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THE USAGE OF MATHEMATICAL MLT MODEL FOR THE CALCULATION OF THERMAL FILTRATION

During the research we used a well-known mathematical MLT model – Muskat–Leverett thermal for the numerical calculation of the thermal filtration of a biphasic liquid in a porous space.

The mathematical theory of thermal influence on filtration of the biphasic liquid for MLT model was offered by V.N. Monakhov and O.B. Bocharov (see [1, 2]). The influence of temperature on the character of liquid motion is possible through the change of viscosity and capillary properties of different liquid components which depends on their temperature and temperature of skeleton in a porous space. The characteristic feature of the thermal model of biphasic filtration – is all equalizations which are necessary for making biphasic filtration, except the laws of Darcy and Laplas, are the results of saving laws of the continuous environment mechanics. In addition, a MLT-model is technological in sense that for its description only the functions-parameters are used, which are defined experimentally.

The basic characteristics of moving liquids are:

s_i – phase saturations (i.e. concentration $s_1 + s_2 = 1$, 1 – water, 2 – oil),
ρ_i – density,
p_i – pressure,
μ_i – viscosity dependent on temperature,
v_i – Darcy velocities of phase (charges),
$\vec{v} = \vec{v}_1 + \vec{v}_2$ – Darcy velocities of a filtration of a mix.

The non-uniform anisotropic porous environment is characterized by porosity $m(x)$, $K_0(x)$ absolute permeability and relative phase permeability's $\bar{k}_i(s_i)$, $\alpha_i = m \cdot s_i$ ($i = 1, 2$), $\alpha_3 = 1 - m$.

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MLT model equations looks like

$$
\left.
\begin{aligned}
& m\frac{\partial}{\partial t}(\alpha_i \rho_i) + div(\rho_i \bar{v}_i) = 0 \\
& -\bar{v}_i = K_0 \frac{\bar{k}_i(s)}{\mu_i(\theta)}(\nabla p_i + \rho_\iota \vec{g}) \\
& p_c(x, \theta, s) = p_2 - p_1 \\
& p_c(x, \theta, s) = \gamma(\theta)\cos\alpha(\theta)\sqrt{\frac{m_0(x)}{K_0(x)}}J(s) \\
& \frac{\partial\theta}{\partial t} = div(\lambda\nabla\theta - \vec{v}\theta), \ s_2 = 1 - s_1
\end{aligned}
\right\} \qquad (1)
$$

where:

$p_c(x, \theta, s)$ – capillary pressure,

$\lambda = \lambda(x, s, \theta)$ – generalized factor of heat conductivity (functions $p_c(x, \theta, s)$ and λ are given, θ – equilibrium temperature).

The resolvability of MLT models with corresponding boundary conditions for a non--stationary case has been considered in works [2], and for a stationary case – in work [3]. By analogy with [1] we shall enter p – average pressure of a mix

$$
p = p_2 + \int_s^1 \frac{k_1}{k_1 + k_2}(\xi, \theta)\frac{\partial}{\partial\xi} p_c(x, \xi, \theta) d\xi \qquad (2)
$$

After corresponding transformations of MLT equation it will be transformed to the following system

$$
\left.
\begin{aligned}
& m\frac{\partial s}{\partial t} div[K(a_1\nabla s - a_2\nabla\theta + \vec{f}_1) - b_1\vec{v} \\
& divK(\nabla p + \vec{f}_2 + a_3\nabla\theta) = 0 \\
& \frac{\partial\theta}{\partial t} = div[\lambda(x, \theta, s)\nabla\theta - \vec{v}\theta) \\
& p_c(x, \theta, s) = p_2 - p_1 = \gamma(\theta)\cos\alpha(\theta)\sqrt{\frac{m_0(x)}{k_0(x)}}J(s)
\end{aligned}
\right\} \qquad (3)
$$

where:

$s = \dfrac{s_1 - s_1^0}{1 - s_1^0 - s_2^0}$ – resulted saturation of a moistening phase, $0 \le s \le 1$, $\theta_* \le \theta \le \theta^*$,

$s_i^0 = \text{const}$ – residual saturations,

$m = m_0(1 - s_1^0 + s_2^0)$ – effective porosity.

$$b_i = k_i (k_1 + k_2)^{-1} \ (i = 1, 2) \ \ K = K_0 (k_1 + k_2); \ \ a_0 = \frac{k_1 k_2}{(k_1 + k_2)^2};$$

$$a_1 = |p_{cs}| a_0; \ \ a_2 = p_{c\theta} a_0; \ \ \vec{f}_1 = [-\nabla_x p_c + (\rho_1 - \rho_2)\vec{g}] a_0;$$

$$a_3 = -b_1 p_{c\theta} - \int_s^1 \frac{\partial}{\partial\theta}\left(b_1 \frac{\partial p_c}{\partial \xi} \right); \ \ f_2 = -b_1 \nabla_x p_c - \int_s^1 \nabla_x \left(\frac{\partial p_c}{\partial \xi} \right) b_1 d\xi + b_1 (\rho_1 - \rho_2)\vec{g};$$

$$k_i (s, \theta) \equiv \bar{k}_i (s) \mu_i^{-1} (\theta); \ \vec{v} = \vec{v}_1 + \vec{v}_2; \ \lambda = \sum_1^3 \alpha_i \lambda_i (\rho_c c_{p_i})^{-1}; \ \alpha_i = m_0 s_i; \ i = 1, 2; \ \alpha_3 = 1 - m_0 .$$

Factors of system (3) are expressed obviously through functional parameters in the initial MLT model and in view of physical possess have following properties:

- $a_0(s_0 > 0, s \in (0, 1); \lambda(s, t) \geq m;$
- $(a_i(s, \theta), \bar{\lambda}(s, \theta)) \in C^1(\overline{Q}), Q = \{s, \theta \mid 0 < s < 1, \theta_* < \theta < \theta^*\}.$

Further the opportunity of the numerical decision stationary MLT models for one-dimensional isotropic porous environment is considered and the algorithm of the decision is offered.

$$\begin{cases} [K(a_1 s_x - a_2 \theta_x) - b_1 \vec{v}]_x = 0 \\ [K(p_x - a_3 \theta_x)]_x = 0 \\ [\lambda \theta_x - v\theta]_x = 0 \\ -v = K(p_x + a_3 \theta_x) \end{cases} \tag{4}$$

With boundary conditions:

$$\begin{aligned} & s|_{x=0} = s^0; \ s|_{x=L} = s^L \\ & \theta|_{x=0} = \theta^0; \ \lambda\theta_x|_{x=L} = \beta(\theta_{env} - \theta) \\ & p|_{x=0} = p^0; \ p|_{x=L} = p^L \end{aligned} \tag{5}$$

where β – the factor of heat exchange.

$$\left. \begin{aligned} & (K a_1 s_x)_x - (K a_2 \theta_x)_x - (b_1 v)_x = 0 \\ & (K p_x)_x - (K a_3 \theta_x)_x = 0 \\ & (\lambda \theta_x)_x - (v\theta)_x = 0 \\ & v = -K(p_x + a_3 \theta_x) \end{aligned} \right\} \tag{6}$$

Let's enter a uniform grid $x_i = x_0 + ih$; $i = \overline{0,N}$, where h = const – a length of the grid block. Then we shall write down difference type of the equations (6) for a constant step of the grid h

$$\frac{1}{h}\left[(Ka_1)_{i+1/2}\frac{s_{i+1}-s_i}{h}-(Ka_1)_{i-1/2}\frac{s_i-s_{i-1}}{h}\right]-$$
$$-\frac{1}{h}\left[(Ka_2)_{i+1/2}\frac{\theta_{i+1}-\theta_i}{h}-(Ka_2)_{i-1/2}\frac{\theta_i-\theta_{i-1}}{h}\right]-\frac{(b_1 v)_{i+1/2}-(b_1 v)_{i-1/2}}{2h}=0$$

$$\left[\lambda_{i+1/2}\frac{\theta_{i+1}-\theta_i}{h}-\lambda_{i-1/2}\frac{\theta_i-\theta_{i-1}}{h}\right]\frac{1}{h}-\frac{1}{2h}\left[v_{i+1/2}\frac{\theta_{i+1}+\theta_i}{2}-v_{i-1/2}\frac{\theta_i+\theta_{i-1}}{2}\right]=0 \qquad (7)$$

$$\frac{1}{h}\left[K_{i+1/2}\frac{p_{i+1}-p_i}{h}-K_{i-1/2}\frac{p_i-p_{i-1}}{h}\right]+\frac{1}{h}\left[(Ka_3)_{i+1/2}\frac{\theta_{i+1}-\theta_i}{h}-(Ka_3)_{i-1/2}\frac{\theta_i-\theta_{i-1}}{2}\right]=0$$

$$v_{i+1/2}=-K_{i+1/2}\left[\frac{p_{i+1}-p_i}{h}+a_{3,i+1/2}\frac{\theta_{i+1}-\theta_i}{h}\right]$$

The equations from the first to the third of the system (7) have accuracy of approximation of the order $O(h^2)$, and the equation for speed of a filtration of a mix has approximation of the order $O(h)$. System of the equations (7) is solved by an implicit iterative method

$$\left.\begin{aligned}
p_i^{n+1} &= p_i^n + \tau_3 L_h^p\,(s_i^n, \theta_i^n, p_i^{n+1}, v_i^n)\\
v_{i+1/2}^{n+1} &= L_h^v\,(s_i^n, \theta_i^n, p_i^{n+1}, v_i^n)\\
s_i^{n+1} &= s_i^n + \tau_1 L_h^s\,(s_i^{n+1}, \theta_i^n, v_i^{n+1})\\
\theta_i^{n+1} &= \theta_i^n + \tau_2 L_h^\theta(s_i^{n+1}, \theta_i^{n+1}, p_i^{n+1}, v_i^{n+1})
\end{aligned}\right\} \qquad (8)$$

where factors at difference of analogues of derivatives calculated on the n layer. Thus, the numerical decision of the initial system of the nonlinear equations (6) turns out with consecutive solves of the equations of the system (8). Factors τ_1, τ_2, τ_3 in system of the equations (8) are iterative parameters for the equations of a saturation, temperature and pressure accordingly.

Boundary conditions are approximated as follows:

$$s_0^n=\overline{s}^0;\ s_N^n=\underline{s}^L;\ \theta_0^n=\overline{\theta}^0;\ \lambda\frac{\theta_N^0-\overline{\theta}_{n-1}^n}{h}=\beta(\theta_{env}-\theta_N^n)\Rightarrow$$
$$\Rightarrow\ \theta_N^n=\frac{\frac{\lambda}{h}}{\frac{\lambda}{h}+\beta}\theta_{N-1}+\frac{\beta}{\frac{\lambda}{h}+\beta}\theta_{env} \qquad (9)$$
$$p_0^n=\overline{p}^0;\ p_N^n=\underline{p}^L;\ v_0^n=\overline{v}^0;\ v_N^n=\underline{v}^L$$

The received system (8) is solved a method of prorace for each equation separately. Further collecting "proracing" factors we are convinced, that they satisfy to a condition of diagonal prevalence, providing stability of a method of prorace.

Influence of a temperature field on character of process of a filtration it was investigated through decrease in viscosity with growth of temperature that has shown increase only the final petrofeedback of the layer. Thus it was supposed, that the temperature of a firm skeleton and liquid phases is leveled instantly. Numerical calculation has shown advantage of use of a non-uniform grid for the decision of system (4) which is caused by fast convergence of decision at insignificant number of iteration and the most adequate conformity of the received results with physics of considered processes.

Further in the given work the research problem of factors of system of the equations (3) is considered at the given statement of problems (4)–(5) for testing the offered algorithm (8)–(9). By means of computing experiment the analysis of influence of factors $\bar{a}(s,\theta) \equiv (a_0, a_1, a_2, a_3)$ and $\lambda \equiv \lambda\,(s,\theta)$ systems of the equations (3) on character of course of researched processes is lead. Initial distributions of saturation, pressure and temperature will be necessary for the organization of iterative process of the account. Hence as initial distribution we shall put for saturation $s_i^0 = 0.37$, temperature $\theta_i^0 = 75$°C, pressure $p_i^0 = 10.5$ MPa. Results of calculation in a case when $\mu_1 = 0.6$ mPa·s, $\mu_2 = 3.5$ mPa·s, $m_0 = 0.225$, $K = 140$ mD, $\bar{s}^0 = 0.8$, $\underline{s}^L = 0.2$, $T_{env} = 70$°C, $p_c(x,\theta,s) = \left(\dfrac{0.36}{3\cdot s+1} - 0.1\cdot(3s+1) + 0.391\right)\cdot\gamma(\theta)\cos\alpha(\theta)\sqrt{\dfrac{m_0}{|K_0|}}$ are resulted in Figure 1.

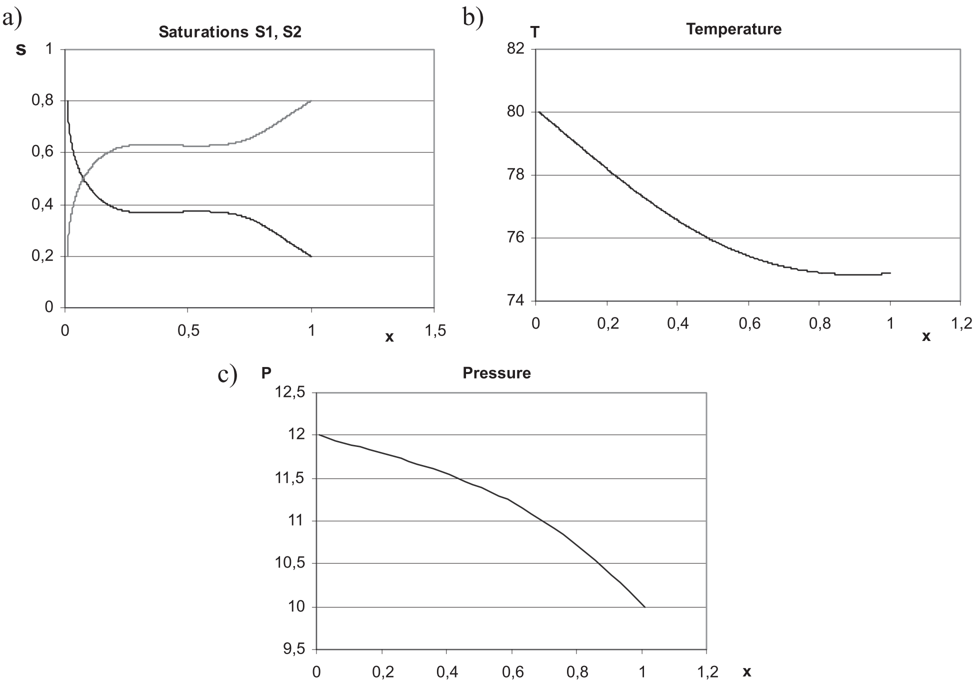

Fig. 1. Results of calculation: a) saturations S_1, S_2; b) temperature; c) pressure

The computational analyses of the influence of thermal processes of joint motion of many-fazed liquids allows to describe the process of filtration of two and more liquids with different physical properties in a porous environment adequately, which represents a large interest in planning and development of gas- and oil fields.

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