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TRANSPORT PLANNING IN CONDITIONS OF DIFFERENT TRANSPORT TARIFFS – APPLICATION OF INTEGER PROGRAMMING

Abstract: This paper presents the application of integer programming to planning transport services carried out in parallel by two different shipping companies using different transport tariffs. Formulation of the problem is based on a set partition model which is used to set transport routes. The paper concludes with the presentation of distribution cost analysis based on the suggested model, as applied in one of local companies of the Wrocław area.

Keywords: transport tariff, set partitioning model, transport planning.

1. Introduction

The rampant growth of transport companies operating on the basis of different tariff types is the reason why decisions related to transport are becoming more and more complicated. On the one hand, the offers extended by large international transport companies guarantee low prices. On the other hand, small transport companies of the countrywide range provide their customers (being often the only ones) with high quality, flexible delivery services.

A list of transport service prices together with rules of their use forms a transport tariff. Considering the criterion for calculating delivery costs, two basic tariff types can be distinguished:

- tariffs based on transport distance and size of consignment,
- tariffs based on the distance covered in order to deliver the consignment.

In a tariff of the first type, the actual distance covered by the consignment is not taken into account, but averaged transport rates dependent on consignment weight and on the distance between the dispatch and reception points are determined in order to standardise and facilitate clearings. Here, consignments on high-volume routes are the most profitable for a transport company. However, by choosing this

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tariff type the transport company undertakes to deliver all consignments. Hence, the determination of appropriate rates is the principal problem when tariffs of this type are used and, initially, high rates are common. Later, depending on various factors, these rates are often reduced considerably for regular customers.

In a tariff of the second type, the cost of transport directly depends upon the actual length of the route covered by a means of transport in order to deliver one or more con-signments. Such tariffs are usually applied by the smallest players in the transport services market. The smaller the dispersal of recipients and the greater the level of vehicle load, the more profitable those tariffs are.

In the vast majority of models designed to help transport decision-makers, the cost of delivery is considered to be identical with the length of the route covered. The widely used tri-index model may be given here as an example (Golden, Assad 1988). Apart from difficulties resulting from the large size of actual tasks, this model can be used exclusively in a situation when the decision-maker only plans the transport routes, not having to choose one of several providers of transportation services. The problem becomes even more complex when the forwarding agents covered by the analysis account on the basis of various different types of transport tariffs.

This change in the nature of problems related to transport decisions forces a change in forms of models used in optimisation of the problems discussed. The formulation which seems most promising in these new conditions, is the recipient set partition model.

2. The set partitioning model

The first formulation of a set partitioning is Balinski and Quant's proposal (Balinski, Quant 1964). This model was proposed in 1964.

Let $\pi = \{Z_1, \dots, Z_m\}$ be a set of all the possible routes fulfilling the conditions of the problem, let w_j^* be the weight of the route Z_j. Let a_{ij} be the coefficient determining the assignment of a recipient i to a route j. It has value of 1, if the recipient i is serviced along the route j or 0 in otherwise. The decision variable x_j takes a value of 1 if and only if the route j belongs to the solution. Balinski and Quandt's proposal in the form of an integer programming model is presented in formulas (1) – (4).

$$\min \sum_{j=1}^{m} w_j^* x_j \tag{1}$$

under the conditions:

$$\sum_{j=1}^{m} a_{ij} x_j = 1 \qquad \text{for each } i \in N \tag{2}$$

$$\sum_{j=1}^{m} x_j = k \tag{3}$$

$$x_j \in \{0,1\} \qquad \text{for each } j = 1,\dots,m \tag{4}$$

Constraint (2) provides fort each recipient being taken into account only once in the solution. Constraint (3) ensures that k routes will be used in the solution.

In the case of the presented formulation the number of variables rises exponentially with the increase of n, which, with large values of n, considerably complicates the problem. Here, the most commonly used approach is the column generation algorithm. The proposals of algorithms serving to solve this problem can be found, inter alia, in the work of Agarwal *et al.* (Agarwal 1989).

The presented approach can be considered as a route selection model. The permissible load, the length of route, as well as the recipients serviced along each route constitute the model parameters. What is crucial from the point of model elasticity is the fact that no conditions regarding routes (route coherence, cyclicity) are made here. This approach also enables many additional constrains to be taken into account already at the stage of model construction (the process of route generation), while providing for the rejection of unfeasible routes.

3. Different kinds of tariffs in the set partitioning model

In order to take into account consignments delivered according to the tariff described earlier as dependent on the transport distance and the size of consignment, a new decisione variable x_j needs to be added to the model. The value of goal function coefficient for the x_j variable will be equal to the delivery fee. As to the coefficient a_{ij}, it will take the value of 1 exclusively in the constraint pertaining to the recipient to whom the consignment should be delivered. In all the other constraints the coefficient in the j-th column will be equal to 0.

Consignments where the delivery cost is determined on the basis of route length (i.e. according to the second tariff type) can be taken into account in the model in the same way, the only difference being that the a_{ij} coefficient may take the value of 1 in more than one constraint. This is due to the fact that more than one consignment may be delivered along one route.

Constraint (3) in the model should take into account exclusively the routes carried out accordingly with the second tariff type (i.e. routes paid for on a basis of the length of transit). This is due to the fact that providers accounting in this way have limited numbers of vehicles at hand and cannot carry out an arbitrary number of routes. If we take into account several transport services providers accounted according to the second tariff type, then the constraint described herein should be introduced for every recipient from this group. The right side of the constraint will be equal to the number of routes which can be carried out by a given recipient.

The presented approach is very flexible. It allows us not only to take into account two transport companies accounted according to different tariff types, but also to choose from among the providers carrying out transport according to different strategies (large or small number of recipients along a route, the use of large or small vehicles etc.).

4. Application example

The presented approach has been applied to an analysis of the transport system (and its potential for development) of one of the Wrocław production companies operating in the construction industry. A sudden increase in the demand for its products in the entire territory of Poland has been the reason for carrying out the analyses presented below.

To the date, the company has been using the services of a large transport company, which performs transport of its products in the territory of Poland. The cost of the services delivered by the transport company is described by the company's board of directors as highly satisfactory. However, due to the consistently growing frequency of orders described as 'priorities', the board of directors considers the possibility of cooperation with a transport company which would mainly carry out priorities, in accordance with the company's arrangements. The fundamental reason for beginning such cooperation is the desire to increase the customer service quality through the introduction of the possibility of carrying out the delivery at a specific time of day as well as by guaranteeing the execution of delivery.

In the described situation, the only way of settling the accounts with the company that is acceptable for the transport company is the application of a fee for every covered kilometre (the second type of the classification presented above). Unfortunately, several different ways of carrying out the deliveries and determining their costs make the decision of beginning the cooperation and its operational aspects more difficult.

In the further proceedings, three areas were determined which should be covered by the analysis enhancing decision making in the situation described. As the first area, we defined the necessity of determining the extent of costs to be incurred by the company if one consignment is delivered as a priority each day. Afterwards, one has to analyse the relevance of extending priority delivery routes to reach additional recipients in order to reduce the overall distribution costs. The third and last part is an attempt at estimating the amount of the transport rate for the new transport company (this rate would have to be competitive with respect to the rate applicable at the current transport company).

Simulation was chosen as the analysis tool. Simulational models are generally more flexible and are not limited by rigid mathematical rules, yet in this case the choice of simulation was mainly determined by the fact that its results should admit detailed analysis in a system corresponding to actual data.

The simulation was performed on the basis of the information disclosed by the company and concerning the fees applied by the two transport companies and the dimensions and places of deliveries over nine business days. The procedure consisted in determination of the optimal distribution plans for each day of the period under analysis, in four variants. Geographical coordinates were used to determine distances.

Variant I is a solution in the situation when one of the packages (priority package) is delivered by the new transport company. In accordance with the decision-maker's suggestion, the consignment of the greatest weight was considered the priority package in the simulation. The other packages are delivered under the standard conditions.

Variant II (a comparative one) corresponds to the currently used way of distribution. No optimisation procedures were required for planning of deliveries for both of the variants described.

Variant III is a case in which a priority consignment route is extended to reach additional recipients of ordinary packages in order to reduce the overall delivery costs. Moreover, this procedure takes into account the maximum capacity, the maximum route length and the maximum number of deliveries along one route initially agreed upon with the new transport company. In the analysed situation, those values are equal to $2,000$ kg, 800 km and 5 deliveries, respectively. The approach presented in the first part of this paper has been used to determine the optimal delivery plan in Variant III.

Variant IV corresponds to the situation when the distribution is carried out exclusively by the new transport company. It was assumed that the new transport company has at least 4 vehicles at hand. In this variant, the determination of the optimal route is carried out using the precise method of solving the route determination problem presented in the reference paper (Hanczar 2005).

The fundamental information obtained during the simulation is the amounts of costs for each variant on each day of the period under analysis. These results are presented in Table 1.

Table 1. *Costs of service of respective variants divided into days*

Date	**Costs [PLN]**			
	I	**II**	**III**	**IV**
Apr 18 2007	1,275.07	689.09	1,219.16	2,330.12
Apr 19 2007	1,340.58	982.32	1,245.81	2,308.70
Apr 20 2007	1,600.50	1,375.93	1,600.50	3,055.32
Apr 23 2007	1,261.10	1,058.42	1,261.10	3,200.74
Apr 24 2007	1,619.19	876.50	1,562.23	2,861.96
Apr 25 2007	1,496.53	1,197.32	1,486.11	3,907.67
Apr 26 2007	935.05	929.42	935.05	3,028.06
Apr 27 2007	1,569.31	815.84	1,446.94	2,785.78
Apr 30 2007	878.22	674.74	859.33	2,352.92
Total	**11,975.55**	**8,599.58**	**11,616.23**	**25,831.27**

Source: *The author's own study based on simulation results*

The first column of Table 1 indicates the day and four others columns show the costs of delivery execution depending on the variant used. From the presented results it is clear that the extension of distribution by priority consignments service would cause a significant increase in delivery costs. It is also clear that savings resulting from possible combined delivery of ordinary consignments with a priority consignment are insignificant. For a clearer image of the decision situation, the results obtained have been expressed as a percent increase in distribution costs when compared with the current situation, i.e. Variant II. This data is presented in Table 2. The last line of this table contains synthetic information on the cost increase. Hence, in the situation of priority packages servicing with individual deliveries the costs will increase by 43% on the average. The use of variant III, consisting in priority packages servicing with more than one recipient along a route will bring insignificant savings when compared with Variant II. Distribution relying exclusively on the services of the new transport company will cost 200% more. This means that this transport company would have to offer prices 2/3 times lower than the current prices in order to be competitive.

Table 2. *Percent increase in distribution cost in respect of Variant II*

Date	**Weight [kg]**	**Cost increase in respect of Variant II [%]**		
		I	**III**	**IV**
Apr 18 2007	561	85	77	238
Apr 19 2007	2,155	36	27	135
Apr 20 2007	2,368	16	16	122
Apr 23 2007	1,075	19	19	202
Apr 24 2007	1,041	85	78	227
Apr 25 2007	941	25	24	226
Apr 26 2007	1,691	1	1	226
Apr 27 2007	1,144	92	77	241
Apr 30 2007	986	30	27	249
Average	1,329	43	39	207

Source: *The author's own study based on simulation results*

Additionally, column 2 of Table 2 shows the overall weight of consignments. This indicates a relation between the increase in delivery costs and the overall size of the consignments delivered on a given day. It is quite clear that in most cases the greater the weight of consignments is, the smaller the increase in delivery costs in comparison with Variant II. When one more closely examines the cases where this dependence does not occur, (e.g. deliveries on April 25th 2007), one can notice that the second factor greatly impacting the cost differences in the analysed situation is the geographical dispersion of the recipients. According to the delivery plan from the day used as an example (April 25th 2007), the new transport company would deliver two large consignments to Kraków besides a priority consignment (to Jasienica in the Kraków Province), which accounts reason for a smaller cost increase.

5. Conclusion

The research conducted has confirmed the usability of simulational methods in planning and analysing a strategy of delivery execution. In the case presented, the simulation has supported an analysis of different ways of product delivery execution. Optimisation methods were used to determine the solutions of partial problems.

The analysis of the obtained results, despite the fact that it covers as few as nine days, will be helpful in making a decision on beginning the cooperation with a new transport company. It also provides the company's board of directors with guidelines allowing for more conscious decision-making in the area of delivery planning.

The presented study is a starting point of simulational research into the application of new logistics ideas. The results obtained are encouragement for continued research, more specifically, for the extension of simulation scope and taking into account a greater number of transport services providers.

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