\epsilon < M(x, y) < \epsilon + \delta \implies d(Sx, Ty) \leq \epsilon, \quad \text{and}$$

- (c') $x, y \in X, M(x, y) > 0 \implies d(Sx, Ty) < M(x, y)$, where

$$M(x, y) := \max\{d(Ix, Jy), d(Ix, Sx), d(Jy, Ty), [d(Ix, Ty) + d(Sx, Jy)]/2\},$$

- (d)

$$d(Ax, By) \leq k[d(Ix, Jy) + d(Ix, Sx) + d(Jy, Ty) + d(Ix, Ty) + d(Jy, Sx)],$$

$$\text{for } 0 \leq k < \frac{1}{2}.$$

Then S, T, I and J have a unique common fixed point.

Proof. It follows by Theorem 4.2 and Example 3.1. □

We point out that Corollary 4.4 improves the main result of [1]. Indeed, in Corollary 4.3 the Lipschitz constant k is allowed to take values in the interval $[0, \frac{1}{2})$ instead of the case studied in [1], where the constant k belongs to the smaller interval $[0, \frac{1}{3})$.

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