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EXISTENCE RESULTS FOR MILD SOLUTIONS OF IMPULSIVE PERIODIC SYSTEMS

YanLong Yang and JinRong Wang

Abstract. By applying the Horn's fixed point theorem, we prove the existence of T_0-periodic PC-mild solution of impulsive periodic systems when PC-mild solutions are ultimate bounded.

Keywords: impulsive periodic systems, T_0-periodic PC-mild solution, Horn's fixed point theorem, existence.

Mathematics Subject Classification: 47J35.

1. INTRODUCTION

It is well known that periodic motion is a very important and special phenomena not only in the natural sciences but also in social sciences such as climate, food supplements, insecticide population and sustainable development. The problem of finding periodic solutions is an important subject in the qualitative study of differential equations. Related studies (such as existence, the relationship between bounded solutions and periodic solutions, stability and robustness) and examples concerning non-autonomous periodic systems on finite dimensional spaces can be found in references such as [5, 6, 13, 16]; periodic systems with time-varying generating operators on infinite dimensional spaces can be found in references such as [1, 7–11].

On the other hand, the theory of impulsive differential equations appears as a natural description of several real processes subject to certain perturbations whose duration is negligible in comparison with the duration of the process. Processes of this type are often investigated in various fields of science and technology: physics, population dynamics, ecology, biological systems, pharmacokinetics, optimal control, etc. For the basic theory on impulsive differential equations on finite dimensional spaces, the reader can see, for instance, the monographs of Bainov and Simeonov [5], Lakshmikantham [12] and Yang [16] et al. For the basic theory on impulsive differential equations on infinite dimensional spaces, the reader can refer to Ahmed's paper (see [3, 4]).

Impulsive periodic systems with time-varying generator operators are basic periodic systems with time-varying generator operators to study the dynamics of processes that are subject to periodic changes in their states. Recently, we have established periodic solutions theory under the existence of a bounded solution for the linear impulsive periodic system with time-varying generator operators on infinite dimensional spaces. Several criteria were obtained to ensure the existence, uniqueness, alternative theorem, Massera's theorem and robustness of the T_0-periodic PC-mild solution for the linear impulsive periodic system with time-varying generator operators (see [14, 15]). However, to our knowledge, we have not seen results about periodic PC-mild solutions of semilinear impulsive periodic system with time-varying generator operators, thus we would like to provide one here.

This paper is concerned with deriving periodic solutions from ultimate boundedness of solutions for the following semilinear impulsive periodic system with time-varying generating operators

$$\begin{cases} \dot{x}(t) = A(t)x(t) + f(t,x), \quad t \neq \tau_k, \\ \Delta x(t) = x(t^+) - x(t^-) = B_k x(t) + c_k, \quad t = \tau_k, \end{cases} \tag{1.1}$$

in the parabolic case on an infinite dimensional Banach space X, where $\{A\,(t),\, t \in [0, T_0]\}$ is a family of closed densely defined linear unbounded operators on X and the resolvent of the unbounded operator $A(t)$ is compact. f is a measurable function from $[0,\infty) \times X$ to X and is T_0-periodic in t. $0 = \tau_0 < \tau_1 < \tau_2 < \ldots < \tau_k \ldots$, $\lim_{k\to\infty} \tau_k = \infty$, $\tau_{k+\delta} = \tau_k + T_0$, $\widetilde{D} = \{\tau_1, \tau_2, \ldots, \tau_\delta\} \subset (0, T_0)$, $\triangle x(\tau_k) := x(\tau_k^+) - x(\tau_k^-)$, where $k \in \mathbb{Z}_0^+ := \{0, 1, 2, \ldots\}$, T_0 is a fixed positive number and $\delta \in \mathbb{N} := \{1, 2, \ldots\}$ denotes the number of impulsive points between 0 and T_0; for each $k \in \mathbb{Z}_0^+$, $B_k \in L_b(X)$, $c_k \in X$ and there exists a $\delta \in \mathbb{N}$ such that $B_{k+\delta} = B_k$ and $c_{k+\delta} = c_k$ where $L_b(X)$ denotes the space of bounded linear operators on X equipped with the usual operator norm.

In the study of periodic solutions, we first construct the new suitable Poincaré operator $\mathcal{P}(\bar{x}) = x(T_0, \bar{x})$, ($T_0$ units along x) by virtue of the impulsive evolution operator corresponding to a homogeneous linear impulsive periodic system with time-varying generator operators and show that $\mathcal{P}$ is compact, where T_0 is the periodic of the system (1.1) and x is the unique solution determined by initial value $\bar{x}$. Then some suitable conditions are given such that Horn's fixed point theorem can be used to to get fixed points for the Poincaré operator $\mathcal{P}$, which give rise to periodic solutions. This extends the study of periodic solutions for the periodic system with time-varying generator operators without impulse to impulsive periodic system with time-varying generator operators on general Banach spaces.

This paper is organized as follows. In section 2, some results of linear impulsive periodic systems with time-varying generator operators and properties of impulsive evolution operators corresponding to homogeneous linear impulsive periodic systems with time-varying generator operators are recalled. In section 3, the Gronwall lemma with an impulse is used and the T_0-periodic PC-mild solution of a semilinear impulsive periodic system with time-varying generator operators (1.1) is introduced. The new Poincaré operator $\mathcal{P}$ is constructed and the relation between T_0-periodic PC-mild solution and the fixed point of Poincaré operator $\mathcal{P}$ is given. After the compactness of Poincaré operator $\mathcal{P}$ are shown, the existence of T_0-periodic PC-mild solutions for semilinear impulsive periodic systems with time-varying generator operators is established by virtue of Horn's fixed point theorem when PC-mild solutions are ultimate bounded. At last, an example is given to demonstrate the applicability of our result.

2. PRELIMINARIES

Throughout this paper, we denote by X a Banach space with the norm $\|\cdot\|$. $L_b(X)$ denotes the Banach space of all bounded linear operators on X equipped with the usual operator norm. Define $\widetilde{D}=\{\tau_1,\ldots,\tau_\delta\}\subset[0,T_0]$, where $\delta\in\mathbb{N}$ denotes the number of impulsive points between 0 and T_0. We introduce $PC([0,T_0];X)\equiv\{x:[0,T_0]\to X \mid x$ is continuous at $t\in[0,T_0]\backslash\widetilde{D}$, x is continuous from the left and has right hand limits at $t\in\widetilde{D}\}$ and $PC^1([0,T_0];X)\equiv\{x\in PC([0,T_0];X)\mid \dot{x}\in PC([0,T_0];X)\}$. Set $\|x\|_{PC}=\sup_{t\in[0,T_0]}\|x(t)\|$ and $\|x\|_{PC^1}=\|x\|_{PC}+\|\dot{x}\|_{PC}$. It can be seen that endowed with the norm $\|\cdot\|_{PC}$ ($\|\cdot\|_{PC^1}$), $PC([0,T_0];X)$ $\big(PC^1([0,T_0];X)\big)$ is a Banach space.

In order to study the semilinear impulsive periodic system with time-varying generating operators, we first recall the following linear impulsive periodic system

$$\begin{cases}\dot{x}\;(t)=A(t)x(t), & t\neq\tau_k,\\ \Delta x(t)=B_kx(t), & t=\tau_k,\end{cases}\tag{2.1}$$

where $\Delta x(t)=x(t^+)-x(t^-)$, $B_k\in L_b(X)$ for each $k\in\mathbb{Z}_0^+$ and $\{A\,(t),\,t\in[0,T_0]\}$ is a family of closed densely defined linear unbounded operators on X satisfying the following assumption.

Assumption 2.1 (A1, [2, p. 158])**.** For $t\in[0,T_0]$, one has:

(P_1) The domain $D(A(t))=D$ is independent of t and is dense on X.

(P_2) For $t\geq0$, the resolvent $R(\lambda,A(t))=(\lambda I-A(t))^{-1}$ exists for all λ with $Re\lambda\leq0$, and there is a constant M independent of λ and t such that

$$\|R\,(\lambda,A(t))\|\leq M(1+|\lambda|)^{-1}\qquad\text{for}\qquad Re\lambda\leq0.$$

(P_3) There exist constants $L>0$ (independent of t,θ,τ) and $0<\alpha\leq1$ such that

$$\big\|\big(A(t)-A(\theta)\big)A^{-1}(\tau)\big\|\leq L|t-\theta|^{\alpha}\qquad\text{for}\qquad t,\theta,\tau\in[0,T_0].$$

Lemma 2.2 ([2, p. 159])**.** *Under the assumption* (A1)*, the Cauchy problem*

$$\dot{x}(t)+A(t)x(t)=0,\;t\in(0,T_0]\quad\textit{with}\quad x(0)=\bar{x}\tag{2.2}$$

has a unique evolution system $\{U(t,\theta)\mid 0\leq\theta\leq t\leq T_0\}$ *on* X *satisfying the following properties:*

(1) $U(t,\theta)\in L_b(X)$ *for* $0\leq\theta\leq t\leq T_0$.

(2) *$U(t,r)U(r,\theta)=U(t,\theta)$ for $0\le\theta\le r\le t\le T_0$;*
(3) *$U(\cdot,\cdot)x\in C(\Delta,X)$ for $x\in X$, $\Delta=\{(t,\theta)\in[0,T_0]\times[0,T_0]\mid 0\le\theta\le t\le T_0\}$;*
(4) *For $0\le\theta<t\le T_0$, $U(t,\theta)\colon X\longrightarrow D$ and $t\longrightarrow U(t,\theta)$ is strongly differentiable on X. The derivative $\frac{\partial}{\partial t}U(t,\theta)\in L_b(X)$ and it is strongly continuous on $0\le\theta<t\le T_0$. Moreover,*

$$\frac{\partial}{\partial t}U(t,\theta)=-A(t)U(t,\theta)\quad \textit{for}\quad 0\le\theta<t\le T_0,$$
$$\left\|\frac{\partial}{\partial t}U(t,\theta)\right\|_{L_b(X)}=\|A(t)U(t,\theta)\|_{L_b(X)}\le\frac{C}{t-\theta},$$
$$\left\|A(t)U(t,\theta)A(\theta)^{-1}\right\|_{L_b(X)}\le C\quad \textit{for}\quad 0\le\theta\le t\le T_0;$$

(5) *For every $v\in D$ and $t\in(0,T_0],U(t,\theta)v$ is differentiable with respect to θ on $0\le\theta\le t\le T_0$*

$$\frac{\partial}{\partial\theta}U(t,\theta)v=U(t,\theta)A(\theta)v,$$

and, for each $\bar{x}\in X$, the Cauchy problem (2.2) has a unique classical solution $x\in C^1([0,T_0];X)$ given by

$$x(t)=U(t,0)\bar{x},\quad t\in[0,T_0].$$

In addition to assumption (A1), we introduce the following assumptions.

Assumption 2.3 (A2)**.** There exits $T_0>0$ such that $A(t+T_0)=A(t)$ for $t\in[0,T_0]$.

Assumption 2.4 (A3)**.** For $t\ge 0$, the resolvent $R(\lambda,A(t))$ is compact.

Then we have the following lemma.

Lemma 2.5. *Let assumptions* (A1)*,* (A2) *and* (A3) *be satisfied. Then evolution system $\{U(t,\theta)\mid 0\le\theta\le t\le T_0\}$ on X has the following two properties:*

(6) *$U(t+T_0,\theta+T_0)=U(t,\theta)$ for $0\le\theta\le t\le T_0$;*
(7) *$U(t,\theta)$ is a compact operator for $0\le\theta<t\le T_0$.*

In order to introduce an impulsive evolution operator and give its properties, we need the following assumption.

Assumption 2.6 (B)**.** For each $k\in\mathbb{Z}_0^+$, $B_k\in L_b(X)$, there exists $\delta\in\mathbb{N}$ such that $\tau_{k+\delta}=\tau_k+T_0$ and $B_{k+\delta}=B_k$.

In order to study system (2.1), we need to consider the associated Cauchy problem

$$\begin{cases}\dot{x}(t)=A(t)x(t), & t\in[0,T_0]\backslash\widetilde{D},\\ \Delta x(\tau_k)=B_kx(\tau_k), & k=1,2,\ldots,\delta,\\ x(0)=\bar{x}.\end{cases}\tag{2.3}$$

For every $\bar{x}\in X$, suppose that the domain D is an invariant subspace of B_k, by using Lemma 2.2, step by step, one can verify that the Cauchy problem (2.3) has a unique classical solution $x \in PC^1([0,T_0];X)$ represented by $x(t) = \mathscr{S}(t,0)\bar{x}$, where $\mathscr{S}(\cdot,\cdot) : \overline{\triangle} = \{(t,\theta) \in [0,T_0] \times [0,T_0] \mid 0 \le \theta \le t \le T_0\} \longrightarrow X$ is given by

$$\mathscr{S}(t,\theta) = \\ = \begin{cases} U(t,\theta), & \tau_{k-1} \le \theta \le t \le \tau_k, \\ U(t,\tau_k^+)(I+B_k)U(\tau_k,\theta), & \tau_{k-1} \le \theta < \tau_k < t \le \tau_{k+1}, \\ U(t,\tau_k^+)\left[\prod_{\theta<\tau_j<t}(I+B_j)U(\tau_j,\tau_{j-1}^+)\right](I+B_i)U(\tau_i,\theta), & \\ & \tau_{i-1} \le \theta < \tau_i \le \ldots < \tau_k < t \le \tau_{k+1}. \end{cases} \tag{2.4}$$

The operator $\mathscr{S}(t,\theta)$ $\big((t,\theta) \in \overline{\triangle}\big)$ is called an impulsive evolution operator associated with $\{B_k;\tau_k\}_{k=1}^{\infty}$.

Now we introduce the PC-mild solution of Cauchy problem (2.3) and T_0-periodic PC-mild solution of system (2.1).

Definition 2.7. For every $\bar{x} \in X$, the function $x \in PC([0,T_0];X)$ given by $x(t) = \mathscr{S}(t,0)\bar{x}$ is said to be the PC-mild solution of the Cauchy problem (2.3).

Definition 2.8. A function $x \in PC([0,+\infty);X)$ is said to be a T_0-periodic PC-mild solution of system (2.1) if it is a PC-mild solution of Cauchy problem (2.3) corresponding to some $\bar{x}$ and $x(t+T_0) = x(t)$ for $t \ge 0$.

The following lemma on the properties of the impulsive evolution operator $\{\mathscr{S}(t,\theta),(t,\theta) \in \overline{\triangle}\}$ associated with $\{B_k;\tau_k\}_{k=1}^{\infty}$ is widely used in this paper.

Lemma 2.9 (see [14, Lemma 2.7])**.** *Assumptions* (A1), (A2), (A3) *and* (B) *hold. The impulsive evolution operator* $\{\mathscr{S}(t,\theta),(t,\theta) \in \overline{\triangle}\}$ *has the following properties:*

(1) *For* $0 \le \theta \le t \le T_0$, $\mathscr{S}(t,\theta) \in L_b(X)$, *and there exists a constant* $M_{T_0} > 0$ *such that*

$$\sup_{0\le\theta\le t\le T_0} \|\mathscr{S}(t,\theta)\| \le M_{T_0};$$

(2) *For* $0 \le \theta < r < t \le T_0$, $r \ne \tau_k$, $\mathscr{S}(t,\theta) = \mathscr{S}(t,r)\mathscr{S}(r,\theta)$;
(3) *For* $0 \le \theta \le t \le T_0$, $\mathscr{S}(t+T_0,\,\theta+T_0) = \mathscr{S}(t,\theta)$;
(4) *For* $0 \le t \le T_0$, $\mathscr{S}(T_0+t,0) = \mathscr{S}(t,0)\mathscr{S}(T_0,0)$;
(5) $\mathscr{S}(t,\theta)$ *is compact operator for* $0 \le \theta < t \le T_0$.

Secondly, we recall nonhomogeneous linear impulsive periodic systems with time-varying generating operators

$$\begin{cases} \dot{x}(t) = A(t)x(t) + f(t), & t \ne \tau_k, \\ \Delta x(\tau_k) = B_k x(\tau_k) + c_k, & t = \tau_k, \end{cases} \tag{2.5}$$

where $f \in L^1([0,T_0];X)$, $f(t+T_0) = f(t)$ and $c_{k+\delta} = c_k$.

Meanwhile, we need to recall the following Cauchy problem

$$\begin{cases} \dot{x}(t) = A(t)x(t) + f(t), & t \in [0, T_0] \backslash \widetilde{D}, \\ \Delta x(\tau_k) = B_k x(\tau_k) + c_k, & k = 1, 2, \ldots, \delta, \\ x(0) = \bar{x}, \end{cases} \tag{2.6}$$

and introduce the PC-mild solution of Cauchy problem (2.6) and T_0-periodic PC-mild solution of system (2.5).

Definition 2.10. A function $x \in PC([0, T_0]; X)$, for finite interval $[0, T_0]$, is said to be a PC-mild solution of the Cauchy problem (2.6) corresponding to the initial value $\bar{x} \in X$ and input $f \in L^1([0, T_0]; X)$ if x is given by

$$x(t) = \mathscr{S}(t, 0)\bar{x} + \int_0^t \mathscr{S}(t, \theta) f(\theta) d\theta + \sum_{0 \le \tau_k < t} \mathscr{S}\left(t, \tau_k^+\right) c_k. \tag{2.7}$$

Definition 2.11. A function $x \in PC([0, +\infty); X)$ is said to be a T_0-periodic PC-mild solution of system (2.5) if it is a PC-mild solution of Cauchy problem (2.6) corresponding to some $\bar{x}$ and $x(t + T_0) = x(t)$ for $t \ge 0$.

Here, we note that system (2.1) has a T_0-periodic PC-mild solution x if and only if $\mathscr{S}(T_0, 0)$ has a fixed point. The impulsive periodic evolution operator $\{\mathscr{S}(t, \theta), (t, \theta) \in \overline{\triangle}\}$ can be used to reduce the existence of T_0-periodic PC-mild solutions for system (2.5) to the existence of fixed points for an operator equation. This implies that we can use the new framework to study the existence of periodic PC-mild solutions for impulsive periodic systems with time-varying generating operators on a Banach space.

3. MAIN RESULTS

In order to derive prior bounds for the solutions, we use the following generalized Gronwall inequality with impulse which can be used in the sequel.

Lemma 3.1. *Let $x \in PC([0, T_0]; X)$ and*

$$\|x(t)\| \le a + b \int_0^t \|x(\theta)\| d\theta + \sum_{0 < \tau_k < t} \zeta_k \|x(\tau_k)\|,$$

where a, b, $\zeta_k \ge 0$, are constants. Then

$$\|x(t)\| \le a \prod_{0 < \tau_k < t} (1 + \zeta_k) e^{bt}.$$

Proof. Define

$$u(t) = a + b\int_0^t \|x(\theta)\|d\theta + \sum_{0<\tau_k<t} \zeta_k \|x(\tau_k)\|,$$

we get

$$\begin{cases} \dot{u}(t) = b\|x(t)\| \le bu(t), \quad t \ne \tau_k, \\ u(0) = a, \\ u(\tau_k^+) = u(\tau_k) + \zeta_k\|x(\tau_k)\| \le (1+\zeta_k)u(\tau_k). \end{cases} \tag{3.1}$$

For $t \in (\tau_k, \tau_{k+1}]$, by (3.1), we obtain

$$u(t) \le u(\tau_k^+)e^{b(t-\tau_k)} \le (1+\zeta_k)u\left(\tau_k\right)e^{b(t-\tau_k)},$$

further,

$$u(t) \le a \prod_{0<\tau_k<t} (1+\zeta_k)e^{bt},$$

thus,

$$\|x(t)\| \le a \prod_{0<\tau_k<t} (1+\zeta_k)e^{bt}.$$

For more details, the reader can refer to Lemma 1.7.1 in [16]. □

Now, we consider the following semilinear impulsive periodic system with time-varying generating operators

$$\begin{cases} \dot{x}(t) = A(t)x(t) + f(t,x), & t \ne \tau_k, \\ \Delta x(t) = B_k x(t) + c_k, & t = \tau_k, \end{cases} \tag{3.2}$$

and introduce the Poincaré operator and study the T_0-periodic PC-mild solution of system (3.2).

In order to study the system (3.2), we need to consider the following associated Cauchy problem

$$\begin{cases} \dot{x}(t) = A(t)x(t) + f(t,x), & t \in [0,T_0]\backslash\widetilde{D}, \\ \Delta x(\tau_k) = B_k x(\tau_k) + c_k, & k = 1,2,\ldots,\delta, \\ x(0) = \bar{x}. \end{cases} \tag{3.3}$$

Definition 3.2. A function $x \in PC([0,T_0];X)$ is said to be a PC-mild solution of the Cauchy problem (3.3) corresponding to the initial value $\bar{x} \in X$ if x satisfies the following integral equation

$$x(t) = \mathscr{S}(t,0)\bar{x} + \int_0^t \mathscr{S}(t,\theta)f(\theta,x(\theta))d\theta + \sum_{0\le\tau_k<t} \mathscr{S}\left(t,\tau_k^+\right)c_k \quad \text{for} \quad t \in [0,T_0].$$

In order to show the existence of the PC-mild solution of Cauchy problem (3.3) and the T_0-periodic PC-mild solution of system (3.2), we introduce the following assumptions.

Assumption 3.3 (F1). $f:[0,\infty)\times X\to X$ is measurable for $t\geq 0$ and for any x, $y\in X$ satisfying $\|x\|$, $\|y\|\leq\rho$ there exists a constant $L_f(\rho)>0$ such that

$$\|f(t,x)-f(t,y)\|\leq L_f(\rho)\|x-y\|.$$

Assumption 3.4 (F2). There exists a positive constant $M_f>0$ such that

$$\|f(t,x)\|\leq M_f(1+\|x\|) \text{ for all } x\in X.$$

Assumption 3.5 (F3). $f(t,x)$ is T_0-periodic in t, i.e. $f(t+T_0,x)=f(t,x)$, $t\geq 0$.

Assumption 3.6 (C). For each $k\in\mathbb{Z}_0^+$ and $c_k\in X$, there exists $\delta\in\mathbb{N}$ such that $c_{k+\delta}=c_k$.

Now, we recall the following result which asserts the existence of PC-mild solutions for Cauchy problem (3.3) and gives a prior bounds of PC-mild solutions for Cauchy problem (3.3) by virtue of Lemma 3.1.

Theorem 3.7. *Let assumptions* (A1), (F1) *and* (F2) *be satisfied, and for each* $k\in\mathbb{Z}_0^+$, $B_k\in L_b(X)$, $c_k\in X$ *be fixed. Let* $\bar{x}\in X$ *be fixed. Then Cauchy problem* (3.3) *has a unique PC-mild solution given by*

$$x(t,\bar{x})=\mathscr{S}(t,0)\bar{x}+\int_0^t\mathscr{S}(t,\theta)f\big(\theta,x(\theta,\bar{x})\big)d\theta+\sum_{0\leq\tau_k<t}\mathscr{S}\left(t,\tau_k^+\right)c_k.$$

Further, suppose $\bar{x}\in\Xi\subset X$, Ξ *is a bounded subset of* X, *then there exits a constant* $M^*>0$ *such that*

$$\|x(t,\bar{x})\|\leq M^* \textit{ for all } t\in[0,T_0].$$

Proof. Under the assumptions (A1), (F1) and (F2), it is well known that Cauchy problem

$$\begin{cases}\dot{x}\ (t)=A(t)x(t)+f(t,x),\ t\in[0,\tau],\\ x(0)=\bar{x}\in X,\end{cases}$$

has a unique mild solution

$$x(t,\bar{x})=U(t,0)\bar{x}+\int_0^t U(t,\theta)f(\theta,x(\theta,\bar{x}))d\theta.$$

In general, for $t\in(\tau_k,\tau_{k+1}]$, Cauchy problem

$$\begin{cases}\dot{x}\ (t)=A(t)x(t)+f(t,x),\ t\in(\tau_k,\tau_{k+1}],\\ x(\tau_k)=x_k\equiv(I+B_k)x(\tau_k)+c_k\in X,\end{cases}$$

has a unique PC-mild solution

$$x(t,x_k)=U(t,\tau_k)x_k+\int\limits_{\tau_k}^{t}U(t,\theta)f(\theta,x(\theta,x_k))d\theta.$$

Combining all solutions on $[\tau_k,\tau_{k+1}]$ $(k=1,\ldots,\delta)$, one can obtain the PC-mild solution of the Cauchy problem (3.3) given by

$$x(t,\bar{x})=\mathscr{S}(t,0)\bar{x}+\int\limits_{0}^{t}\mathscr{S}(t,\theta)f\big(\theta,x(\theta,\bar{x})\big)d\theta+\sum_{0\le\tau_k<t}\mathscr{S}\left(t,\tau_k^{+}\right)c_k.$$

Further, by assumption (F2) and (1) of Lemma 2.9, we obtain

$$\|x(t,\bar{x})\| \quad\le\quad M_{T_0}\|\bar{x}\|+M_{T_0}M_fT_0+M_{T_0}\sum_{k=1}^{\delta}\|c_k\|+M_{T_0}\int\limits_{0}^{t}\|x(\theta,\bar{x})\|d\theta.$$

From $\bar{x}\in\Xi\subset X$, Ξ is a bounded subset of X and Lemma 3.1, one can verify that there exists a constant $M^*>0$ such that

$$\begin{aligned}\|x(t,\bar{x})\|&\le\Bigg(M_{T_0}\|\bar{x}\|+M_{T_0}M_fT_0+M_{T_0}\sum_{0\le\tau_k<T_0}\|c_k\|\Bigg)e^{M_{T_0}T_0}\equiv\\&\equiv M^*,\ \text{ for all } t\in[0,T_0].\end{aligned}$$

□

Now, we can introduce the T_0-periodic PC-mild solution of system (3.2).

Definition 3.8. A function $x\in PC([0,+\infty);X)$ is said to be a T_0-periodic PC-mild solution of system (3.2) if it is a PC-mild solution of the Cauchy problem (3.3) corresponding to some $\bar{x}$ and $x(t+T_0)=x(t)$ for $t\ge 0$.

In order to study the periodic solutions of system (3.2), we construct a new Poincaré operator from X to X as follows.

$$\mathcal{P}(\bar{x})=x(T_0,\bar{x})=\mathscr{S}(T_0,0)\bar{x}+\int\limits_{0}^{T_0}\mathscr{S}(T_0,\theta)f\left(\theta,x(\theta,\bar{x})\right)d\theta+\sum_{0\le\tau_k<T_0}\mathscr{S}\left(T_0,\tau_k^{+}\right)c_k \tag{3.4}$$

where $x(\cdot,\bar{x})$ denotes the PC-mild solution of the Cauchy problem (3.3) corresponding to the initial value $x(0)=\bar{x}$.

We note that a fixed point of $\mathcal{P}$ gives rise to a T_0-periodic PC-mild solution as follows.

Lemma 3.9. *System* (3.2) *has a T_0-periodic PC-mild solution if and only if $\mathcal{P}$ has a fixed point.*

Proof. Suppose $x(\cdot,\bar{x})$ is a T_0-periodic PC-mild solution of system (3.2), then $x(\cdot,\bar{x}) = x(\cdot+T_0,\bar{x})$, which implies $\bar{x} = x(0,\bar{x}) = x(T_0,\bar{x}) = \mathcal{P}(\bar{x})$. This shows that $\bar{x}$ is a fixed point of $\mathcal{P}$. On the other hand, if $\mathcal{P}x_0 = x_0$, $x_0 \in X$, then for the PC-mild solution $x(\cdot,x_0)$ of the Cauchy problem (3.3) corresponding to the initial value $x(0) = x_0$, we can define $y(\cdot,y(0)) = x(\cdot+T_0,x_0)$, then $y(0) = x(T_0,x_0) = \mathcal{P}x_0 = x_0$. Now, for $t > 0$, we can use (2), (3) and (4) of Lemma 2.9 and assumptions (A2), (B), (F3) and (C) to obtain

$$\begin{aligned}
&y(t,y(0)) = x(t+T_0,x_0) = \\
&= \mathscr{S}(t+T_0,0)x_0 + \int_0^{t+T_0} \mathscr{S}(t+T_0,\theta) f\left(\theta, x(\theta,x_0)\right) d\theta + \sum_{0\le\tau_k<t+T_0} \mathscr{S}\left(t+T_0,\tau_k^+\right) c_k = \\
&= \mathscr{S}(t+T_0,T_0)\mathscr{S}(T_0,0)x_0 + \int_0^{T_0} \mathscr{S}(t+T_0,T_0)\mathscr{S}(T_0,\theta) f\big(\theta, x(\theta,x_0)\big) d\theta + \\
&\quad + \sum_{0\le\tau_k<T_0} \mathscr{S}(t+T_0,T_0)\mathscr{S}\left(T_0,\tau_k^+\right) c_k + \\
&\quad + \int_{T_0}^{t+T_0} \mathscr{S}(t+T_0,\theta) f\big(\theta, x(\theta,x_0)\big) d\theta + \sum_{T_0\le\tau_{k+\delta}<t+T_0} \mathscr{S}\left(t+T_0,\tau_{k+\delta}^+\right) c_{k+\delta} = \\
&= \mathscr{S}(t,0)\left\{ \mathscr{S}(T_0,0)x_0 + \int_0^{T_0} \mathscr{S}(T_0,\theta) f\big(\theta, x(\theta,x_0)\big) d\theta + \sum_{0\le\tau_k<T_0} \mathscr{S}\left(T_0,\tau_k^+\right) c_k \right\} + \\
&\quad + \int_0^t \mathscr{S}(t+T_0,s+T_0) f\big(s+T_0, x(s+T_0,x_0)\big) ds + \sum_{0\le\tau_k<t} \mathscr{S}\left(t,\tau_k^+\right) c_k = \\
&= \mathscr{S}(t,0)y(0) + \int_0^t \mathscr{S}(t,s) f\big(s, y(s,y(0))\big) ds + \sum_{0\le\tau_k<t} \mathscr{S}\left(t,\tau_k^+\right) c_k.
\end{aligned} \tag{3.5}$$

This implies that $y(\cdot,y(0))$ is a PC-mild solution of Cauchy problem (3.3) with initial value $y(0) = x_0$. Thus the uniqueness implies that $x(\cdot,x_0) = y(\cdot,y(0)) = x(\cdot+T_0,x_0)$, so that $x(\cdot,x_0)$ is T_0-periodic. □

In the sequel, we need to prove the compactness of the operator $\mathcal{P}$ defined by (3.4). We also list the following definition.

Definition 3.10. An operator $\mathcal{P}$: $X \to X$ is called compact on X if $\mathcal{P}$ maps a bounded set into precompact set.

Lemma 3.11. *Let assumptions* (A1), (A3), (F1) *and* (F2) *be satisfied. Then the operator* $\mathcal{P}$ *defined by* (3.4) *is a compact operator.*

Proof. We only verify that $\mathcal{P}$ takes a bounded set into a precompact set on X. By Theorem 3.1, Cauchy problem (3.3) has a unique PC-mild solution given by

$$x(t,\bar{x}) = \mathscr{S}(t,0)\bar{x} + \int_0^t \mathscr{S}(t,\theta) f\big(\theta, x(\theta,\bar{x})\big)d\theta + \sum_{0\le\tau_k<t} \mathscr{S}\left(t,\tau_k^+\right)c_k \quad \text{for} \quad t\in[0,T_0].$$

Let Γ be a bounded subset of X. Define $K=\mathcal{P}\Gamma$ and $K=\mathcal{P}\Gamma=\{\mathcal{P}(\bar{x})\in X \mid \bar{x}\in\Gamma\}$.

For $0<\varepsilon<t\le T_0$, define

$$K_\varepsilon = \mathcal{P}_\varepsilon\Gamma = \mathscr{S}(T_0,T_0-\varepsilon)\big\{x(T_0-\varepsilon,\bar{x}) \mid \bar{x}\in\Gamma\big\}.$$

Next, we show that K_ε is precompact on X. In fact, for $\bar{x}\in\Gamma$ fixed, we have

$$\begin{aligned}
&\|x(T_0-\varepsilon,\bar{x})\| = \\
&= \left\| \mathscr{S}(T_0-\varepsilon,0)\bar{x} + \int_0^{T_0-\varepsilon} \mathscr{S}(T_0-\varepsilon,\theta) f\left(\theta,x(\theta,\bar{x})\right)d\theta + \sum_{0\le\tau_k<T_0-\varepsilon} \mathscr{S}\left(T_0-\varepsilon,\tau_k^+\right)c_k \right\| \le \\
&\le M_{T_0}\|\bar{x}\| + M_{T_0}M_fT_0 + \int_0^{T_0} \|x(\theta,\bar{x})\|d\theta + M_{T_0}\sum_{0\le\tau_k<T_0}\|c_k\| \le \\
&\le M_{T_0}\|\bar{x}\| + M_{T_0}M_fT_0 + T_0\rho + M_{T_0}\sum_{k=1}^{\delta}\|c_k\|.
\end{aligned}$$

This implies that the set $\big\{x(T_0-\varepsilon,\bar{x}) \mid \bar{x}\in\Gamma\big\}$ is totally bounded.

By assumption (A3) and (5) of Lemma 2.9, $\mathscr{S}(T_0,T_0-\varepsilon)$ is a compact operator. Thus, K_ε is precompact on X.

On the other hand, for arbitrary $\bar{x}\in\Gamma$,

$$\mathcal{P}_\varepsilon(\bar{x}) = \mathscr{S}(T_0,0)\bar{x} + \int_0^{T_0-\varepsilon} \mathscr{S}(T_0,\theta) f\left(\theta,x(\theta,\bar{x})\right)d\theta + \sum_{0\le\tau_k<T_0-\varepsilon} \mathscr{S}\left(T_0,\tau_k^+\right)c_k.$$

Thus, combining with (3.4), using assumption (F2), we have

$$\begin{aligned}
\big\|\mathcal{P}_\varepsilon(\bar{x}) - \mathcal{P}(\bar{x})\big\| &\le \left\| \int_0^{T_0-\varepsilon} \mathscr{S}(T_0,\theta)f(\theta,x(\theta))d\theta - \int_0^{T_0} \mathscr{S}(T_0,\theta)f(\theta,x(\theta))d\theta \right\| + \\
&+ \left\| \sum_{0\le\tau_k<T_0-\varepsilon} \mathscr{S}\left(T_0,\tau_k^+\right)c_k - \sum_{0\le\tau_k<T_0} \mathscr{S}\left(T_0,\tau_k^+\right)c_k \right\| \le \\
&\le \int_{T_0-\varepsilon}^{T_0} \|\mathscr{S}(T_0,\theta)\|\|f(\theta,x(\theta))\|d\theta + M_{T_0}\sum_{T_0-\varepsilon\le\tau_k<T_0}\|c_k\| \le \\
&\le 2M_{T_0}M_f(1+\rho)\varepsilon + M_{T_0}\sum_{T_0-\varepsilon\le\tau_k<T_0}\|c_k\|.
\end{aligned}$$

It shows that the set K can be approximated to an arbitrary degree of accuracy by a precompact set K_ε. Hence K itself is precompact set in X, that is, $\mathcal{P}$ takes a bounded set into a precompact set in X. As a result, $\mathcal{P}$ is a compact operator. □

After showing the compactness of the operator $\mathcal{P}$, we can follow and derive periodic PC-mild solutions for system (3.2). The following definitions are standard, we state them here for convenient reference. Note that uniform boundedness and uniform ultimate boundedness are not required to obtain the periodic solutions here, so we only define (local) boundedness and ultimate boundedness.

Definition 3.12. We say that PC-mild solutions of Cauchy problem (3.3) are bounded if for each $B_1 > 0$, there is a $B_2 > 0$ such that $\|\bar{x}\| \leq B_1$ implies $\|x(t, \bar{x})\| \leq B_2$ for $t \geq 0$.

Definition 3.13. We say that PC-mild solutions of Cauchy problem (3.3) are locally bounded if for each $B_1 > 0$ and $k_0 > 0$, there is a $B_2 > 0$ such that $\|\bar{x}\| \leq B_1$ implies $\|x(t, \bar{x})\| \leq B_2$ for $0 \leq t \leq k_0$.

Definition 3.14. We say that PC-mild solutions of Cauchy problem (3.3) are ultimate bounded if there is a bound $B > 0$, such for each $B_3 > 0$, there is a $k > 0$ such that $\|\bar{x}\| \leq B_3$ and $t \geq k$ imply $\|x(t, \bar{x})\| \leq B$.

We also list the following result as a reference.

Lemma 3.15 ([10, Theorem 3.1])**.** *{Local boundedness and ultimate boundedness} implies {boundedness and ultimate boundedness}.*

Lemma 3.16 (Horn's Fixed Point Theorem)**.** *Let $E_0 \subset E_1 \subset E_2$ be convex subsets of a Banach space X, with E_0 and E_2 compact subsets and E_1 open relative to E_2. Let $\mathcal{P}\colon E_2 \to X$ be a continuous map such that for some integer m, one has*

$$\mathcal{P}^j(E_1) \subset E_2,\ 1 \leq j \leq m-1,$$

$$\mathcal{P}^j(E_1) \subset E_0,\ m \leq j \leq 2m-1.$$

Then $\mathcal{P}$ has a fixed point in E_0.

With these preparations, we can prove our main result of this paper.

Theorem 3.17. *Let assumptions* (A1)–(A3), (B), (C) *and* (F1)–(F3) *be satisfied. If the PC-mild solutions of Cauchy problem* (3.3) *are ultimately bounded, then system* (3.2) *has a T_0-periodic PC-mild solution on* $[0, +\infty)$.

Proof. Let $x(\cdot, \bar{x})$ be a PC-mild solution of Cauchy problem (3.3) corresponding to the initial value $x(0) = \bar{x}$. From Lemma 3.11, we know that $\mathcal{P}(\bar{x}) = x(T_0, \bar{x})$ on X is compact. By Theorem 3.7 and Definition 3.13, $x(\cdot, \bar{x})$ is locally bounded, and from Lemma 3.15, $x(\cdot, \bar{x})$ is bounded. Next, let $B > 0$ be the bound in the definition of ultimate boundedness. Then by boundedness, there is a $B_1 > B$ such that $\|\bar{x}\| \leq B$ implies $\|x(t, \bar{x})\| \leq B_1$ for $t \geq 0$. Furthermore, there is a $B_2 > B_1$ such that $\|\bar{x}\| \leq B_1$ implies $\|x(t, \bar{x})\| \leq B_2$ for $t \geq 0$. Now, using ultimate boundedness, there is a positive integer m such that $\|\bar{x}\| \leq B_1$ implies $\|x(t, \bar{x})\| \leq B$ for $t \geq (m-2)T_0$.

Define $y(\cdot, y(0)) = x(\cdot + T_0, \bar{x})$. Then $y(0) = x(T_0, \bar{x}) = P(\bar{x})$. Using (3.5) in Lemma 3.9, we see that $y(t, y(0)) = x(t + T_0, \bar{x})$ for $t \geq 0$, is also a PC-mild solution of Cauchy problem (3.3) corresponding to the initial value $y(0) = x(0) = \bar{x}$. Further, $P(y(0)) = y(T_0, y(0)) = x(2T_0, \bar{x})$. Thus, the uniqueness implies that $P^2(\bar{x}) = P(P(\bar{x})) = P(y(0)) = x(2T_0, \bar{x})$. Suppose that there exists integer $m-1$ such that $P^{m-1}(\bar{x}) = x\big((m-1)T_0, \bar{x}\big)$. By introduction, we arrive at

$$P^m(\bar{x}) = P^{m-1}(P(\bar{x})) = P^{m-1}(y(0)) = y\big((m-1)T_0, y(0)\big) = x(mT_0, \bar{x}) \text{ for } \bar{x} \in X.$$

Thus, we have

$$\big\|\mathcal{P}^{j-1}(\bar{x})\big\| = \big\|x\big((j-1)T_0, \bar{x}\big)\big\| < B_2, \quad j = 1, 2, \ldots, m-1 \text{ and } \|\bar{x}\| < B_1; \tag{3.6}$$

$$\big\|\mathcal{P}^{j-1}(\bar{x})\big\| = \big\|x\big((j-1)T_0, \bar{x}\big)\big\| < B, \quad j \geq m \text{ and } \|\bar{x}\| < B_1. \tag{3.7}$$

Now let

$$\begin{aligned} H &= \{\bar{x} \in X \colon \|\bar{x}\| < B_2\},\ E_2 = \mathrm{cl.}\big(\mathrm{cov.}(\mathcal{P}(H))\big), \\ W &= \{\bar{x} \in X \colon \|\bar{x}\| < B_1\},\ E_1 = W \cap E_2, \\ G &= \{\bar{x} \in X \colon \|\bar{x}\| < B\},\ E_0 = \mathrm{cl.}\big(\mathrm{cov.}(\mathcal{P}(G))\big), \end{aligned}$$

where $\mathrm{cov.}(Y)$ is the convex hull of the set Y defined by $\mathrm{cov.}(Y) = \{\sum_{i=1}^n \lambda_i y_i \mid n \geq 1,\ y_i \in Y,\ \lambda_i \geq 0,\ \sum_{i=1}^n \lambda_i = 1\}$, and $\mathrm{cl.}(Y)$ denotes the closure of Y. Then we see that $E_0 \subset E_1 \subset E_2$ are convex subsets of X with E_0, E_2 compact subsets and E_1 open relative to E_2 and from (3.6) and (3.7), one has

$$\begin{aligned} &\mathcal{P}^j(E_1) \subset \mathcal{P}^j(W) = \mathcal{P}\mathcal{P}^{j-1}(W) \subset \mathcal{P}(H) \subset E_2,\ j = 1, 2, \ldots, m-1; \\ &\mathcal{P}^j(E_1) \subset \mathcal{P}^j(W) = \mathcal{P}\mathcal{P}^{j-1}(W) \subset \mathcal{P}(G) \subset E_0,\ j = m, m+1, \ldots, 2m-1. \end{aligned}$$

Finally, we verify the continuity of the map $\mathcal{P}$. Let $\bar{x}$, $\bar{y} \in E_2$, E_2 is a bounded subset of X. Suppose $x(\cdot, \bar{x})$ and $x(\cdot, \bar{y})$ are the PC-mild solutions of Cauchy problem (3.3) corresponding to the initial value $\bar{x}$ and $\bar{y} \in E_2$, respectively and are given by

$$x(t, \bar{x}) = \mathscr{S}(t, 0)\bar{x} + \int_0^t \mathscr{S}(t, \theta) f\big(\theta, x(\theta, \bar{x})\big) d\theta + \sum_{0 \leq \tau_k < t} \mathscr{S}\left(T_0, \tau_k^+\right) c_k;$$

$$x(t, \bar{y}) = \mathscr{S}(t, 0)\bar{y} + \int_0^t \mathscr{S}(t, \theta) f\big(\theta, x(\theta, \bar{y})\big) d\theta + \sum_{0 \leq \tau_k < t} \mathscr{S}\left(T_0, \tau_k^+\right) c_k.$$

Thus, by assumption [F2], we obtain

$$\|x(t, \bar{x})\| \leq M_{T_0} \|\bar{x}\|_{E_2} + M_{T_0} M_f T_0 + M_{T_0} \sum_{0 \leq \tau_k < T_0} \|c_k\| + M_{T_0} \int_0^t \|x(\theta, \bar{x})\| d\theta;$$

$$\|x(t, \bar{y})\| \leq M_{T_0} \|\bar{y}\|_{E_2} + M_{T_0} M_f T_0 + M_{T_0} \sum_{0 \leq \tau_k < T_0} \|c_k\| + M_{T_0} \int_0^t \|x(\theta, \bar{y})\| d\theta.$$

By Lemma 3.1, one can verify that there exist constants M_1^* and $M_2^* > 0$ such that

$$\|x(t,\bar{x})\| \leq M_1^* \text{ and } \|x(t,\bar{y})\| \leq M_2^*.$$

Let $\rho = \max\{M_1^*, M_2^*\} > 0$, then $\|x(\cdot,\bar{x})\|$, $\|x(\cdot,\bar{y})\| \leq \rho$. By assumptions [F1] and (1) of Lemma 2.9, we obtain

$$\begin{aligned}\|x(t,\bar{x}) - x(t,\bar{y})\| &\leq \|\mathscr{S}(t,0)\|\|\bar{x}-\bar{y}\|_{E_2} + \\ &\quad + \int_0^t \|\mathscr{S}(t,\theta)\| f(\theta, x(\theta,\bar{x})) - f(\theta, x(\theta,\bar{y}))\| d\theta \leq \\ &\leq M_{T_0}\|\bar{x}-\bar{y}\|_{E_2} + M_{T_0} L_f(\rho)\int_0^t \|x(\theta,\bar{x}) - x(\theta,\bar{y})\| d\theta.\end{aligned}$$

By Lemma 3.1 again, one can verify that there exists a constant $M_0 > 0$ such that

$$\|x(t,\bar{x}) - x(t,\bar{y})\| \leq M_0 M_{T_0}\|\bar{x}-\bar{y}\|_{E_2} \equiv \widetilde{L}\|\bar{x}-\bar{y}\|_{E_2}, \text{ for all } t \in [0, T_0],$$

which implies that

$$\|\mathcal{P}(\bar{x}) - \mathcal{P}(\bar{y})\| = \big\|x(T_0,\bar{x}) - x(T_0,\bar{y})\big\| \leq \widetilde{L}\|\bar{x}-\bar{y}\|_{E_2}.$$

Hence, $\mathcal{P}$: $E_2 \to X$ is a continuous map. Consequently, from Horn's fixed point theorem, we know that the operator $\mathcal{P}$ has a fixed point $x_0 \in E_0 \subset X$. By Lemma 3.9, we know that the PC-mild solution $x(\cdot, x_0)$ of Cauchy problem (3.3) corresponding to the initial value $x(0) = x_0$, is just T_0-periodic. Therefore $x(\cdot, x_0)$ is a T_0-periodic PC-mild solution of system (3.2). This completes the proof. □

Remark 3.18. For the impulsive differential equations on a general Banach space, if the following conditions are satisfied:

1. The PC-mild solutions of the impulsive differential equations are locally bounded and ultimate bounded;
2. The main equation is T_0-periodic at time $t \neq \tau_k$;
3. Impulsive perturbations are also T_0-periodic at time $t = \tau_k$;
4. The map $\mathcal{P}$ that maps an initial value along the PC-mild solution by T_0 units is compact.

Then the impulsive differential equations has at least one T_0-periodic PC-mild solution.

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YanLong Yang
yylong1980@163.com

Guizhou University
College of Technology
Guiyang, Guizhou 550004, P.R. China

JinRong Wang
wjr9668@126.com

Guizhou University
College of Science
Guiyang, Guizhou 550025, P.R. China

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